PHY140Y

31 Implications of Total Angular Momentum

31.1 Overview

- Energy Considerations
- Transition Rules

31.2 Energy Considerations

We now know that the hydrogen atom can come in states of a given orbital angular momentum specified by the orbital quantum number l, and a given total angular momentum specified by the total angular momentum quantum number j. This has two important effects.

First, let's take a look at the energies of the two different configurations that give us two possible values of j for each value of l (this only occurs when $l \neq 0$). In one case, the electron's intrinsic spin is pointing in the same direction as the orbital angular momentum leading to the largest value of $j = l + 1/2$. If we now look at the electron in its rest frame, we see that the electron would see the proton orbiting around it in a counter-clockwise manner as shown in Fig. 1. This orbiting charge would create a current, and associated with that current would be a magnetic field pointing in the opposite direction to the electron spin vector. Since the electron has a magnetic dipole moment, the presence of the magnetic field pointing in the opposite direction causes this configuration to have a higher energy than would be predicted by the Schrödinger equation alone.

Similarily, the other orientation of angular momentum and spin would give us a spin vector pointing downward in the electron rest frame, yielding a quantum state with lower energy than otherwise expected.

How large is this effect? It turns out that for the $n = 2$ and $l = 1$ states of the hydrogen atom, the 4 quantum states with $2P_{3/2}$ and the two quantum states with $2P_{1/2}$ will have an energy difference $\Delta E = 5 \times 10^{-5}$ eV. This is about 10^5 times smaller than the energy of the states themselves, leading to a a very small "splitting" in the energy levels of these states. This splitting is shown in the energy level diagram in Fig. 2, and is known as the "fine structure" of the hydrogen atom spectrum.

31.3 Transition Rules

The second implication of these total angular momentum states is that we can now incorporate our knowledge of angular momentum when it comes to looking at "optical transitions" of these states, ie, the transition of the hydrogen atom from an excited state to a lower energy state by emitting a photon.

The photon itself also has an intrinsic spin equal to $j = 1$ – ie., it has total angular momentum $|\vec{J}| = \sqrt{j(j+1)}\hbar = \sqrt{2}\hbar$. Since the total angular momentum has to be conserved when a hydrogen

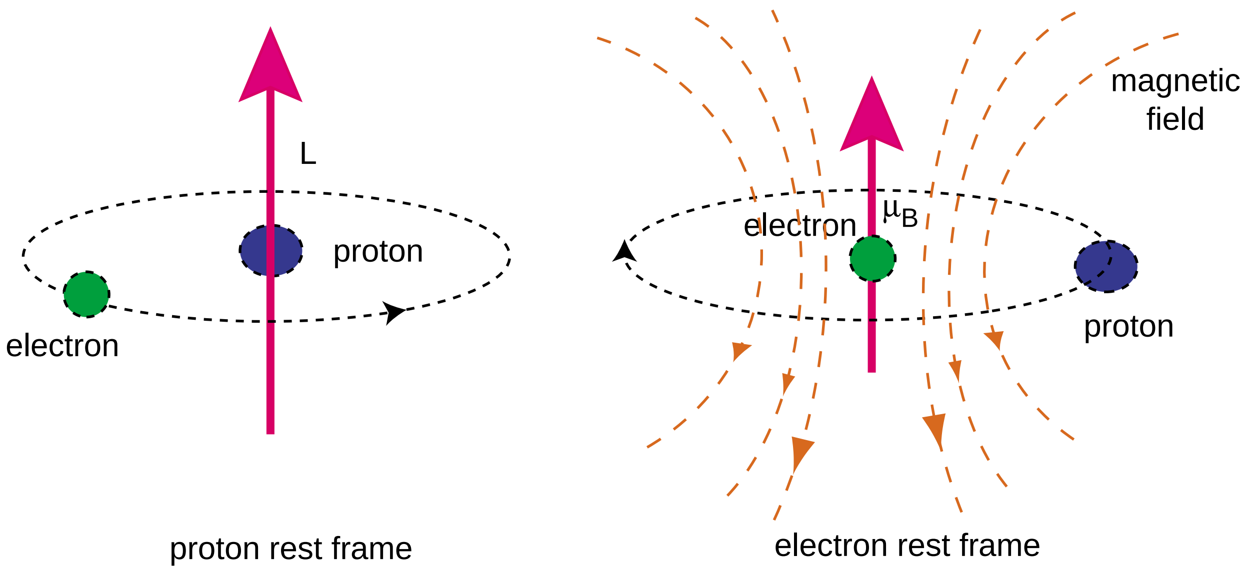

Figure 1: An electron in a state where $\vec{L}$ and $\vec{S}$ are aligned with each other in the proton rest frame, and the same configuration as seen in the electron rest frame.

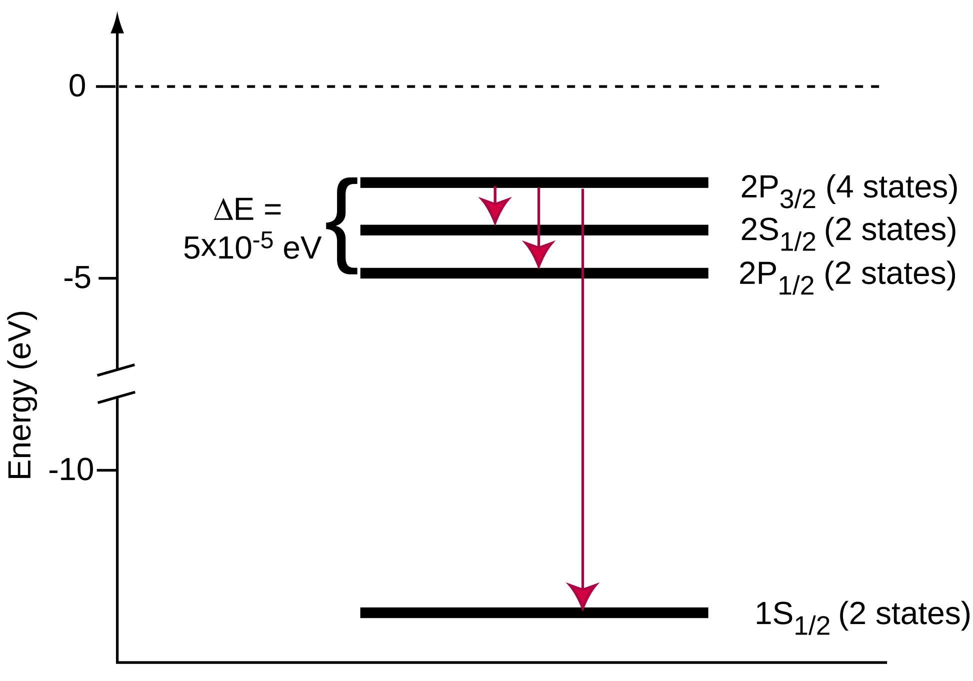

Figure 2: The energy level diagrams for the $n = 1$ and $n = 2$ states of the hydrogen atom showing (in a distorted scale) the spin-orbit splitting.

atom emits a photon, that means that the total angular momentum of the state that the hydrogen atom decays to must differ from the initial total angular momentum by one unit of $\hbar$, ie., $\Delta J = 1$. This means that there are only certain transitions allowed between all the quantum states. Fig. 2 shows the allowed transitions between the $n = 1$ and $n = 2$ excited states of the hydrogen atom.