PHY140Y

29 Electron Spin and Angular Momentum

29.1 Overview

- Stern-Gerlach Experiment
- Electron Spin

29.2 Stern-Gerlach Experiment

The angular momentum of the wave functions of the hydrogen atom seemed to be well-defined by just allowing for solutions of Schrödinger's equation that had angular-dependence. How could this be tested? In the case of the radial solutions, they each had difference energies so the emission or absorption spectrum of the hydrogen atom was a good test of the theory.

It turns out that a study in 1922, known as the Stern-Gerlach experiment after the two physicists who performed it, already had provided some strong evidence for the angular momentum structure of the atom. A beam of silver atoms had been passed through a non-uniform magnetic field. The beam had split into several separate beams as it passed through the field. The interpretation of this experiment was that the silver atoms seemed to have at leat two different possible magnetic dipole moments.

29.2.1 Magnetic Dipole Moments

A magnetic dipole moment, you say? Since we haven't talked about magnetic forces, let me review a few basics of magnetism. A compass needle is the prototypical magnetic dipole. It has the property that it would like to align itself with the magnetic field around it, with one direction as being preferred over the other. Thus we can define a vector associated with the compass needle that we call the magnetic dipole moment. The magnitude of the dipole moment vector reflects the strength of the dipole, and is expressed as current × area, or in SI units as Ampere-metres^2. It turns out that a loop of electrical current also creates a magnetic dipole moment, as shown in Fig. 1, where the magnitude of the dipole moment is given by the current flow times the surface area subtended by the current loop.

A hydrogen atom in a quantum state with the electron having non-zero angular momentum around the proton forms such a "current loop." It turns out that the magnitude of the magnetic dipole moment in the z direction associated with this loop of current is given by

$$\begin{aligned} M_z &= \frac{e}{2m_e} L_z && (1) \\ &= \left(\frac{e\hbar}{2m_e}\right) m_l = 9.27 \times 10^{-24}\ \mathrm{A\,m^2} \equiv \mu_B\, m_l. && (2) \end{aligned}$$

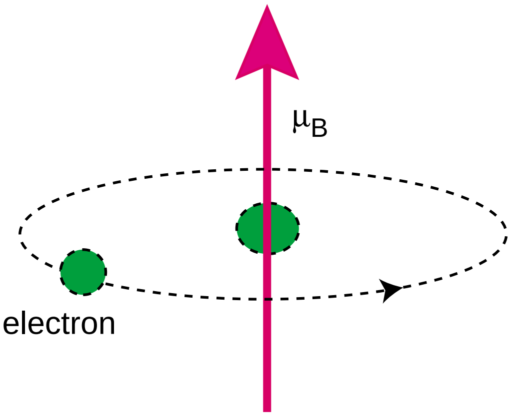

Figure 1: The magnetic dipole moment created by a loop of electrical current, similar to what would be formed by an electron orbitting a proton.

The quantity μ_B is known as the ‘Bohr magneton” and it sets the scale for the size of the magnetic dipole moments associated with the electron’s motion in an atom.

A final note about magnetic dipole moments is that when placed in a non-uniform magnetic field, the dipole moments will experience a force that accelerates them in the direction that the magnetic field is changing. This is the effect used in the Stern-Gerlach experiment to measure the magnetic moments of an atom.

29.2.2 The Experiment

As mentioned earlier, the Stern-Gerlach experiment takes a beam of atoms and passes them through a non-uniform magnetic field. What was in fact expected was that if an atom had a magnetic dipole moment, it would be oriented in essentially every possible way and therefore each atom would be accelerated a different amount in the magnetic field. Thus, the beam would just “smear” out in the direction defined by the gradient of the magnetic field.

What was observed was quite startling, as the atomic beam was split into only a few discrete separate beams. The experiment was repeated with hydrogen atoms in 1927, and the same effect was apparent. Here, only two beams were produced, with about half of the hydrogen atoms being accelerated in one direction with a given force, and the other half being accelerated in the opposite direction. Given that these were hydrogen atoms in their ground state $n = 0$, what would you have expected for this experiment?

29.3 Electron Spin

Since the Stern-Gerlach experiment showed that the hydrogen atom seemed to come with two possible magnetic dipole moments, and it couldn’t be ascribed to the orbital angular momentum of the electron relative to the proton (since the hydrogen was in the ground state), the only other explanation was that the electron (or possible the proton) had a magnetic dipole itself. This implied that we had to think of the electron as having its own internal angular momentum, or “intrinsic spin.”

The Stern-Gerlach experiment gave a measure of the size of the magnetic dipole moment, and therefore the size of the intrinsic spin of the electron. It turned out that the spin along the $\hat{z}$ axis needed to be equal to $\hbar/2$. In order for quantum mechanics to accommodate this particular value, it required the electron to have a total angular momentum

$$\begin{aligned} |\vec{S}| &= \sqrt{s\,(s+1)}\hbar && (3) \\ &= \frac{\sqrt{3}}{2}\hbar, && (4) \end{aligned}$$

where we now have to consider the electron to have an angular momentum quantum number of $s = 1/2$. This means that any electron can be found in two possible internal spin states, one state with the z component of the spin aligned in the $+\hat{z}$ direction and the other aligned in the $-\hat{z}$ direction. These two configurations are often labelled “spin up” and “spin down” states.