PHY140Y

Spring Term – Tutorial 14 Solutions

January 17, 2000

1. (a) The centre of mass of the system is the point exactly between the two suns. We could have been pedantic about it and use the formal definition of the centre of mass, $\vec{R_{cm}}$,

$$M_{tot}\vec{R_{cm}} = \sum_i m_i \vec{r_i}, \tag{1}$$

where $M_{tot} = \sum_i m_i$ is the total mass of the system.

(b) To solve this problem, note that each sun is in a circular orbit of radius R so the normal relationships for uniform circular motion must be satisfied. In particular, the centripetal acceleration should satisfy

$$a_c = \frac{v^2}{R}. \tag{2}$$

This has to be equal to the gravitational attraction of the other sun, which is a distance $2R$ away:

$$a_g = \frac{GM}{4R^2}. \tag{3}$$

Using the relationship $vT = 2\pi R$ for the period T, we get the result

$$T^2 = \frac{8\pi^2 R^3}{GM}. \tag{4}$$

2. (a) We want to determine the radius at which the escape velocity, v_{esc}, would equal the speed of light. We use the standard formula derived in class:

$$\begin{aligned} v_{esc} &= \sqrt{\frac{2GM_b}{r_b}} && (5)\\ \Rightarrow r_b &= \frac{2GM_b}{v_{esc}^2} && (6)\\ &= \frac{2(6.67\times10^{-11})(5\times1.99\times10^{30})}{(3.00\times10^8)^2} = 1.48\times10^4 \text{ m}. && (7)\end{aligned}$$

The event horizon, ie. the surface from which not even light can escape, has a radius of only 15 km. Scary...

(b) In order to escape the gravitational pull of the black hole, he has to have a total energy of at least 0. Thus, at any radius r from the black hole, his speed v is required to satisfy

$$\begin{aligned} E \equiv K + U &> 0 && (8)\\ \Rightarrow K &> -U && (9)\\ \Rightarrow \frac{1}{2}mv^2 &> \frac{GM_b m}{r} && (10)\\ \Rightarrow v &> \sqrt{\frac{2GM_b}{r}}, && (11)\end{aligned}$$

where m is the mass of the adventurer and his space ship (and is totally irrelevant to this problem). For a minimum orbital radius $r = 20$ km, we find

$$v = \sqrt{\frac{2(6.67 \times 10^{-11})(5 \times 1.99 \times 10^{30})}{2 \times 10^4}} = 2.6 \times 10^8 \text{ m/s.} \tag{12}$$

As we might have expected, this is a good fraction of the speed of light.

(c) As we noted in class, the tidal forces acting in the radial direction from a massive object will vary as $4a/r$, where $2a$ is that separation between the points across which the tidal acceleration is evaluated. Our adventurer should want to minimize the total tidal force acting across his body, and therefore want to minimize a. Hence, he should align himself so that his body is perpendicular to the vector separating him from the black hole.

(d) Assume that his height is about 2 m, which corresponds to $2a$. Then lined up as described, the tidal acceleration acting across his body is

$$a_t = \frac{GM_b}{r_c^2}\left(\frac{4a}{r_c}\right) \tag{13}$$

$$= \frac{(6.67 \times 10^{-11})(5 \times 1.99 \times 10^{30})}{(5 \times 10^3)^2}\left(\frac{4}{5 \times 10^3}\right) \tag{14}$$

$$= 3.3 \times 10^8 \text{ m/s}^2. \tag{15}$$

Uh, oh. Mission Control, I think we have a problem.

3. (a) In order for the force of gravity between the Earth and Moon and the electrostatic force created by equal charges q_M on both the Earth and Moon be equal, we must have

$$\frac{GM_E M_M}{r_M^2} = \frac{kq_M^2}{r_M^2} \tag{16}$$

$$\Rightarrow q_M = \sqrt{\frac{GM_E M_M}{k}} \tag{17}$$

$$= \sqrt{\frac{(6.67 \times 10^{-11})(5.97 \times 10^{24})(7.35 \times 10^{22})}{9 \times 10^9}} = 5.7 \times 10^{13} \text{ C,} \tag{18}$$

where M_E is the mass of the Earth, M_M is the mass of the Moon and r_M is the radius of the Moon's orbit.

(b) We don't need to know the radius of the Moon's orbit. Since both forces have the same radial dependence, the radius cancels in the answer to part a) above.

(c) A gram of ionized hydrogen has $N_A = 6.03 \times 10^{23}$ protons in it (N_A is Avogradro's number or the number of atoms in a mole). Thus, the total charge in a gram of hydrogen is

$$q_H = N_A q_e \tag{19}$$

$$= (6.03 \times 10^{23})(1.6 \times 10^{-19}) = 9.6 \times 10^4 \text{ C,} \tag{20}$$

where q_e is the charge of a proton. Note, we have ignored the mass of the electron in all this, since it represents less than 1/1000th the mass of a hydrogen atom. The total mass of ionized hydrogen required to produce q_M charge is therefore

$$M_H = \frac{q_M}{q_H} \tag{21}$$

$$= \frac{5.7 \times 10^{13}}{9.6 \times 10^4} = 5.9 \times 10^8 \text{ g}. \tag{22}$$

This is 590 tonnes of ionized hydrogen.

As a bonus, estimate how deep an ocean of liquid hydrogen this would create on the Moon (the density of liquid hydrogen is about 0.07 g/l).