PHY140Y

Spring Term – Tutorial 23 Discussion Solutions

27 March, 2000

1. (a) Since the magnitude of the z component of the spin vector can take on values of $\pm 1/2\hbar$, the magnitude of the z component of the magnetic dipole moment of the electron is

$$\begin{aligned} M_z &= \frac{e\hbar}{2m_e} && (1)\\ &= \frac{(1.60\times10^{-19})(1.054\times10^{-34})}{2(9.109\times10^{-31})} = 9.26\times10^{-24}\ \mathrm{A}\cdot\mathrm{m}^2. && (2) \end{aligned}$$

Check the units!

(b) The difference in energy would arise from the difference in the magnetic energy of the two possible spin states of the electron. This is given by

$$\begin{aligned} \Delta E &= 2\mu_B B && (3)\\ &= 2(9.26\times10^{-24})(1.5) = 2.78\times10^{-23}\ \mathrm{J} = 1.7\times10^{-4}\ \mathrm{eV} && (4) \end{aligned}$$

(c) In Figure 40-30, we see that the application of a 2.38 T magnetic field generates three lines where there was one before. The energy of the photon associated with the lower line at $\lambda_l = 404.625$ nm is

$$\begin{aligned} E_l &= \frac{2\pi c\hbar}{\lambda_l} && (5)\\ &= \frac{2\pi(3.00\times10^8)(6.582\times10^{-16})}{4.04625\times10^{-7}} = 3.06624\ \mathrm{eV} && (6) \end{aligned}$$

while the energy of the photon associated with the upper line at $\lambda_u = 404.710$ nm is

$$\begin{aligned} E_u &= \frac{2\pi c\hbar}{\lambda_u} && (7)\\ &= \frac{2\pi(3.00\times10^8)(6.582\times10^{-16})}{4.04710\times10^{-7}} = 3.06560\ \mathrm{eV}. && (8) \end{aligned}$$

The difference in energy is

$$\Delta E = E_u - E_l = 6.4\times10^{-4}\ \mathrm{eV} \qquad (9)$$

which is of the same magnitude as what we had calculated earlier after we scale for the difference in magnetic field strengths (2.38 T versus 1.5 T). However, there is a factor of 2 missing! This is the famous "Thomas factor" that turns out to be required by special relativity.

(d) This difference can be measured by looking for the optical transitions between the two different spin states and either lower states (using emission spectra) or higher energy states (using absorption spectra).

2. (a) Let's look at this in the rest frame of the colliding electron and helium atom. In order to excite the He atom into a 20.61 eV energy level, the minimum kinetic energy of the electron would have to be such that after the collision, the electron has transferred all of its energy to the helium atom in the rest frame of the colliding helium. Since this rest frame is almost the same as the rest frame of the helium atom itself, we can in effect ignore the motion of the helium atom and say that the kinetic energy of the electron must all go to exciting the helium atom, or $K_e = 20.61$ eV. If we assume the electron is moving non-relativistically, this means that the velocity of the electron is

$$v_e = \sqrt{\frac{2E}{m}} \tag{10}$$

$$= \sqrt{\frac{2(20.61)(1.602 \times 10^{-19})}{9.109 \times 10^{-31}}} = 2.69 \times 10^6 \text{ m/s.} \tag{11}$$

Since this is less than 1% of the speed of the light, we are justified in assuming that the electron is moving nonrelativistically.

(b) The energy of the intermediate state is

$$E_3 = E_2 - \frac{2\pi c\hbar}{\lambda_e} \tag{12}$$

$$= 20.66 - \frac{2\pi(3.00 \times 10^8)(6.58 \times 10^{-16})}{6.328 \times 10^{-7}} = 18.67 \text{ eV} \tag{13}$$

This is in the red portion of the spectrum.

(c) The wavelength of the resulting photon is

$$\lambda_s = \frac{2\pi c\hbar}{E_s} \tag{14}$$

$$= \frac{2\pi(3.00 \times 10^8)(6.58 \times 10^{-16})}{18.67} = 66.4 \text{ nm.} \tag{15}$$

This photon is well into the ultraviolet.

(d) Each coherent photon has an energy of 1.99 eV, whereas the electron has transferred an energy of 20.61 eV to the helium atom. Thus the maximum fraction of energy put into the laser beam is

$$f = \frac{1.99}{20.61} = 0.096. \tag{16}$$

(e) The minimum amount of electrical power required to form a laser beam with power $P_l = 1$ mW beam would be

$$P_c = \frac{P_l}{f} \tag{17}$$

$$= \frac{1 \times 10^{-3}}{0.096} = 0.104 \text{ W.} \tag{18}$$

This would require a minimum current I_{min} passing through a voltage difference of $\Phi = 120$ V of

$$\begin{aligned} I_{min} &= \frac{P_c}{\Phi} && (19) \\ &= \frac{0.104}{120} = 8.6 \times 10^{-5} \text{ A.} && (20) \end{aligned}$$

This is of course the theoretically minimum current. In fact, the real current load is much more because we haven't taken into account the inefficiency of the energy-transfer process between the electrons and the helium atoms and optical losses in the laser itself.