PHY140Y

10 More on Gauss's Law

10.1 Overview

- Gauss's Law and Planar Charge Distributions

10.2 Planar Surface Charge

Another good example where Gauss's Law can be employed is to calculate the electric field associated with a uniformly charged large sheet, or a planar charge distribution. We can assume that the sheet is so large that we can ignore any edge effects. If the total charge on the sheet is Q and its surface area is A_{sheet}, we can define a **surface charge density**

$$\sigma \equiv \frac{Q}{A_{sheet}}, \tag{1}$$

where σ (the Greek letter "sigma") has units of C/m^2.

In this problem, since the symmetry is planar, the electric field must be perpendicular to the plane at all points (the normal to the sheet is the only unique direction in this problem that points away from the charge distribution). Although we know the electric field direction, we don't know its strength at any given distance z above or below the charged sheet. We do know, however, that the electric field at $-z$ (ie. below the sheet) must be the same strength as the field at z, again from symmetry.

Having established the symmetry properties of the electric field, to apply Gauss's Law, we have to carefully chose a Gaussian surface that allows us to determine the electric field strength $E(z)$ at a distance z from the surface of the sheet. A good choice is the "pill box" shown in Fig. 1. The electric field $\vec{E}$ is parallel to the sides of the cylinder and therefore perpendicular to the vector normal to the sides of the cylinder. Hence the flux of the electric field through the sides is zero. The only net flux is therefore through the top and bottom of the cylinders.

The total flux of the electric field can therefore be written as

$$\begin{aligned} \phi &= \int_S \vec{E} \cdot d\vec{A} && (2) \\ &= \int_{top} \vec{E}(z) \cdot d\vec{A} + \int_{bottom} \vec{E}(-z) \cdot d\vec{A} && (3) \\ &= (E(z)\hat{z}) \cdot (A_{top}\hat{z}) + (-E(-z)\hat{z}) \cdot (-A_{top}\hat{z}) && (4) \\ &= 2E(z)A_{top}, && (5) \end{aligned}$$

since the area of the top and bottom of the cylinders is equal, the electric field at $-z$ and the normal to the bottom of the cylinder are in the $-\hat{z}$ direction, and $|\vec{E}(z)| = |\vec{E}(-z)|$.

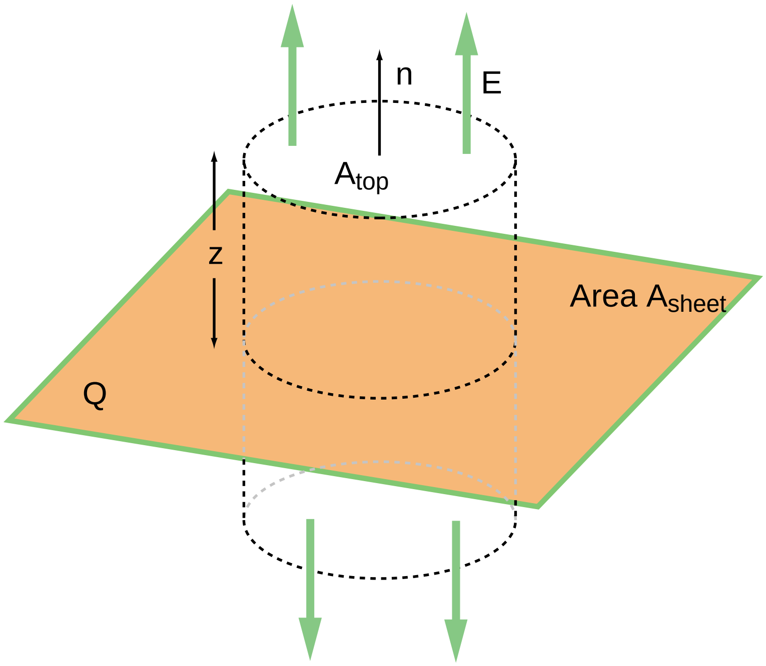

Figure 1: The calculation of the electric field associated with a large planar surface charge distribution. The sheet itself is considered so large that we can ignore any "edge-effects" that would ruin the planar symmetry. The "pill-box" Gaussian cylinder penetrates the sheet and extends a distance z above and below the sheet.

The charge enclosed in the Gaussian cylinder is the charge on the sheet that is contained in the cross-sectional area of the cylinder. This is just

$$\int_V \rho(\vec{r}')dV' = A_{top}\sigma, \tag{6}$$

so Gauss's Law tells us

$$\begin{aligned} \phi &= 2E(z)A_{top} && (7) \\ &= \frac{1}{\epsilon_0}A_{top}\sigma && (8) \\ \Rightarrow E(z) &= \frac{\sigma}{2\epsilon_0}. && (9) \end{aligned}$$

Check the units so that they make sense.

Note that this electric field doesn't depend on the distance you are away from the charged sheet, and the electric field strength is proportional to the surface charge density.