PHY140Y

Spring Term – Tutorial 24 Discussion Solutions

3 April, 2000

1. (a) The number of fissions (each releasing ΔE of energy) that must occur per second, R, in order to achieve a particular power rating P is

$$R_{235} = \frac{P}{\Delta E} \quad (1)$$

$$= \frac{3.2 \times 10^9}{(2 \times 10^8)(1.60 \times 10^{-19})} = 1 \times 10^{20} \text{ fissions/s.} \quad (2)$$

(b) Since each fission consumes 1 ^{235}U nucleus, with represents $f = 0.72\%$ of natural uranium, the total weight of uranium fuel that would be consumed in one year, W_{235}, would be

$$W_{235} = \left(\frac{R_{235} \times 365 \times 24 \times 60 \times 60}{f}\right)\left(\frac{238.03}{6.03 \times 10^{23}}\right) \quad (3)$$

$$= 1.73 \times 10^8 \text{ g}, \quad (4)$$

or about 173 tonnes of natural uranium consumed as fuel in one year.

(c) The rate of fuel consumption would decrease, since the ^{235}U fissions only in the presence of slow neutrons. Since the number of neutrons has been reduced by a factor of 20, the fuel consumption would also change by the same factor. In other words, the fuel consumption would drop to 8.64×10^6 g/year.

(d) For the same power output from ^{235}U fission, the amount of ^{238}U in the reactor core is smaller by the enrichment ratio:

$$\frac{0.72\%}{3.0\%} = 0.24. \quad (5)$$

Since ^{239}Pu is produced by neutron bombardment of the ^{238}U, the rate of production of this isotope in the General Electric reactor would only be 24% of the isotope production in the CANDU reactor.

2. (a) A blast of 150 kT would involve the fissioning of

$$N_{fission} = \frac{(150)(4.18 \times 10^{12})}{(1.60 \times 10^{-19})(2 \times 10^8)(0.30)} = 6.53 \times 10^{25} \text{ nuclei,} \quad (6)$$

assuming 30% yield. The mass of ^{235}U required would be

$$m_{235} = N_{fission} \times \left(\frac{235}{6.03 \times 10^{23}}\right) = 2.54 \times 10^4 \text{ g.} \quad (7)$$

If R is the radius of the pit, then

$$m_{235} = \frac{4}{3}\pi R^3 \rho_{235} \tag{8}$$

$$R = \left(\frac{3m_{235}}{4\pi\rho_{235}}\right)^{1/3} \tag{9}$$

$$= \left(\frac{3(2.54 \times 10^4)}{4\pi(18.7)}\right)^{1/3} = 6.9 \text{ cm.} \tag{10}$$

3. Since the half-lives of the principle isotopes of uranium are $t_{1/2,235} = 7.04 \times 10^8$ years and $t_{1/2,238} = 4.46 \times 10^9$ years, if we have a mass of uranium at today's isotopic content, we can extrapolate back 4.5×10^9 years to determine the ratio of isotopes then:

$$\frac{(0.0072)/2^{-(4.5\times10^9)/(7.04\times10^8)}}{(0.9928)/2^{-(4.5\times10^9)/(4.46\times10^9)}} = \frac{0.605}{2.00}. \tag{11}$$

Thus, the fraction of ^{235}U at the time the Earth was formed was

$$\frac{0.605}{2.605} = 23\%. \tag{12}$$