PHY140Y

Spring Term – Tutorial 15 Solutions

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1. (a) Since the field is oriented at right angles to the surface, $\vec{F} \cdot \hat{n} = F$. Thus the flux is

$$\phi = \int_S (F(\vec{x}) \cdot \hat{n}) dS = FA. \tag{1}$$

When the surface is oriented with its normal perpendicular to the field, $\vec{F} \cdot \hat{n} = 0$. Thus, the flux is zero.

(b) Orient the cube so that one set of faces have their unit normals parallel to the field vector. In this orientation, four of the six surfaces have $\vec{F} \cdot \hat{n} = 0$. For the other two,

$$\vec{F} \cdot \hat{n} = FA \text{ and} \tag{2}$$
$$\vec{F} \cdot \hat{n} = -FA. \tag{3}$$

Thus, the total flux through this object is zero in this constant field.
For the sphere, we can use symmetry arguments similar to what we used in the case of the cube to show that the flux has to be zero.

(c) The electric field of a point charge at a distance r is given by

$$\vec{E}(\vec{r}) = \frac{kq}{r^2}\hat{r}, \tag{4}$$

where $k = 9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}$. The normal to the sphere of radius r is also $\hat{r}$. Thus the flux of the electric field through the sphere is

$$\phi = \int_S (\vec{E} \cdot \hat{n}) dS \tag{5}$$
$$= \frac{kq}{r^2} 4\pi r^2 = 4\pi kq. \tag{6}$$

Note that this is independent of the size of the sphere.

2. (a) First we calculate the electric field associated with the top plate. By Gauss's Law, we can determine that this field is

$$\vec{E}_t(z) = +\frac{Q}{2\epsilon_0 A}\,\hat{z} \text{ for } z > d, \text{ and} \tag{7}$$
$$\vec{E}_t(z) = -\frac{Q}{2\epsilon_0 A}\,\hat{z} \text{ for } z < d, \tag{8}$$

where ϵ_0 is the permittivity. In the same way, the field associated with the bottom plate is

$$\vec{E}_b(z) = -\frac{Q}{2\epsilon_0 A}\,\hat{z} \text{ for } z > 0, \text{ and} \tag{9}$$
$$\vec{E}_b(z) = +\frac{Q}{2\epsilon_0 A}\,\hat{z} \text{ for } z < 0. \tag{10}$$

The total field is, by the superposition principle, the sum of these two. Thus,

$$\vec{E}(z) = 0 \text{ for } z > d, \tag{11}$$

$$\vec{E}(z) = -\frac{Q}{\epsilon_0 A}\,\hat{z} \text{ for } 0 > z > d, \text{ and} \tag{12}$$

$$\vec{E}(z) = 0 \text{ for } z < 0. \tag{13}$$

(b) The work performed by the field on a charge q that is moved from the bottom to the top plate is

$$W = \int_0^d q(\vec{E}\cdot d\vec{x}) \tag{14}$$

$$= -\frac{qQd}{\epsilon_0 A}. \tag{15}$$

Thus, we define the potential difference between the two plates to be

$$\Delta V = -W/q \tag{16}$$

$$= \frac{Qd}{\epsilon_0 A}. \tag{17}$$

(c) The force acting on the top plate is

$$\vec{F}_t = Q\vec{E}_b(z = d) = -\frac{Q^2}{8\pi\epsilon_0^2 A}\hat{z}. \tag{18}$$

There is an equal and opposite force acting on the bottom plate.

(d) The capacitance of this system is

$$C \equiv \frac{Q}{\Delta V} \tag{19}$$

$$= \frac{\epsilon_0 A}{d}. \tag{20}$$

3. (a) See Fig. 25-22 in the text.

(b) The field at a point $z\hat{z}$ is

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\left[\frac{q}{\left(z-\frac{d}{2}\right)^2} - \frac{q}{\left(z+\frac{d}{2}\right)^2}\right]\hat{z} \tag{21}$$

$$= \frac{q}{4\pi\epsilon_0 z^2}\left[\frac{1}{\left(1-\frac{d}{2z}\right)^2} - \frac{1}{\left(1+\frac{d}{2z}\right)^2}\right]\hat{z} \tag{22}$$

$$\simeq \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1+2\frac{d}{2z}\right) - \left(1-2\frac{d}{2z}\right)\right]\hat{z} \tag{23}$$

$$= \left[\frac{2qd}{4\pi\epsilon_0 z^3}\right]\hat{z}. \tag{24}$$

In this calculation, we have made the approximation

$$(1+\delta)^{-2} = 1 - 2\delta + O(\delta^2) \quad (25)$$

$$\simeq 1 - 2\delta, \quad (26)$$

where we assume that $\delta \ll 1$. This is just a Taylor series expansion truncated to the first order term.

4. The only way this could work is if the three charges were placed in some form of symmetrical arrangement. The only one possible is where the charge $-q$ is placed directly between the two charges $+4q$. It turns out that it doesn't matter how far away the two positive charges are as long as they are equidistant from the charge $-q$ (check this!).