PHY140Y

8 Electric Potential

8.1 Overview

- Electric Potential
- Potential of a Uniform Field and a Point Charge
- van de Graaff Generator

8.2 Electric Potential

Because two charges act on each other with an electrostatic force, when a charge q_t (we will refer to this as a "test charge" as it is being used to probe the strength of the electric field) is moved from point $\vec{r_1}$ to point $\vec{r_2}$, the electric field will do work W on the test charge. The energy that represents the work done comes from somewhere–we atrribute it to the electric field. Thus, the electric field has associated with it a potential energy.

The work done W by the electric field in moving the test charge is therefore associated with the negative of the difference of the potential energy between the points $\vec{r_2}$ and $\vec{r_1}$. Thus,

$$\begin{aligned} W &= \int_{\vec{r_1}}^{\vec{r_2}} \left(\vec{F} \cdot \vec{dl}\right) & (1) \\ &= -\left(U(\vec{r_2}) - U(\vec{r_1})\right) & (2) \end{aligned}$$

is the expression for the work done and its relationship with the change in the potential energy $U(\vec{r})$. For this test charge, we can rewrite this as

$$\begin{aligned} U(\vec{r_2}) - U(\vec{r_1}) &= -\int_{\vec{r_1}}^{\vec{r_2}} \left(\vec{F} \cdot \vec{dl}\right) & (3) \\ &= -q_t \left[\int_{\vec{r_1}}^{\vec{r_2}} \left(\vec{E} \cdot \vec{dl}\right)\right]. & (4) \end{aligned}$$

The term in the square brackets depends only on the electric field itself and is what we call the **electric potential**. It has units of Joules per Coulomb, which we introduced earlier as the Volt.

I note that the electric potential does not depend on the path taken between the two points – you always get the same answer for the potential regardless of which path you have taken to evaluate it. This is a general property of **conservative forces**.

8.2.1 Uniform Field

In a uniform electric field $\vec{E}$ (ie., $\vec{E}(\vec{r})$ is a constant vector independent of $\vec{r}$), the electric potential difference between two points is

$$
\begin{aligned}
V(\vec{r_2}) - V(\vec{r_1}) &= -\int_{\vec{r_1}}^{\vec{r_2}} \left(\vec{E} \cdot \vec{dl}\right) && (5) \\
&= -\vec{E} \cdot \int_{\vec{r_1}}^{\vec{r_2}} \vec{dl} && (6) \\
&= -\vec{E} \cdot (\vec{r_2} - \vec{r_1}) \,. && (7)
\end{aligned}
$$

Since the electric field is constant, we can take it outside the integral, which then becomes the vector difference between the two points.

8.2.2 Point Charge

If we place a point charge q at the origin, we can calculate the electric potential between two points $\vec{r_1}$ and $\vec{r_2}$. To do this, we choose a path over which it is most convenient to calculate the path integral

$$
\begin{aligned}
V(\vec{r_2}) - V(\vec{r_1}) &= -\int_{\vec{r_1}}^{\vec{r_2}} \left(\vec{E} \cdot \vec{dl}\right) && (8) \\
&= -\int_{\vec{r_1}}^{\vec{r_2}} \frac{kq}{r^2}\,\hat{r} \cdot \vec{dl}. && (9)
\end{aligned}
$$

The most "convenient" path is one where we move from $\vec{r_1}$ radially to the radius of $\vec{r_2}$ and then move perpendicular to $\hat{r}$ to reach $\vec{r_2}$. The second part of the path integral is zero since the electric field is perpendicular to the $\vec{dl}$ along it. The first integral gives us

$$
\begin{aligned}
V(\vec{r_2}) - V(\vec{r_1}) &= -\int_{r_1}^{r_2} \frac{kq}{r^2}\,\hat{r} \cdot \hat{r} dr && (10) \\
&= -\int_{r_1}^{r_2} \frac{kq}{r^2}\, dr && (11) \\
&= \left[\frac{kq}{r}\right]_{r=r_1}^{r=r_2} && (12) \\
&= \frac{kq}{r_2} - \frac{kq}{r_1}. && (13)
\end{aligned}
$$

This means that the position dependence of the electric potential can only be

$$
V(r) = \frac{kq}{r} + C \qquad (14)
$$

where C is some constant that doesn't have any r dependence. We find it convenient to define $C = 0$ in most of our calculations, which implies that $V(r) \to 0$ when $r \to \infty$ and $V(r) \to \infty$ when $r \to 0$.

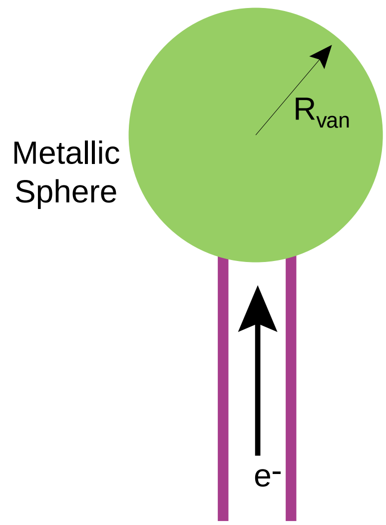

Figure 1: A schematic of a van de Graaff generator, showing the charged hollow sphere sitting on an insulated stalk. Electrons are placed on the sphere by a motor-driven belt (not shown).

8.3 Example: van de Graaff Generator

An interesting system is the van de Graaff generator, which can create electric potentials as large as 400 000 V (at least the one we use for demonstrations). Figure 1 illustrates schematically a van de Graaff generator, showing that it consists of a metallic sphere that is charged up by having electrons transferred to it via a belt driven by a motor. Since the metallic surface is approximately spherical, the charge on it forms an approximately spherical charge distribution. We showed that in the case of gravity, a spherical mass distribution generates a force of gravity that appears to act from the centre of the spherical distribution. A spherical charge distribution has the same property, something that one can show in exactly the same way as we showed this for gravity. Thus, the electric potential associated with the van de Graaff generator will behave like the case of the point charge we examined in the previous section.

Thus, if $V_{van} = 400\ 000$ V, then the charge Q on the metallic sphere of the generator will satisfy

$$V_{van} = \frac{kQ_{van}}{R_{van}}, \tag{15}$$

where Q_{van} is the total charge on the sphere and R_{van} is the radius of the sphere (note that Eq. 15 also holds for any radius larger than R_{van}). From Eq. 15, we can solve for the charge

$$\begin{aligned} Q_{van} &= \frac{V_{van}R_{van}}{k} && (16) \\ &= \frac{(4\times 10^5)(0.1)}{9\times 10^9} = 4.4\times 10^{-6}\ \text{C}. && (17) \end{aligned}$$

This is actually a fair amount of charge.

After the lecture, someone came up to me and asked why a van de Graaff generator doesn't work well in a damp climate. Can you figure it out? Hint: Does the sphere on the van de Graaff generator remain charged? How does charge leak away from it?