PHY140Y

Spring Term – Tutorial 18 Discussion Solutions

21 February 2000

1. (a) The acceleration on the surface of Eros is

$$g_E = \frac{GM_E}{R_E^2} \tag{1}$$

$$= \frac{(6.67\times10^{-11})(5.0\times10^{15})}{(7.0\times10^3)^2} = 6.81\times10^{-3}\ \mathrm{kg\,m/s^2}. \tag{2}$$

(b) A 2000-kg truck would appear to be weigh as much as an object on Earth with mass scaled by the ratio of the accelerations of gravity:

$$M_{equiv} = M\frac{g_E}{g} = (2000)\frac{6.81\times10^{-3}}{9.8} = 1.39\ \mathrm{kg}. \tag{3}$$

So it would not be difficult to pick up this truck.

(c) On earth, a jump of $h = 0.5$ m means that you have to accelerate your centre of mass to a velocity v_i such that from the time you leave the surface to the time you reach your maximum height, Δt,

$$h = \frac{1}{2}g(\Delta t)^2 \qquad \text{and} \qquad v_i = g\Delta t \tag{4}$$

$$\Rightarrow v_i = \sqrt{2hg} \tag{5}$$

$$= \sqrt{2(0.5)(9.8)} = 3.13\ \mathrm{m/s}. \tag{6}$$

On Eros, the escape velocity is given by

$$v_E = \sqrt{\frac{2GM_E}{R_E}} = 9.76\ \mathrm{m/s}, \tag{7}$$

so you will not jump off Eros (but you will go very high!).

(d) The density of Eros is

$$\rho_E = \frac{M_E}{4\pi R_E^3/3}, \tag{8}$$

so the mass of an asteroid with radius r would be

$$m = \rho_E\left(4\pi r^3/3\right) = M_E\left(\frac{r}{R_E}\right)^3. \tag{9}$$

The escape velocity off this object would be

$$v_{es} = \sqrt{\frac{2Gm}{r}} \tag{10}$$

$$\begin{aligned}
&= \sqrt{\frac{2GM_E r^2}{R_E^3}} && (11)\\
&= \sqrt{\frac{2GM_E}{R_E}}\frac{r}{R_E} && (12)\\
&= v_E\frac{r}{R_E}. && (13)
\end{aligned}$$

Thus, the escape velocity scales with the radius of the asteroid. In order for v_{es} to equal v_i, we have

$$\begin{aligned}
v_i &= v_E\frac{r}{R_E} && (14)\\
\Rightarrow r &= R_E\frac{v_i}{v_E} = (7.0\times 10^3)\,\frac{3.13}{9.76} = 2.3 \text{ km}. && (15)
\end{aligned}$$

Given that the killer is much larger than the minimum radius calculated in the previous section, "Armageddon" gets this part of its physics about right (we can argue about some of the rest though).

2. (a) The moment of inertia of each wheel is

$$I_w = \frac{1}{2}M_w R_w^2 = (0.5)(50)(0.6)^2 = 9.0 \text{ kg m}^2. \qquad (16)$$

(b) The angular velocity of the wheel at cruising speed is

$$\omega_c = \frac{v_c}{R_c}, \qquad (17)$$

and this is reached in time $t_1 = 60$ s under uniform linear and angular acceleration. Thus, the angular acceleration is

$$\alpha = \frac{\omega_c}{t_1} = \frac{v_c}{R_c t_1} = \frac{(1.2\times 10^5)/(3600)}{(0.6)(60)} = 0.926 \text{ rad/s}^2. \qquad (18)$$

(c) The kinetic energy of the centre of mass of the truck is

$$K_{cm} = \frac{1}{2}M_t v_c^2 = (0.5)(2\times 10^4)\left(\frac{1.2\times 10^5}{3600}\right)^2 = 1.11\times 10^7 \text{ J}. \qquad (19)$$

The kinetic energy of rotation of each wheel about its axis, times the number of wheels is

$$\begin{aligned}
K_{rot} &= 18\times\frac{1}{2}I_w\omega_c^2 && (20)\\
&= 18\times\frac{1}{2}I_w\left(\frac{v_c}{R_c}\right)^2 && (21)\\
&= 18\times(0.5)(9.0)\left(\frac{1.2\times 10^5}{(3600)(0.6)}\right)^2 = 2.5\times 10^5 \text{ J}. && (22)
\end{aligned}$$

The fraction of energy in the form of rotational motion is

$$\frac{K_{rot}}{K_{rot}+K_{cm}} = \frac{2.5\times 10^5}{2.5\times 10^5 + 1.11\times 10^7} = 0.022. \tag{23}$$

(d) The flying wheel comes off with both its rotational and kinetic energy. This is

$$\begin{aligned} E_w &= \frac{1}{2}M_w v_c^2 + \frac{1}{2}I_w\omega_c^2 && (24)\\ &= (0.5)(50)\left(\frac{1.2\times 10^5}{3600}\right)^2 + (0.5)(9.0)\left(\frac{1.2\times 10^5}{(3600)(0.6)}\right)^2 && (25)\\ &= 2.78\times 10^4 + 1.39\times 10^4 = 4.16\times 10^4\ \text{J}. && (26)\end{aligned}$$

This is why wheels flying off of transport rigs are so dangerous...

3. (a) The photoelectric effect is caused by a photon colliding with a free electron in the metal and "kicking" it out of the surface. The work function is the minimum energy required to liberate an electron, and corresponds to the effective potential energy of the electron in the surface of the metal. The work function can be measured by shining light at a given energy on the surface, and then determining the potential difference between the grid and metal surface when electrons liberated by the light can be longer reach the grid because they do not have enough kinetic energy once released from the surface to cross the potential gap.

(b) The energy of a photon, E_γ, is related to its frequency, ν, by the relationship

$$E_\gamma = h\nu \tag{27}$$

where $h = 6.62\times 10^{-34}$ J s $= 4.13\times 10^{-15}$ eV s. Thus, the minimum and maximum frequencies of the source should be

$$\begin{aligned} \nu_{min} &= (1.0)/(4.13\times 10^{-15}) = 2.42\times 10^{14}\ \text{s}^{-1} \quad \text{and} && (28)\\ \nu_{max} &= (10.0)/(4.13\times 10^{-15}) = 2.42\times 10^{15}\ \text{s}^{-1}. && (29)\end{aligned}$$

(c) The electric field E between the two plates has a magnitude

$$E = \frac{V}{d} = \frac{100}{0.02} = 5000\ \text{V/m} \tag{30}$$

and is directed from the grid to the metal surface.

(d) Since the work functions range from 1.0 to 10.0 eV, and the photon energies we are working with have the same range, we want to be able to measure photo-electrons with energies ranging from 0 eV (corresponding to the maximum work function of 10 eV and the maximum photon energy) to 9.0 eV (corresponding to the minimum work function of 1.0 eV and the maximum photon energy of 10 eV).

4. (a) The moment of inertia of the chisel rolling about its longitudinal axis is

$$I_c = \frac{1}{2}M_c R_c^2 = (0.5)(0.25)(1.0\times 10^{-2})^2 = 1.25\times 10^{-5}\ \text{kg m}^2. \tag{31}$$

(b) The chisel, treated as a rolling cylinder, has total kinetic energy

$$E_{tot} = \frac{3}{4} M_c v_c^2, \tag{32}$$

where v_c is the velocity of the chisel when it leaves the roof. This is equal to the change in gravitational potential energy, which is

$$\Delta U = g M_c h = g M_c L \sin\theta, \tag{33}$$

where $h = L\sin\theta$ is the vertical drop as it rolls down the roof. Thus,

$$\frac{3}{4} M_c v_c^2 = g M_c L \sin\theta \tag{34}$$

$$\Rightarrow v_c = \sqrt{\frac{4gL\sin\theta}{3}} \tag{35}$$

$$= \sqrt{\frac{4(9.8)(10.0)(0.5)}{3}} = 8.08 \text{ m/s}. \tag{36}$$

(c) The chisel is launched from the roof with a trajectory parallel to the roof. This is given by

$$v_x = v_c \cos\theta \quad \text{and} \quad v_y = -v_c \sin\theta. \tag{37}$$

Thus, the equations for the motion in the x and y directions is

$$x(t) = x_o + v_x t \tag{38}$$

$$y(t) = y_o - v_y t - \frac{1}{2} g t^2, \tag{39}$$

where $x_o = 0$, $y_o = 10$ m, $v_x = v_c\cos\theta = 4.95$ m/s and $v_y = -v_c \sin\theta = -2.86$ m/s. We first solve for the time, t_1, the chisel hits the ground:

$$0 = y_o + v_y t_1 - \frac{1}{2} g t_1^2 \tag{40}$$

$$\Rightarrow t_1 = \frac{-v_y \pm \sqrt{v_y^2 + 2y_o g}}{-g} \tag{41}$$

$$= 0.471 \text{ s}. \tag{42}$$

We have used the quadratic formula and have discarded the second solution that gives a negative time. The chisel hits the ground a distance

$$v_x t_1 = v_c \cos\theta t_1 = 3.30 \text{ m} \tag{43}$$

away from the house.

(d) The total energy of the chisel is given by the change in potential energy from where it was dropped, or

$$E_{tot} = M_c g(L\sin\theta + h) = (0.25)(9.8)(5.0 + 3.0) = 19.6 \text{ J}. \tag{44}$$

This is equivalent to a very well thrown baseball. I'd start packing...

5. (a) The temperature T_g that emits most strongly at that frequency is

$$T \quad = \quad \frac{2.898 \times 10{-}3}{\lambda_{max}} = 5,269^\circ \text{ K}. \tag{45}$$

(b) This is very close to the temperature of the sun. Why? Because our eyes have evolved to be most effective in the environment in which we live, namely one illuminated by the sun. Thus, the very good match between the performance of our eyes and the light output of the sun.

(c) The brightness of an object scales as T^4 (the Stefan-Boltzmann law). Thus, the relative brightness on a planet as described would be

$$\left(\frac{3000}{5800}\right)^4 = 0.072. \tag{46}$$

(d) Since the apparent brightness goes like (distance)$^{-2}$, the planet would have to be in orbit whose radius is

$$\sqrt{\frac{1}{0.072}} = 0.27 \tag{47}$$

of that of the original orbit.