PHY140Y

Spring Term – Tutorial 16 Discussion Solutions

January 31, 2000

1. The net force acting on the block along the incline is

$$\sum F = m_b g \sin\phi - T - \mu F_N \tag{1}$$
$$= = m_b a, \tag{2}$$

where F_N is the normal force acting out of the plane, T is the tension on the string, $\phi = 30°$ is the slope of the plane and μ is the coefficient of kinetic friction. The torque acting on the drum is

$$\tau = -rT \tag{3}$$
$$= I_d \frac{d^2\theta}{dt^2}, \tag{4}$$

where $r = 5.0$ cm is the radius of the drum. This torque produces an angular acceleration that translates into a linear acceleration on the string (and hence the block). Thus, we can relate

$$r\frac{d^2\theta}{dt^2} = -a, \tag{5}$$

where the negative sign comes in from assuming that a positive rotation will cause the drum to wind up the string. These latter two relationships can be combined so that

$$rT = I_d \left(\frac{a}{r}\right), \tag{6}$$

which we can use to eliminate the tension T in the first equation. Thus,

$$T = I_d \left(\frac{a}{r^2}\right) \tag{7}$$
$$= \frac{1}{2} m_d a, \tag{8}$$

where we have used the moment of inertia of a solid drum

$$I_d = \frac{1}{2} m_d r^2. \tag{9}$$

With this expression for T, we can solve the first equation for μ:

$$m_b a = m_b g \sin\phi - \frac{1}{2} m_d a - \mu m_b g \cos\phi \tag{10}$$
$$\mu = \tan\phi - \frac{a}{g\cos\phi}\left(\frac{m_d}{2m_b} + 1\right) \tag{11}$$
$$= 0.36. \tag{12}$$

2. Angular momentum is conserved so that the angular momentum of the wheel (all there is initially) has to equal the angular momentum of the student, turntable and the wheel after being turned upside down. Thus,

$$I_{wh}\omega_{wh} = -I_{wh}\omega_{wh} + (I_t + I_{st})\,\omega_{st}, \qquad (13)$$

where ω_{wh} is the angular speed of the wheel, ω_{st} is the angular speed of the turntable and student, and I_{st} is the angular inertia of the student. Solving this for I_{st}, we find

$$\begin{aligned} I_{st} &= I_{wh}\left(\frac{2\omega_{wh}}{\omega_{st}}\right) - I_t & (14)\\ \Rightarrow m_{st} &= \frac{2}{r_{st}^2}\left[I_{wh}\left(\frac{2\omega_{wh}}{\omega_{st}}\right) - I_t\right] & (15)\\ &= 45.1 \text{ kg.} & (16) \end{aligned}$$