PHY140Y

Spring Term – Tutorial 17 Discussion Solutions

7 February 2000

1. (a) The forces acting on the ladder are:

- the force of gravity, $\vec{F}_g$, acting through the centre of mass of the ladder,
- the normal force, $\vec{F}_N$, responding to the weight of the ladder on the ground,
- the force of friction, $\vec{F}_f$, preventing the ladder from sliding on the ground, and
- the normal force, $\vec{F}_w$, exerted by the wall on the ladder to keep it stationary.

These forces are shown in Fig. 1.

(b) Choose a coordinate system with $\hat{x}$ pointing toward the wall and $\hat{y}$ pointing upwards along the wall. The net force in the $\hat{y}$ direction is

$$\begin{aligned} 0 &= F_N + F_g && (1) \\ &= F_N - mg. && (2) \end{aligned}$$

The net force in the horizontal direction is

$$\begin{aligned} 0 &= F_f - F_w && (3) \\ &= \mu F_N - F_w. && (4) \end{aligned}$$

(c) The best axis to choose for this problem is the one located at the base of the ladder. This is because two of the four forces act through this point, so that the torque they generate on the ladder is zero. The net torque is then given by the force of gravity acting on the centre of mass and the force exerted by the wall at the end of the ladder:

$$\begin{aligned} \tau_{net} &= \vec{r}_{cm} \times \vec{F}_g + \vec{r}_{end} \times \vec{F}_w && (5) \\ &= \left(-\frac{LF_g \cos\phi}{2} + LF_w \sin\phi \right) \hat{z}, && (6) \end{aligned}$$

where $\vec{r}_{cm}$ and $\vec{r}_{end}$ are vectors from the axis to the centre of mass and the end of the ladder, respectively. The axis $\hat{z}$ points out of the page.

(d) We can use the force relations to find

$$\begin{aligned} F_N &= mg && (7) \\ \Rightarrow F_f &= \mu F_N = \mu mg, && (8) \end{aligned}$$

and

$$F_w = \mu F_N = \mu mg. \quad (9)$$

Substituting that into the expression for net torque, we find

$$\begin{aligned} 0 &= -\frac{LF_g \cos\phi}{2} + LF_w \sin\phi && (10) \\ &= -\frac{Lmg \cos\phi}{2} + L\mu mg \sin\phi && (11) \\ \Rightarrow \mu &= \frac{\cos\phi}{2\sin\phi} = \frac{1}{2}\cot\phi. && (12) \end{aligned}$$

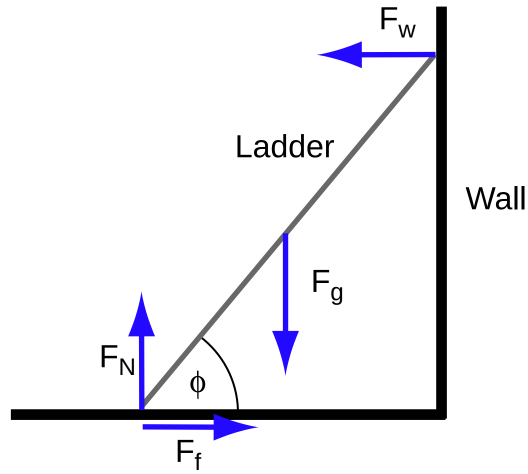

Figure 1: The forces acting on the ladder.

This is pretty remarkable, in that it doesn't depend on the length of the ladder, it's mass or the force of gravity. So that is why there is a "universal" best angle at which a ladder should be placed leaning up against the wall. We also see that the force of friction has to rise very rapidly as ϕ decreases, especially once you have angles less than $\pi/4$.

Also note that we didn't even have to compute a moment of inertia in this problem!

2. The conservation of energy requires that the work done by the string on the yo-yo, W_{unwind}, be equal to the change in mechanical energy. So, when the string is being pulled, the work done is

$$\begin{aligned} W_{unwind} &= \tau\Delta\theta & (13) \\ &= (Tr)\,(\Delta l/r) & (14) \\ &= T\Delta l, & (15) \end{aligned}$$

where T is the tension on the string, r is the radius of the narrow shaft the string is wound upon (and the distance over which the tension acts from the axis of rotation), and Δl is the total length of the string. The total arc length over which the tension is applied is the length of the string divided by the radius of the shaft on which it is coiled (which cancels in the expression for the work done).

All of this work goes into the rotation of the yo-yo, since the tension on the string is equal to the gravitational force and keeps the centre-of-mass at rest while the string unwinds. Thus, the rotational energy of the yo-yo is W_{unwind}, or

$$\begin{aligned} K_{rot} &= T\Delta l & (16) \\ &= (0.98)(0.8) = 0.78\text{ J}. & (17) \end{aligned}$$

Once the string slips off, the yo-yo free-falls to the floor, so that the kinetic energy of the centre-of-mass when it hits the floor is equal to the change in gravitational potential energy, or

$$
\begin{aligned}
K_{kin} &= mgh && (18) \\
&= (0.1)(9.8)(0.5) = 0.49 \text{ J.} && (19)
\end{aligned}
$$

Since there is no torque acting on the yo-yo in free fall, K_{rot} remains unchanged.