UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL/MAY EXAMINATIONS 1999

PHY140Y

Duration - 3 hours

The only allowed aids are hand-held calculators and an attached appendix of formulae. Answer BOTH sections. Show all derivations, and justify each significant step in your answer.

Section A (20%)

Each question is of worth 2%. Give BRIEF answers to all questions.

1. Define the concept of work. A team of movers wishes to load a truck using a frictionless ramp from the ground to the rear of the truck. One of the movers claims that less work would be required to load the truck if the length of the ramp were increased, reducing the angle of the ramp with respect to the horizontal. Is this claim valid? Explain.
2. Define simple harmonic motion. Is uniform circular motion an example of simple harmonic motion? Explain why or why not.
3. Describe driven harmonic motion. What is resonance and how does it arise?
4. State the Principle of Conservation of Mass-Energy. Describe how it can be derived from a thought experiment.
5. Is an astronaut on the Space Shuttle weightless? Explain why or why not.
6. Given two massive objects, describe the possible types of motion of the two-body system based on the initial conditions (relative velocity and position of the two masses) of the system.
7. Define the quantum mechanical concept of complementary variables. Give at least one example of this. You may want to bring in the Heisenberg Uncertainty Principle.
8. In what ways do the alkali metals and inert gases differ chemically? Why do they have such different properties?
9. The fission of a ^{235}U nucleus results in several radioactive nuclei, many neutrons and perhaps some other radioactivity. Net energy results from this fission process. Where does this energy come from and in what form is it?
10. How many fundamental forces are there? Briefly describe their differences.

Section B (80%)

Answer any 6 of the 8 questions. Each question is of equal value.

1. Choose one of the two following topics and write a short ($\sim$ 500 word) essay on it. Structure your answer! A "stream-of-consciousness" response will not receive full marks. A brief outline should be part of your answer.

   (a) State the Special Theory of Relativity (STR). Why was its "discovery" so important? What does the STR imply for the speed of light? Describe how time dilation and length contraction result from the STR. Explain the twin paradox.

   (b) The use of nuclear fission as a source of energy is controversial today. Proponents of this technology argue that it is one of the cleanest types of fuel with the least environmental impact. Opponents point to the alarming events at Three Mile Island and Chernobyl. Outline why fission can be viewed as being so beneficial by some and so dangerous by others. Can these points of view be reconciled?

2. A new deep-space probe that is being launched by the Canadian Space Agency from Earth is intended to explore the region between Earth and Mars. The mass of the probe is 50.0 kg.

   (a) The probe is placed initially in a circular orbit exactly halfway between the orbit of Earth (radius of orbit $R_E = 1.50 \times 10^8$ km) and the orbit of Mars (radius of orbit $R_M = 2.28 \times 10^8$ km). What is the probe's speed?

   (b) What is the minimum energy that must be expended (by rockets primarily) to get the probe into this orbit from Earth (the masses of the sun and Earth are $M_s = 1.99 \times 10^{30}$ kg and $M_E = 5.98 \times 10^{24}$ kg, respectively)? Don't forget you need to escape from Earth's gravitational field.

   (c) The mission design calls for the probe to rendevous with a large asteroid, with a mass $M_A = 2.00 \times 10^{20}$ kg and a radius $R_A = 2.00 \times 10^5$ km. What is the acceleration due to gravity on the surface of the asteroid?

   (d) The probe is to place itself in a circular orbit around the asteroid at a height of 200. m off its surface. What is the orbital period?

3. An electron with mass $m_e = 9.11 \times 10^{-31}$ kg is moving in a one-dimensional square well potential formed by a thin metal plate sandwiched between a slab of insulator and a semiconductor. The square well potential is described as

$$U(x) = \begin{cases} \infty \text{ for } x < 0, \\ 0 \text{ for } 0 < x < L, \quad \text{and} \\ U_\circ \text{ for } x \geq L, \end{cases} \tag{1}$$

where $L = 1.00 \times 10^{-10}$ m.

(a) Write down the Schrödinger equation that the wave function for an electron with total energy E must satisfy both inside and outside the well.

(b) What boundary conditions does the wave function $\psi(x)$ satisfy when $E < U_\circ$?

(c) What is the form for $\psi(x)$ for the ground state of this system? You don't have to solve for the parameters that define the spatial dependence – just solve for the equations that these parameters must satisfy.

(d) Sketch the behaviour of the wave function $\psi(x)$ over the interval $x \in [0, 2L]$ if we assume that $E \ll U_\circ$. In the limit where $U_\circ \to \infty$, what is the ground state energy of the electron.

4. We have a gas of N spin-1/2 fermions that are confined in a box, where each particle can be in energy levels denoted by the quantum number n, $n = 1, 2, 3, ...$ The energy of each level, E_n, and its degeneracy, m_n, are given by

$$E_n = n\epsilon \tag{2}$$

$$m_n = n, \tag{3}$$

where $\epsilon = 1.5$ eV.

(a) What is the ground state of a system with $N = 5$? What is its energy?

(b) What are the possible quantum states when the gas consists of only two fermions with total energy 4ϵ?

(c) The energy required to add an additional particle to this gas is given by the lowest energy unfilled quantum state in this system. Plot this energy as a function of the number of particles from $N = 1$ to $N = 12$.

(d) If instead of fermions, we had a gas of N spin-1 bosons, what would be the ground state energy of this new system?

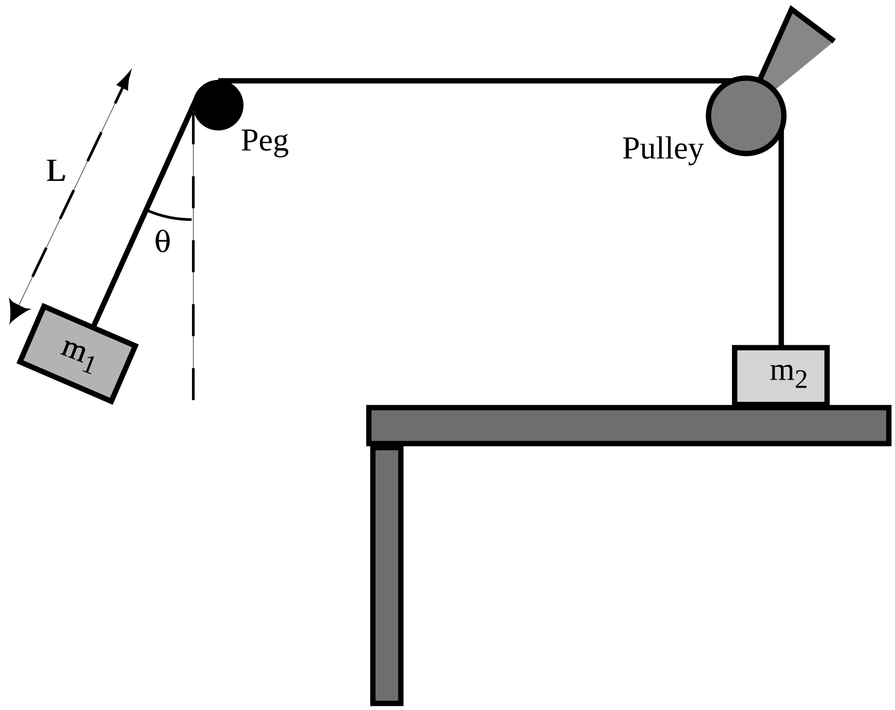

Figure 1: The two masses connected by a cord and pulley arrangement.

5. Two blocks are connected by a massless cord that passes over a frictionless pulley and a frictionless peg as shown in Fig. 1. One end of the cord is attached to mass $m_1 = 3.00$ kg that is a distance $L = 1.20$ m from the peg. The other end of the cord is connected to a block of mass $m_2 = 6.00$ kg resting on a table. Mass m_1 is released from rest with the cord at an angle θ from the vertical (as shown in the figure).

   (a) Derive an expression for the speed, v, of mass m_1 at the bottom of its circular path in terms of L and θ.

   (b) Derive an expression for the tension force, T, in the cord attached to mass m_1 when m_1 is at the bottom of its circular path.

   (c) From what angle θ (measured from the vertical) must mass m_1 be released in order to just begin to lift mass m_2 off the table?

   (d) What are v and T for this value of θ?

6. The radius of a galaxy is L.

   (a) How fast would a spaceship have to travel to cross the entire galaxy in a time T, as measured from within the spaceship? Express the speed in terms of the speed of light c. Assume that we can neglect the effects of acceleration and deceleration.

   (b) How much time would elapse on Earth during the traversal?

   (c) Derive general expressions for the time intervals in (a) and (b), and then evaluate these expressions for $L = 3.00 \times 10^{20}$ m and $T = 300$ years.

7. A mass of 220 g is connected to a light spring having force constant of $K = 5.4$ N/m. It is free to oscillate in one dimension on a horizontal frictionless surface.

   (a) If the mass is displaced 7.0 cm from the equilibrium position and released from rest, what are the natural frequency and the period of its motion?

   (b) What are the maximum speed and acceleration of the mass?

   (c) What is the total energy of the system? What are the kinetic and potential energies of the system when the mass has a displacement of 2.0 cm from equilibrium?

   (d) If the surface is no longer frictionless, but instead gives rise to a damping force with damping constant $b = 1.7$ kg/s, is the motion of the mass underdamped, critically damped, or overdamped? Justify your answer.

8. There are 6 known quark flavours, up (u), down (d), strange (s), charm (c), bottom (b) and top (t). They can only decay via the weak force, usually through β decay.

   (a) Mesons are formed by combining a quark together with an antiquark. This two-body system, like the hydrogen atom, can be found in the ground state, or in an infinite number of excited states. List the quark flavours of the ground state mesons that can be formed from combinations of the up, down, and strange mesons (for bonus marks, label the names of the particles associated with each combination!).

   (b) We believe the top quark decays to a bottom quark and a W^+ boson almost all the time. In the rest frame of the top quark, what is the momentum of the W^+ and bottom quark after the decay (the mass of the top quark is $M_t = 175$ GeV/c^2, the mass of the bottom quark is $M_b = 4.5$ GeV/c^2, and the mass of the W^+ boson is $M_W = 80$ GeV/c^2).

   (c) We know that most of the energy released in the fission of a ^{235}U nucleus comes from the resulting energy caused by the electrostatic repulsion of the decay products. If the top quark decay products are produced 1 fm from each other, where does most of the energy from this decay come from?