PHY140Y

Spring Term – Tutorial 20 Discussion

6 March 1999

1. Let's look at a 2-D single particle in a box. Assume that we now have a square of sides of length L, containing a single electron with mass m. The two-dimensional form for Schrödinger's equation is

$$\frac{-\hbar^2}{2m}\left(\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}\right)+U(\vec{x})\psi = E\psi. \tag{1}$$

   (a) Using Schrödinger's equation, show that

$$\psi(x,y) = A\sin(k_x x)\sin(k_y y) \tag{2}$$

   could be a solution to Schrödinger's equation.

   (b) What are the possible values that k_x and k_y can take on?

   (c) What are the expressions for the 3 lowest lying energy levels? Sketch an energy level diagram illustrating these states. What is their *degeneracy* (ie., the number of distinct quantum states having the same energy)?

   (d) How large would L have to be for the energy of the ground state to be equal to 10 eV?

   (e) Describe what the probability distribution for the ground state would look like?

2. Suppose we have a semi-infinite potential barrier as shown in Figure 40-34 of the text, with a particle of mass m inside it. The height of the "barrier" is U_0. Assume that the total energy of the particle $E < U_0$.

   (a) Show by direct substitution into Schrödinger's equation that the following are solutions:

$$\psi_1(x) = A\sin(k_1 x) \text{ for } 0 \le x \le L, \tag{3}$$

$$\psi_2(x) = B\exp(k_2 x) \text{ for } x > L. \tag{4}$$

   (b) How are k_1 and k_2 related to the energy of the particle?

   (c) If we want the particle to tunnel as far as possible past the barrier, what is the optimal value for E? What is the optimal value for L?

   (d) The "other" boundary condition we have to take into account is that the wave function be continuous at $x = L$ and that it's derivative be continuous at $x = L$. What do these conditions imply about A and B?

3. An electron beam is accelerated at the back of a TV tube and then heads toward the center of the screen with horizontal velocity 2.2×10^7 m/s. As the electrons leave the acceleration region, their vertical position is known to within ± 45 nm. Find the mimimum angular spread of the beam, as set by the uncertainty principle.