PHY140Y

1 Newton's Gravity

1.1 Overview

- A brief history of Newton
- Concept of a universal gravitational force
- Features of gravity

1.2 Newton's Law of Universal Gravitation

Our view of gravity evolved from a body of observation that goes back many millennia. The idea that there was something that attracted us to the solid earth is a concept that goes back much earlier than the 16th century (as implied by Wolfson and Passachoff.[1] Greeks talked about it. So did the Chinese. And my guess is that every culture that supported philosophical inquiry made contributions.

What makes the 17th century unique is that we can trace back our own understanding of gravity, and the empirical basis for it, to the work of specific natural philosophers or scientists, in particular the work of Nicolaus Copernicus, Tycho Brahe and subsequently to Galileo and Newton. Based on very detailed and precise observations by Brahe, Copernicus was able to summarize these data into three laws:

1. Planets orbit the sun in ellipses, with the sun at one focus.
2. The line joining the sun and earth sweeps out equal areas at equal times.
3. The square of the orbit's period, T, is proportional to the cube of the semimajor axis of its orbit, R, *i.e.*

$$T^2 \propto R^3. \tag{1}$$

I emphasize that these were "laws," with nothing behind them to explain why they were just so (see the end of this lecture for some comments on why I emphasize this point).

Galileo's principle contribution to this evolution in thought was the discovery that other planets have objects such as our moon. This observation indicated that the moon was not a unique object and that such bodies apparently orbitted a "parent" due to some mysterious attraction.

Newton made the leap when he realized that the effects of gravity as we see them on the surface of the earth could also explain the behaviour of the moon and its orbit. With his knowledge of mechanics (remember $F = ma$, etc.) and his own development of calculus (concurrent with

[1] From now on, I will refer to the primary text by the initials of the authors, **W&P**.

Lemarqe in France), he was able to show that a gravitational force between two *point particles* that had a magnitude of

$$F = \frac{Gm_1m_2}{r^2}, \tag{2}$$

explained it all, where, m_1 and m_2 are the masses of the two point particles, r is their separation, and G is some constant that depends on which units you choose. We call this the **law of universal gravitation**, though it is often called **Newton's law of gravity**.

The force is always attractive, and is lined up along the unit vector $\hat{r_{12}}$, where the vector $\vec{r_{12}}$ is the separation vector between the two particles (pointing at particle 2).

1.2.1 Radial Dependence

The $1/r^2$ depedendence of Newton's Law has been tested over distance scales from about 1 cm to several thousand light years by direct observation. However, at the time of Newton, this was a revolutionary idea, and one that had to be tested by measuring the force at short and long distances.

The $1/r^2$ dependence is not unique to gravity. The electrostatic force has the same dependence, and suggests a number of connections that we don't have time to go into here. But it does give gravity a number of unique properties, which we will see more clearly in the systems we will investigate later.

1.2.2 Mass Dependence

The mass-dependence of Newton's law is of interest in and of itself. The most interesting point is that the force of gravity is determined by the same *mass* as appears in Newton's second and third law of motion. This makes gravity in and of itself unique. The other forces in nature (namely, the electroweak and strong forces) depend on other properties of the object. How they react to a given force is then dictated by Newton's Laws of Motion

1.3 Uniqueness of G

A significant aspect of Newton's Law of Gravity is that G is hypothesized to be a *unique* number, so that the only property of an object that gravity cares about is the mass of the object. This is not necessarily the only way gravity could (and perhaps does work). For example, most of the mass around us is in the form of what we call baryons (protons and neutrons are the most ubiquitous) and leptons (electrons are in this class). Gravity might only act on baryons and not leptons. Since for the most part, the number of electrons is roughly proportional to the number of neutrons and protons in ordinary atoms, and the mass of electrons are about 1/2000th of that of a proton or neutron, it is possible that gravity could only work on baryons and the effect of this on Newton's Law would only be a slight shift in the value of G.

To test this, Eetvos performed a series of experiments where he measured the relative force of gravity on different substances. His results, although somewhat controversial more recently,[2] demonstrated that gravity really did appear to be universal and only cared about mass.

The most precise estimate of G is

$$G = (6.67259 \pm 0.00085) \times 10^{-11} \text{N m}^2/\text{kg}^2. \tag{3}$$

The method by which is performed will be discussed later.

1.4 Comparison with the Electric Force

In order to get some sense of how small the force of gravity is, it is perhaps easiest to compare it to the electrostatic force, which many of us should be familiar with. The force of electromagnetism is proportional to the electric charge on an object, and the electric field that the object is in. We can make a direct comparison using a pair of charged pith balls, as shown in Fig. 1, where we have deposited about equal amounts of static charge on each ball. If we let the F_E and F_G be the electrostatic and gravitational forces on the left-most ball, then the components of those forces perpendicular to the string must be equal and opposite for the pith ball to be motionless. Thus, we have

$$\begin{aligned} F_E \cos\theta &= F_g \sin\theta && (4) \\ \Rightarrow F_E &= mg\frac{\sin\theta}{\cos\theta} && (5) \\ &= mg\tan\theta, && (6) \end{aligned}$$

where θ is the angle of the string from the vertical. When we did this experiment, the pith balls were suspended about $d = 2$ cm apart while the string was about 20 cm long. Thus, we found that $\theta = 0.05$ (let's forget uncertainties for the moment!). Furthermore, the masses of the pith balls were $m = 2.7 \times 10^{-5}$ kg. Thus, we find

$$\begin{aligned} F_E &= (2.7 \times 10^{-5})(9.8)\tan(0.05) && (7) \\ &= 1.3 \times 10^{-5} \text{ N}. && (8) \end{aligned}$$

The corresponding gravitational force on the two pith balls is

$$\begin{aligned} F_G &= \frac{Gm^2}{d^2} && (9) \\ &= \frac{(6.67 \times 10^{-11})(2.7 \times 10^{-5})^2}{(0.02)^2} && (10) \\ &= 1.2 \times 10^{-16} \text{ N}. && (11) \end{aligned}$$

Thus, we see that the force of gravity is only about 10^{-11} times that of electromagnetism in this example, a profoundly large difference in strengths.

[2] His experiments have been reproduced, although not exactly in identical fashion, seemed to indicate the existence of a "fifth" force. Further studies have only confirmed Eotvos original conclusions and the difficulty of making precise tests of Newton's force at short ranges.

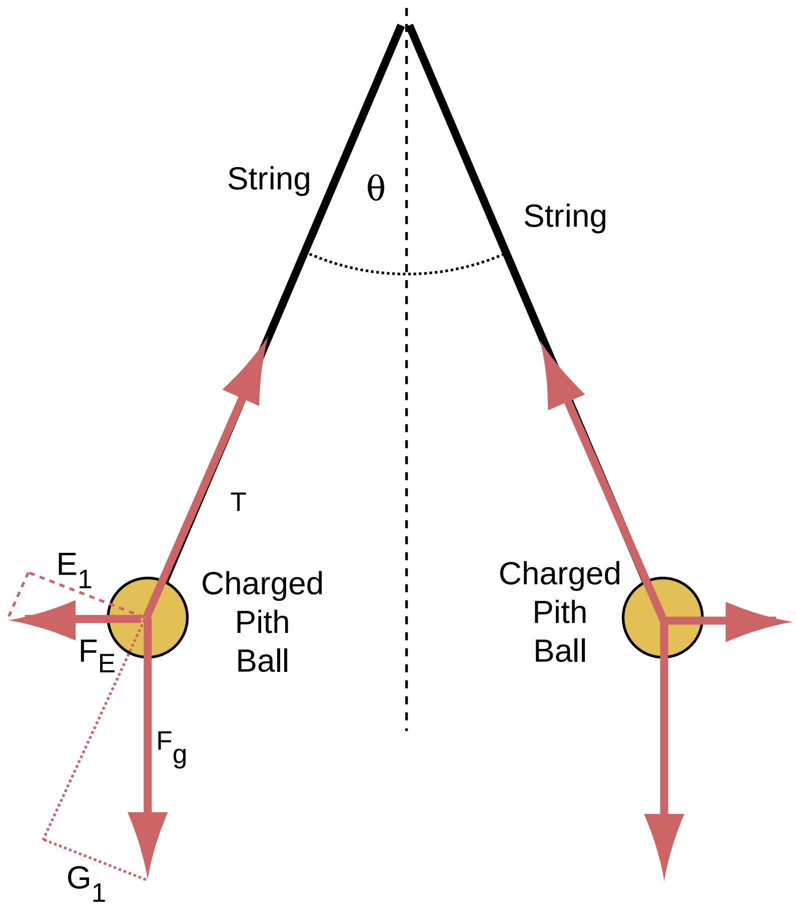

Figure 1: A pair of charged pith balls, suspended on strings. The force diagrams for each ball are shown, and the electric and gravitational forces, F_E and F_g, respectively, are resolved into components parallel and perpendicular to the string. The force of tension, T, balances the components of the other two forces in the same anti-parallel to it, while the forces perpendicular to the string, E_1 and G_1, are equal and opposite.

The bottom line is that gravity is a very weak force. The only reason it is so evident a part of our life is that we sit on an emormous mass known as the earth, and all that mass comes together to attract us to the earth's surface. It is, after all, the force that brought the earth together into the object that it is now.

1.5 Spherically Symmetric Objects

The law of universal gravitation only applies to point particles. However, the concept of a point particle with mass is something that is hard to conceive of, and is quite foreign to our everyday experience. We can show with a little bit of calculus that the this law holds even with extended objects, so long as these objects are spherically symmetric.

The calculus is straightforward, but laborious, and eventually leads you to consult tables of integrals if you try to solve this using the law of univeral gravitation. We'll be able to show this much more easily once we have introduced the idea of the gravitational potential energy.