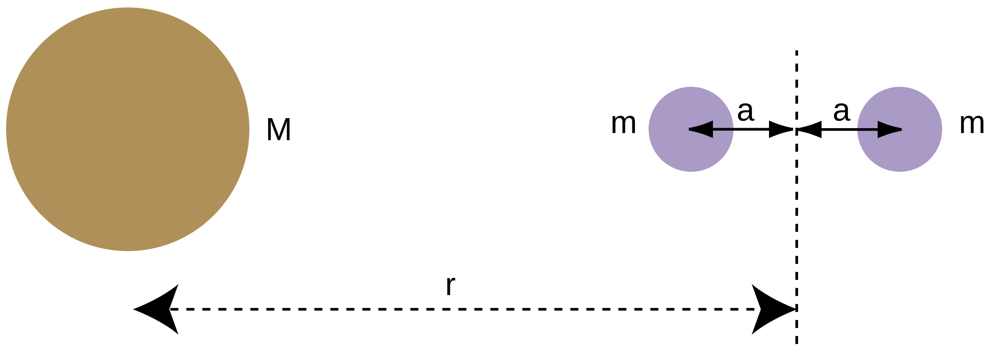

Figure 1: A conceptual example illustrating the effect of tidal forces. The diagram shows a large mass M and two smaller masses m that are experience different gravitational acceleration.

PHY140Y

6 Tidal Forces and the Equivalence Principle

6.1 Overview

- Tidal Forces
- Equivalence Principle

6.2 Tidal Forces

Since the gravitational field $\vec{g}(\vec{r})$ can be non-uniform, an extended object will experience different gravitational accelerations at different points. These differential accelerations can cause stresses inside an object, and lead to some unusual phenomena.

Let's take a simple example: Suppose we have a large spherical mass M, and then take two smaller spheres, each of mass m and lined up along side each other, as shown in Fig. 1. Let's work out the relative acceleration felt be the two small spheres due to the mass M.

This would be

$$\begin{aligned}
\Delta\vec{g} &= \vec{g}(\vec{r_1}) - \vec{g}(\vec{r_2}) && (1)\\
&= -GM\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{r} && (2)\\
&= \frac{-GM}{r^2}\left[\frac{1}{(1-a/r)^2} - \frac{1}{(1+a/r)^2}\right]\hat{r}. && (3)
\end{aligned}$$

We can use a Taylor Series expansion to approximate the terms in this last expression, ie.

$$(1 - a/r)^{-2} = 1 + 2\frac{a}{r} + O\left(\left[\frac{a}{r}\right]^2\right) \tag{4}$$

$$\simeq 1 + 2\frac{a}{r} \quad \text{and} \tag{5}$$

$$(1 + a/r)^{-2} \simeq 1 - 2\frac{a}{r}. \tag{6}$$

With this, we get

$$\Delta\vec{g} \simeq \frac{-GM}{r^2}\left(1 + 2\frac{a}{r} - 1 + 2\frac{a}{r}\right)\hat{r} \tag{7}$$

$$= \frac{-GM}{r^2}\left(\frac{4a}{r}\right)\hat{r}. \tag{8}$$

We see that this differential force – what we call the "tidal force" exerted by the mass M is the gravitational force multiplied by the factor $4a/r$. Let's look at how we can now understand the effect of the moon on the surface of the earth. First, we note that the gravitational acceleration of the Moon acting on the Earth at the centre of the Earth is

$$\vec{g_M} = \frac{GM_M}{r_M^2} = 3.31 \times 10^{-5}\ \text{m/s}^2. \tag{9}$$

If we now calculate the difference in the moon's gravitational field on opposite sides of the Earth (ie., what we would call the tidal force of the moon), we find

$$\Delta g = \vec{g}\left(\frac{4r_E}{r_M}\right) \tag{10}$$

$$= \vec{g_M}\,(0.032)\,, \tag{11}$$

so the tidal acceleration is 3.2% of the total gravitational acceleration experienced by the Earth. This is a rather small force difference. However, given that 3/4 of the Earth is covered by a fluid, this subtle acceleration can cause quite a significant amount of "sloshing" of water.

In particular, if we think of the water as free to move about on the surface of the Earth, we see that the water on the side nearest the Moon will feel a stronger attraction towards the moon than the Earth itself, resulting in a "bulge" of water on that side, and creating what we call a high tide. On the opposite side of the Earth, we find that the water is feeling less of a gravitational acceleration than either the Earth itself or the water on the side facing the Moon. Thus, this creates a second bulge of water and creates a second high tide. Since the Earth rotates once every 24 hours, while it takes the Moon 27 days to rotate once, these two bulges of water create two high tides in a period slightly less than 24 hours.

As an exercise, how large are the tides associated with the Sun?

6.3 Equivalence Principle

There is no apparent connection between the two types of masses that you have now been introduced to, namely the "inertial mass" found in

$$\vec{F} = m_i\vec{a}, \tag{12}$$

and the gravitational mass

$$\vec{g} \;=\; \frac{-Gm_g}{r^2}\hat{r}. \tag{13}$$

Observationally, these two concepts of mass seemed to coincide. The most precise demonstration of this has been credited to Baron von Eötvös, who showed in 1909 that for a large variety of materials, the ratio m_i/m_g was the same to within a few parts in a billion.

However, Einstein had pursued this mystery in his development of a more complete theory of gravity. It was already known in the late 19th century that the law of universal gravitation didn't correctly predict the behaviour of Mercury's orbit, which slowly precessed. Einstein made the bold stroke that the reason that m_i and m_g were the same is that the force of gravity is really a distortion of space-time, and that what we saw as a curved path of an object in a gravitational field was really the result of the object taking the shortest path from one point to the next in a universe that had curvature in it. In this way, he was able to identify the inertial mass and the gravitational mass as one and the same quantity, with both being a measure of the energy associated with a given type of matter.

Where does this curvature arise? It comes from the presence of mass itself. If we have a large mass concentrated in one place, space-time will be more curved in the region around this mass, and will therefore create a larger apparent distortion in the path of an object passing near the mass.

An implication of Einstein's theory is that it isn't actually mass that matters, but energy. Thus, particles of light, which we know to be massless, still had energy and thus could be affected by this curvature. The observation by Sir Eddington in May 1919 of the bending of starlight in the sun's gravitational field[1] was the first of many triumphs of Einstein's theory of general relativity that have reinforced our confidence in this idea of gravity.

[1]The Eddington expedition was already planned in 1917, when it was realized that the May 1919 solar eclipse would occur at a time when there would be a field of bright starts right behind the Sun whose light would pass very near the Sun and would therefore be most sensitive to any bending. Eddington considers his observations that month to have been the most important contribution he made to science!