PHY140Y

Spring Term – Tutorial 22 Discussion Solutions

20 March, 2000

1. (a) To get the hydrogen atom into an $n = 2$ state, it must absorb a photon with energy

$$\begin{aligned} E_\gamma &= h\nu && (1)\\ &= \frac{\hbar^2}{2m_e a_0^2}\left(1 - \frac{1}{2^2}\right) && (2)\\ \Rightarrow \nu &= \frac{\hbar^2}{2m_e a_0^2 h}\left(1 - \frac{1}{2^2}\right) && (3)\\ &= \frac{(13.6)(0.75)}{(6.63\times10^{-34})(1.60\times10^{19})} = 9.62\times10^{14}\ \mathrm{s}^{-1}. && (4)\end{aligned}$$

(b) The atoms can possibly be in angular momentum states with orbital quantum numbers $l = 0$ and $l = 1$ after absorbing a photon. The selection rule $\Delta l = \pm 1$ doesn't allow us to populate the $l = 0$ state from the ground state of hydrogen, so only $l = 1$ states are populated. There are three of these, with $m_l = +1, 0$ and -1. Their angular momenta have absolute values of

$$\begin{aligned} |\vec{L}| &= \sqrt{l(l+1)}\hbar && (5)\\ &= \sqrt{2}\hbar && (6)\end{aligned}$$

and the projections of this angular momentum on any arbitrarily chosen $\hat{z}$ axis would be

$$L_z = -\hbar, 0 \text{ and } +\hbar. \quad (7)$$

(c) We would expect to see three beams formed coming out of the Stern-Gerlach apparatus, separating the atoms by their value of L_z.

(d) Now we have to take into account the total angular momentum of the hydrogen atom $\vec{J}$, whose quantum number can take on values of $j = 3/2$ and $j = 1/2$ when we add the spin of the electron to the orbital angular momentum $\vec{L}$. This means that we have the following possible quantum states:

$$\begin{aligned} j &= 3/2 \quad \text{and} \quad m_j = +3/2 && (8)\\ j &= 3/2 \quad \text{and} \quad m_j = +1/2 && (9)\\ j &= 3/2 \quad \text{and} \quad m_j = -1/2 && (10)\\ j &= 3/2 \quad \text{and} \quad m_j = -3/2 && (11)\\ j &= 1/2 \quad \text{and} \quad m_j = +1/2 && (12)\\ j &= 1/2 \quad \text{and} \quad m_j = -1/2 && (13)\end{aligned}$$

So we would expect to have 4 beams form when passing through the inhomogeneous magnetic field of the Stern-Gerlach apparatus. Since there are two $m_j = \pm 1/2$ states, we would expect these beams to have twice the intensity of the other two beams.

2. (a) We know that we can put two electrons into the ground state with $E_0 = \hbar\omega/2$. We can then put two electrons into the next excited state with $E_1 = 3\hbar\omega/2$. This allows us to place one more electron into the $E_2 = 5\hbar\omega/2$ energy level. Thus, there are 5 electrons in the potential well.

   (b) The energy of the most energetic electron is $E_2 = 5\hbar\omega/2$.

   (c) Since we have fully occupied ground state and first excited state, we have the following possibilities:

   - An electron from the first excited state is promoted to the second excited level (where we have an empty state). This would require a light frequency of $\nu_1 = 2\pi\omega$.
   - An electron from the ground state can be promoted to the second excited state. This would require a frequency $\nu_2 = 4\pi\omega$. This frequency of light could also promote an electron from the first excited state to the third excited state.