PHY140Y

Solutions to Term Test 3

February 24, 1999

Section A

1. Gravitational mass is the quantity associated with an object that is proportional to the force of gravity on the object. Inertial mass is the quantity that relates the acceleration of an object to the force applied to it. Gravitational mass is introduced by Newton's universal law of gravitation, whereas inertial mass is introduced by Newton's second law of motion. We cannot "prove" these are identical. We can demonstrate their equivalence (to a certain degree of accuracy) by experiment (which leads us to the Equivalence Principle - a postulate and not a result that we have proven using a set of more fundamental axioms).

2. The moment of inertia of the dumbell is

$$I = 2(1.0)(0.125)^2 = 0.0313 \text{ kg m}^2. \tag{1}$$

   The linear analogue of the moment of inertia is **mass**.

3. Gravity is considered the weakest of all forces because by any measure of comparison it generates the weakest forces. This is most obvious when you compare the effects of static electricity on small objects, which in normal day life reflects a minute imbalance in positive and negative charges on an object, to the force of gravity between the same objects. One easily finds that the electrostatic force can be many orders of magnitude larger the gravitational one. The fact that gravity is only an attractive force means that the weakness of the force can be compensated by having a large amount of it nearby (eg., the force of gravity generated by the earth).

4. Tides result from the force of gravity falling off as $1/r^2$. This means that for an extended object in the gravitational field of a point object, the force of attraction on points farthest from the object is less than on points nearest to the object, creating a differential force between these two points. In the case of the earth-moon system, the tidal force of the moon causes the water nearest the moon to accelerate away from the centre of the earth, and causes the centre of the earth to accelerate away from the water on the side of the earth farthest from the moon. Thus, we have two "bulges" of water that effectively rotate about the earth every 24 hours since in the course of one earth revolution, the moon moves only about 1/28 of an orbit.

5. A "black body" is an object that absorbs all radiation incident upon it. When it is in thermal equilibrium, it emits radiation with a characteristic black-body spectrum that depends only on its temperature. The frequency where the spectrum is its most intense is linearly proportional to the temperature of the object (in K degrees).

Section B

1. Full marks on this question required you to write a coherent short essay addressing all or most of the issues outlined below:

   (a) This topic is properly treated if you successfully identify the mass and electric charge as being the properties of the object that are responsible for the resulting force fields. The properties of these forces then only depend on the amount of mass or charge present and the distance from these. Gauss's Law gives a geometric picture of the relationship between the "flux" associated with the force through a closed surface and the mass or charge enclosed within the surface. This is the easiest way of making the connection: the flux through a spherical surface of a point charge or mass is a constant independent of the size of the sphere.

   (b) One argument here is to first note that rigid-body rotation can be analyzed using the concepts we already have introduced in studying linear motion of simple systems, with the simplification that we have factored away any common motion associated with the centre of mass of the object. By defining the concept of an angular moment of inertia, one can go from Newton's second law,

$$\vec{F} = \frac{d\vec{p}}{dt} \tag{2}$$

   to the analogous relationship for angular motion,

$$\vec{\tau} = \frac{d\vec{L}}{dt}. \tag{3}$$

   This gives us two **conserved** quantities when there is no external force or external torque on the system. These are then constants of motion that allow us to understand the behaviour of systems more readily. In particular, the application of angular momentum conservation assists in explaining a host of phenomena that are often counter-intuitive to our rather linear thinking.

   An essay is supposed to lead the reader through an argument, presented in a logically consistent way. There needs to be some form of introduction, giving the reader the "thesis" of writer, then a body that gives the arguments supporting the thesis, and finally a conclusion summarizing the argument and perhaps pointing out the bigger picture, if there is one.

2. (a) The acceleration on the surface of Eros is

$$g_E = \frac{GM_E}{R_E^2} \tag{4}$$

$$= \frac{(6.67 \times 10^{-11})(5.0 \times 10^{15})}{(7.0 \times 10^3)^2} = 6.81 \times 10^{-3}\ \mathrm{kg\,m/s^2}. \tag{5}$$

   (b) A 2000-kg truck would appear to be weigh as much as an object on Earth with mass scaled by the ratio of the accelerations of gravity:

$$M_{equiv} = M\frac{g_E}{g} = (2000)\frac{6.81 \times 10^{-3}}{9.8} = 1.39\ \mathrm{kg}. \tag{6}$$

   So it would not be difficult to pick up this truck.

(c) On earth, a jump of $h = 0.5$ m means that you have to accelerate your centre of mass to a velocity v_i such that from the time you leave the surface to the time you reach your maximum height, Δt,

$$h = \frac{1}{2} g (\Delta t)^2 \quad \text{and} \quad v_i = g \Delta t \tag{7}$$

$$\Rightarrow v_i = \sqrt{2hg} \tag{8}$$

$$= \sqrt{2(0.5)(9.8)} = 3.13 \text{ m/s}. \tag{9}$$

On Eros, the escape velocity is given by

$$v_E = \sqrt{\frac{2GM_E}{R_E}} = 9.76 \text{ m/s}, \tag{10}$$

so you will not jump off Eros (but you will go very high!).

(d) The density of Eros is

$$\rho_E = \frac{M_E}{4\pi R_E^3/3}, \tag{11}$$

so the mass of an asteroid with radius r would be

$$m = \rho_E \left(4\pi r^3/3\right) = M_E \left(\frac{r}{R_E}\right)^3. \tag{12}$$

The escape velocity off this object would be

$$v_{es} = \sqrt{\frac{2Gm}{r}} \tag{13}$$

$$= \sqrt{\frac{2GM_E r^2}{R_E^3}} \tag{14}$$

$$= \sqrt{\frac{2GM_E}{R_E}} \frac{r}{R_E} \tag{15}$$

$$= v_E \frac{r}{R_E}. \tag{16}$$

Thus, the escape velocity scales with the radius of the asteroid. In order for v_{es} to equal v_i, we have

$$v_i = v_E \frac{r}{R_E} \tag{17}$$

$$\Rightarrow r = R_E \frac{v_i}{v_E} = (7.0 \times 10^3) \frac{3.13}{9.76} = 2.3 \text{ km}. \tag{18}$$

Given that the killer is much larger than the minimum radius calculated in the previous section, "Armageddon" gets this part of its physics about right (we can argue about some of the rest though).

3. (a) The moment of inertia of each wheel is

$$I_w = \frac{1}{2} M_w R_w^2 = (0.5)(50)(0.6)^2 = 9.0 \text{ kg m}^2. \tag{19}$$

(b) The angular velocity of the wheel at cruising speed is

$$\omega_c = \frac{v_c}{R_c}, \tag{20}$$

and this is reached in time $t_1 = 60$ s under uniform linear and angular acceleration. Thus, the angular acceleration is

$$\alpha = \frac{\omega_c}{t_1} = \frac{v_c}{R_c t_1} = \frac{(1.2 \times 10^5)/(3600)}{(0.6)(60)} = 0.926 \text{ rad/s}^2. \tag{21}$$

(c) The kinetic energy of the centre of mass of the truck is

$$K_{cm} = \frac{1}{2} M_t v_c^2 = (0.5)(2 \times 10^4) \left(\frac{1.2 \times 10^5}{3600} \right)^2 = 1.11 \times 10^7 \text{ J}. \tag{22}$$

The kinetic energy of rotation of each wheel about its axis, times the number of wheels is

$$\begin{aligned} K_{rot} &= 18 \times \frac{1}{2} I_w \omega_c^2 && (23) \\ &= 18 \times \frac{1}{2} I_w \left(\frac{v_c}{R_c} \right)^2 && (24) \\ &= 18 \times (0.5)(9.0) \left(\frac{1.2 \times 10^5}{(3600)(0.6)} \right)^2 = 2.5 \times 10^5 \text{ J}. && (25) \end{aligned}$$

The fraction of energy in the form of rotational motion is

$$\frac{K_{rot}}{K_{rot} + K_{cm}} = \frac{2.5 \times 10^5}{2.5 \times 10^5 + 1.11 \times 10^7} = 0.022. \tag{26}$$

(d) The flying wheel comes off with both its rotational and kinetic energy. This is

$$\begin{aligned} E_w &= \frac{1}{2} M_w v_c^2 + \frac{1}{2} I_w \omega_c^2 && (27) \\ &= (0.5)(50) \left(\frac{1.2 \times 10^5}{3600} \right)^2 + (0.5)(9.0) \left(\frac{1.2 \times 10^5}{(3600)(0.6)} \right)^2 && (28) \\ &= 2.78 \times 10^4 + 1.39 \times 10^4 = 4.16 \times 10^4 \text{ J}. && (29) \end{aligned}$$

This is why wheels flying off of transport rigs are so dangerous...

4. (a) The photoelectric effect is caused by a photon colliding with a free electron in the metal and "kicking" it out of the surface. The work function is the minimum energy required to liberate an electron, and corresponds to the effective potential energy of the electron

in the surface of the metal. The work function can be measured by shining light at a given energy on the surface, and then determining the potential difference between the grid and metal surface when electrons liberated by the light can be longer reach the grid because they do not have enough kinetic energy once released from the surface to cross the potential gap.

(b) The energy of a photon, E_γ, is related to its frequency, ν, by the relationship

$$E_\gamma = h\nu \tag{30}$$

where $h = 6.62 \times 10^{-34}$ J s $= 4.13 \times 10^{-15}$ eV s. Thus, the minimum and maximum frequencies of the source should be

$$\begin{aligned} \nu_{min} &= (1.0)/(4.13 \times 10^{-15}) = 2.42 \times 10^{14}\ \mathrm{s}^{-1} \quad \text{and} && (31) \\ \nu_{max} &= (10.0)/(4.13 \times 10^{-15}) = 2.42 \times 10^{15}\ \mathrm{s}^{-1}. && (32) \end{aligned}$$

(c) The electric field E between the two plates has a magnitude

$$E = \frac{V}{d} = \frac{100}{0.02} = 5000\ \mathrm{V/m} \tag{33}$$

and is directed from the grid to the metal surface.

(d) Since the work functions range from 1.0 to 10.0 eV, and the photon energies we are working with have the same range, we want to be able to measure photo-electrons with energies ranging from 0 eV (corresponding to the maximum work function of 10 eV and the maximum photon energy) to 9.0 eV (corresponding to the minimum work function of 1.0 eV and the maximum photon energy of 10 eV).

5. (a) The moment of inertia of the chisel rolling about its longitudinal axis is

$$I_c = \frac{1}{2}M_c R_c^2 = (0.5)(0.25)(1.0 \times 10^{-2})^2 = 1.25 \times 10^{-5}\ \mathrm{kg\,m^2}. \tag{34}$$

(b) The chisel, treated as a rolling cylinder, has total kinetic energy

$$E_{tot} = \frac{3}{4}M_c v_c^2, \tag{35}$$

where v_c is the velocity of the chisel when it leaves the roof. This is equal to the change in gravitational potential energy, which is

$$\Delta U = gM_c h = gM_c L\sin\theta, \tag{36}$$

where $h = L\sin\theta$ is the vertical drop as it rolls down the roof. Thus,

$$\begin{aligned} \frac{3}{4}M_c v_c^2 &= gM_c L\sin\theta && (37) \\ \Rightarrow v_c &= \sqrt{\frac{4gL\sin\theta}{3}} && (38) \\ &= \sqrt{\frac{4(9.8)(10.0)(0.5)}{3}} = 8.08\ \mathrm{m/s}. && (39) \end{aligned}$$

(c) The chisel is launched from the roof with a trajectory parallel to the roof. This is given by

$$v_x = v_c \cos\theta \quad \text{and} \quad v_y = -v_c \sin\theta. \tag{40}$$

Thus, the equations for the motion in the x and y directions is

$$x(t) = x_o + v_x t \tag{41}$$

$$y(t) = y_o - v_y t - \frac{1}{2}gt^2, \tag{42}$$

where $x_o = 0$, $y_o = 10$ m, $v_x = v_c \cos\theta = 4.95$ m/s and $v_y = -v_c \sin\theta = -2.86$ m/s. We first solve for the time, t_1, the chisel hits the ground:

$$0 = y_o + v_y t_1 - \frac{1}{2}gt_1^2 \tag{43}$$

$$\Rightarrow t_1 = \frac{-v_y \pm \sqrt{v_y^2 + 2y_o g}}{-g} \tag{44}$$

$$= 0.471 \text{ s.} \tag{45}$$

We have used the quadratic formula and have discarded the second solution that gives a negative time. The chisel hits the ground a distance

$$v_x t_1 = v_c \cos\theta t_1 = 3.30 \text{ m} \tag{46}$$

away from the house.

(d) The total energy of the chisel is given by the change in potential energy from where it was dropped, or

$$E_{tot} = M_c g(L\sin\theta + h) = (0.25)(9.8)(5.0 + 3.0) = 19.6 \text{ J.} \tag{47}$$

This is equivalent to a very well thrown baseball. I'd start packing...

6. (a) The temperature T_g that emits most strongly at that frequency is

$$T = \frac{2.898 \times 10{-}3}{\lambda_{max}} = 5,269^\circ \text{ K.} \tag{48}$$

(b) This is very close to the temperature of the sun. Why? Because our eyes have evolved to be most effective in the environment in which we live, namely one illuminated by the sun. Thus, the very good match between the performance of our eyes and the light output of the sun.

(c) The brightness of an object scales as T^4 (the Stefan-Boltzmann law). Thus, the relative brightness on a planet as described would be

$$\left(\frac{3000}{5800}\right)^4 = 0.072. \tag{49}$$

(d) Since the apparent brightness goes like (distance)$^{-2}$, the planet would have to be in orbit whose radius is

$$\sqrt{\frac{1}{0.072}} = 0.27 \tag{50}$$

of that of the original orbit.