

PHY140Y

32 The Pauli Exclusion Principle

32.1 Overview

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32.2 Pauli Exclusion Principle

Our focus on the hydrogen atom should have given some insight into how one electron can interact with a proton. However, the behaviour of multi-electron systems was quite enigmatic. What was observed was that multi-electron atoms seemed to have periodic properties. As you added electrons to the atom, one found that the electrons appeared to occupy increasingly higher-energy “shells,” with two electrons per shell. This led to the following principle: You could not have two or more electrons occupying the same quantum state.

Known as the “Pauli Exclusion Principle,” this ansatz proved to be right on the mark. It turns out that it not only applies to electrons, but to any set of identical fermions. It was first observed in electronic systems because of the ubiquitous nature of atoms. However, it has been shown to be true for all sorts of fermionic systems that it forms a fundamental aspect of quantum mechanics.

There is in fact a more fundamental reason for why the Pauli Exclusion Principle is required. It has to do with the quantum mechanics description of spin states, and the properties of the associated wave function. What we can show is that for any two fermions, the wave function that describes their quantum state must actually be multiplied by -1 if we interchange the two fermions. However, since these two fermions are identical, if the two fermions were in the same quantum state, we would have just the same state as before. However, we would not have the same wave function **unless** the wave function for this two fermion system was identically zero! Thus, we see that the probability of having two fermions in the same quantum state must be zero as well. QED.

32.3 Example: The Periodic Table

As the most obvious concrete example, let’s look at some of the lightest atoms and see what Pauli’s Exclusion Principle implies about the quantum numbers (or “electronic structure”) of the electrons in these atoms. These are shown in Table 1. We see that as we add electrons to the atom, higher electronic quantum states are occupied. The exact order in which these states are filled has to do with the energy of the state: in general, the lowest-energy states are occupied first.

This sort of filling process explains the regularity in the chemical properties of the atoms, which are defined by the electronic structure. For example, all the alkali metals (eg., Li, Na, K) have one electron in their S shell. Thus, they prefer to “give up” this electron to another atom if it

Table 1: The electronic configuration of some of the lowest lying atoms.

Atomic Number	Electronic Structure
1	$1S^1$
2	$1S^2$
3	$1S^2 2S^1$
4	$1S^1 2S^2$
5-10	$1S^1 2S^2 2P^{N-4}$

helps fill a shell in the other atom, making them highly covalent and willing to form strong bonds with other elements. Similarly, the Noble gases all have filled electron shells and therefore are very unreactive chemically.

32.4 Example: Neutron Stars

Another consequence of the Pauli Exclusion Principle is the behaviour of neutron stars. The standard life cycle of a star has a number of stages. Near the end of its life, as it consumes the last of its hydrogen fuel, it begins to consume heavier elements, creating in successive steps, He, Be, Li, etc. Eventually, it no longer is generating sufficient power to support the weight of its atmosphere and it begins to shrink. If the mass of the star is somewhere between about 1.4 and 5 times the mass of our sun, then this contraction process does not end with the sun contracted into a white dwarf or similar object, but will continue until all the electrons and protons in the nuclei of the atoms fuse and form a ball of neutrons. This object is said to be a “neutron star.”

These neutrons are also spin-1/2 particles, which implies that they also have to obey the Pauli Exclusion Principle. This means that these neutrons are in all different energy states, some of which can be quite energetic. They give the neutron star a number of unusual properties. Too bad these are virtually impossible with today’s technology to observe. They are truly unusual objects.