PHY140Y

Spring Term – Tutorial 21 Discussion

13 March, 2000

1. Suppose we have a particle of mass m in a square well potential, where

$$U(x) = 0 \qquad \text{for } |x| < \frac{L}{2} \text{ and} \tag{1}$$

$$= U_\circ \qquad \text{for} |x| \geq \frac{L}{2}. \tag{2}$$

This is a finite square well. Suppose further that

$$\sqrt{\frac{mU_\circ L^2}{2\hbar^2}} = 4, \tag{3}$$

in which case detailed analysis shows that there is a ground state and its energy is given approximately by $E_1 = 0.098U_\circ$. The wave function for this state is

$$\psi(x) = \psi_1(x) \equiv 1.26\ \frac{1}{\sqrt{L}} \cos\left(2.50x/L\right) \qquad \text{for } |x| < \frac{L}{2} \text{ and} \tag{4}$$

$$= \psi_2(x) \equiv 17.9 \frac{1}{\sqrt{L}} e^{-7.60|x|/L} \qquad \text{for} |x| \geq \frac{L}{2}. \tag{5}$$

   (a) Show that the wave function satisfies the appropriate boundary conditions at the points $|x| = L/2$.

   (b) Calculate the probabilty of the particle being **outside** the well.

   (c) Calculate the average distance the particle "lives" outside the well when it is indeed out of the well. Assume that $L = 10$ Å.

   (d) If I do a measurement and can claim that the particle is in the well, give a minimum estimate for the momentum of the particle.

2. Suppose a hydrogen atom is in its ground state. Calculate

   (a) What is the probability of finding it at least ten Bohr radii away form the proton?

   (b) Why is this not even as probable as this calculation suggest?