# PHY140Y

## 17 Beginnings of Quantum Mechanics

### 17.1 Overview

- Black Body Radiation
- Problems with the Classical Interpretation
- Planck's Quantization Hypothesis

### 17.2 Black Body Radiation

Where does "colour" come from? We see an object by being aware of the light that is reflected off it, which then forms an image on our retina. The "colour" of an object comes from the particular wavelengths of light that the object preferentially reflects. Thus, an object that looks red is one that reflects particularly strongly in the longer wavelength part of the spectrum and preferentially absorbs light of shorter wavelengths. Similarly, an object that looks white reflects visible light of all wavelenths equally strongly.[1]

Thus, a "black object" is one that reflects no light at all: It absorbs essentially all light, and we detect it's "blackness" by the **lack** of light reflected off its surface.

But what happens to objects that heat up? We find that, as is the case with an element on an electric stovetop, an object that is black will start to emanate visible light if it is hot enough. In fact, we have a rough idea of what sort of light corresponds to what temperatures. Glowing "red" implies that the element is hot, but glowing "white" implies that it is even hotter. Thus, if we heat up a black body to a sufficiently high temperature, it will give off light with a spectrum that is characteristic of the temperature of the object. This is what is known as a "black body spectrum" and early attempts to understand it provided the important first steps in the development of our modern theory of matter.

What constitutes a good black body? A black object such as a traditional cast iron frying is not such a bad approximation. However, perhaps the best black body is one that is formed from taking a hollow object and making a small hole in it. Any radiation that hits the hole will enter the interior of the object and bounce around till it is absorbed. It very likely would not re-emerge from the hole. Thus, the radiation that does emerge from the hole comes from the hollow interior of the container, and is perhaps the best practical "black body" that you can create in a laboratory.

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[1] It is useful to make the distinction between an object that has a mirror surface, which reflects light equally in all wavelengths and has a smooth surface, and a white surface, which reflects light equally in all wavelengths but whose surface is rough.

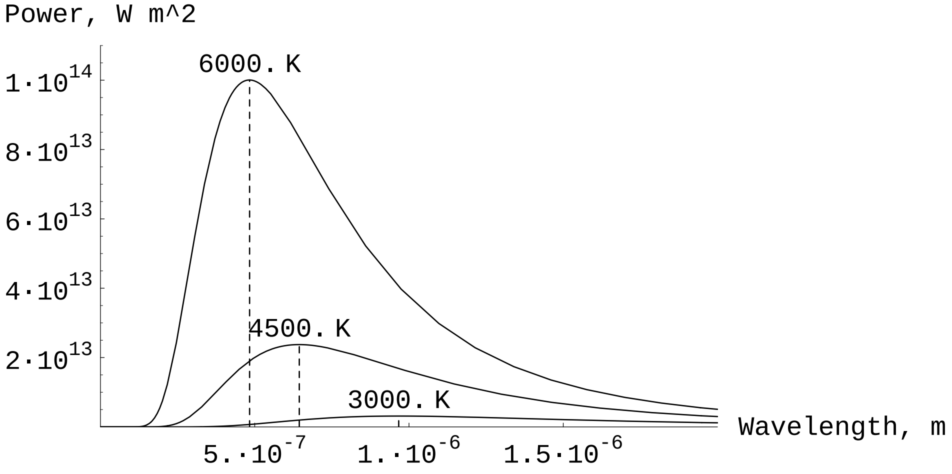

Figure 1: The spectral density for a black body with several different temperatures.

### 17.3 The Classical Interpretation

The early attempt to describe how an object can begin to give off radiation once it is hot enough tried to identify a mechanism by which matter could give rise to light waves with a spectrum characteristic of a black body. The initial attempts predicted the power that would be produced by a black body at a given wavelength, or what we call the spectral density $R(\lambda, T)$.

The exact definition of this function is that

$$R(\lambda, T)\, d\lambda \tag{1}$$

is the total power of light produced within the frequency interval $\lambda$ to $\lambda + d\lambda$. Experiments in the late 1800's had shown that the black body spectrum had a spectral density described by a parametrization

$$R(\lambda, T) \quad = \quad \frac{2\pi hc^2}{\lambda^5 \left(e^{hc/\lambda kT} - 1\right)}, \tag{2}$$

where the new constants are $h = 6.63 \times 10^{-34}$ J · s and $k = 1.39 \times 10^{-23}$ J/K. This spectrum is shown in Fig. 1 for a number of different temperatures.

The spectrum has a number of features:

- As the temperature increases, the power associated with the radiation also increases. The total power (which once can determine by integrating the spectral density) is given by the Stefan-Boltzmann Law

  $$P \quad = \quad \sigma A T^4 \tag{3}$$

  where $A$ is the surface area of the object and the constant $\sigma = 5.67 \times 10^{-8}$ W/m$^2$K$^4$.

- The peak of the power spectrum occurs at a wavelength $\lambda_{max}$ that satisfies the relationship

  $$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m K}. \tag{4}$$

  Known as the Wein Displacement Law, this was a reflection of the fact that as the temperature of the object increased, it emitted increasingly shorter wavelengths.

- The spectrum is universal. It only depends on the temperature of the black body and not on what it was made of.

The classical prediction was that

$$R(\lambda, T) = \frac{2\pi ckT}{\lambda^4}, \tag{5}$$

which was known as the Rayleigh-Jeans Law. This formula predicted that as the wavelength decreased, one would expect to find an increasing amount of power at these wavelengths, such that integrating over all wavelengths resulted in the prediction that any black body should be radiating with infinite power. This prediction was known as the "ultraviolet catastrophe."

Various attempts were made to understand what was wrong with this prediction. It seemed to be soundly based on the fundamentals of classical physics, where it was assumed that the light arose from the vibration between the atoms of matter, and that the number of possible frequencies rose rapidly with decreasing wavelength. It took a bold step to get out of this conundrum.

## 17.4 Planck's Quantum Hypothesis

Max Planck in 1900 suggested a new model that could describe the black body spectrum. He postulated that the vibrational energy states of matter had discrete energy levels given by

$$E_n = nhf, \tag{6}$$

where $n$ is a positive integer and $f$ is a fundamental frequency characteristic of the molecule that is vibrating. When a transition from a higher energy state to a lower energy state is made, a photon is released that has an energy equal to the difference in these discrete energy levels. The energy difference, $hf$, is considered a "quanta." With this hypothesis, one predicts the observed black body spectrum, including both the wavelength and temperature dependence. The new parameter $h = 6.63 \times 10^{-34}$ J s is called Planck's constant, and is a fundamental constant of nature.

Planck was reluctant to believe that he had the right theory given the need to assume these mysterious quanta of energy. However, subsequent developments made it clear that this was indeed the correct model. The reason this works correctly is that although the number of the discrete energy states does increase with energy, the probability that the object could be in such a high energy vibrational state decreases very rapidly with increasing energy. The problem with the classical treatment was that you could have a much larger number of energy states with similar energies, leading the ultraviolet catastrophe.

Planck was awarded the 1918 Nobel Prize in Physics for this breakthrough.