PHY140Y

20 Bohr Atom and Matter Waves

20.1 Overview

- Implications of the Bohr Atom
- Problems with the Bohr Model
- de Broglie and his Matter Waves

20.2 Implications of the Bohr Atom Model

Now that we have the energy levels of the hydrogen atom predicted by Bohr's model, namely

$$E_n = -\left(\frac{m_e k^2 e^4}{2\hbar^2}\right)\frac{1}{n^2}, \tag{1}$$

where $n = 1, 2, 3, \ldots$, we can predict how light might interact with the atom. In particular, we can now see that a hydrogen atom has a lowest energy state given by

$$E_1 = -\left(\frac{m_e k^2 e^4}{2\hbar^2}\right) = -13.6 \text{ eV}. \tag{2}$$

If an atom is in a higher energy state determined by its angular momentum quantum number n_2, then to make a transition to a lower energy state given by the quantum number n_1, the difference in energy is predicted to be

$$\Delta E = \left(\frac{m_e k^2 e^4}{2\hbar^2}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right). \tag{3}$$

If this energy is carried away by a photon of energy $E_\gamma = h\nu = hc/\lambda = \hbar c/(2\pi\lambda)$, we predict that the photon's wavelength should satisfy

$$\frac{\hbar c}{2\pi\lambda} = \left(\frac{m_e k^2 e^4}{2\hbar^2}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \tag{4}$$

$$\Rightarrow \frac{1}{\lambda} = \left(\frac{2\pi m_e k^2 e^4}{2c\hbar^3}\right)\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right). \tag{5}$$

The constant in the brackets is in fact equal to R_H, a triumph of Bohr's atomic theory. An energy level diagram of these transitions is illustrated in Fig. 1.

It looked like the theory of the atom had been solved.

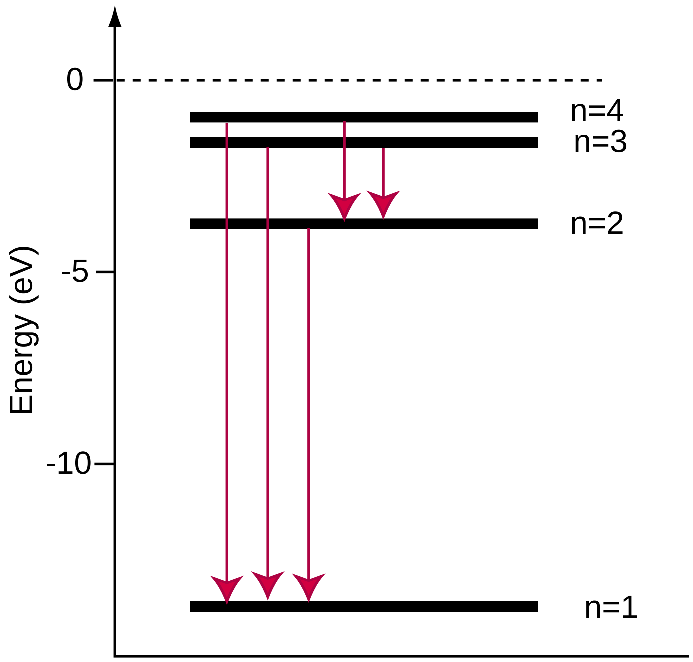

Figure 1: The energy level diagram for the Bohr atom, showing the first four energy levels and some of the transitions between these levels.

20.3 Problems with Bohr's Model

There were a number of problems with Bohr's model. The most difficult problem was that his model predicted that the electron would be experiencing a continuous centripetal acceleration. Measurements had shown that such an acceleration would cause the electron to give off radiation and lose energy, thereby falling into the nucleus. Since this clearly wasn't the case, there was something missing in the explanation.

Other problems also presented themselves. Bohr's model could not be readily extended to predict the behaviour of more complex atoms, even that of helium with only two electrons. It also could not predict some of the more subtle effects observed in the atomic energy states (for example, the behaviour of atoms in magnetic fields). Thus, there was ample concern that this was not the full picture.

20.4 de Broglie and Matter Waves

The unusual wave-particle duality of light and the partial success of the Bohr model led to speculation that matter might itself have some sort of wave properties. This line of thinking led to the postulate in 1923 by Louis de Broglie that matter itself had associated with it a wavelength. He suggested that the wavelength of a particle is related directly to its momentum, much in the same way that a photon's wavelength is related to its momentum. Specifically, he assumed

$$\lambda = \frac{h}{p}, \tag{6}$$

where p is the momentum of the particle.

This was on the one hand an incredible speculation, based largely on analogy with the photon than any clear observation. However, it did have one feature in that it gave one possible mechanism for why the Bohr model worked. If one required that orbit of an electron around an atom had to have a circumference that equalled an integral number of de Broglie wavelengths of the electron, one found that

$$\begin{aligned} 2\pi r &= n\lambda_e && (7)\\ \Rightarrow r\frac{h}{\lambda_e} &= n\hbar && (8)\\ \Rightarrow L = rp &= n\hbar, && (9) \end{aligned}$$

which is exactly the Bohr quantization condition that leads to the correct energy levels.

The de Broglie wavelength for a particle is typically very small. For example, an electron in a hydrogen atom with the atom in its lowest energy state would have a de Broglie wavelength of

$$\lambda_e = 2\pi r = 2\pi a_0 = 3.3 \times 10^{-10} \text{ m}. \quad (10)$$

The de Broglie wavelength of a well-thrown baseball (assume a mass of 150 g and a velocity of 45 m/s), one finds that its de Broglie wavelength is

$$\lambda_b = \frac{h}{p} = 9.8 \times 10^{-35} \text{ m}. \quad (11)$$

A very small length, indeed!