Top Mass Measurement

- Introduction
  - Importance of measuring the top quark mass
  - Systematics, statistics and combinatoric issues
- Top mass reconstruction: the χ^2 method
  - Taking the best χ^2
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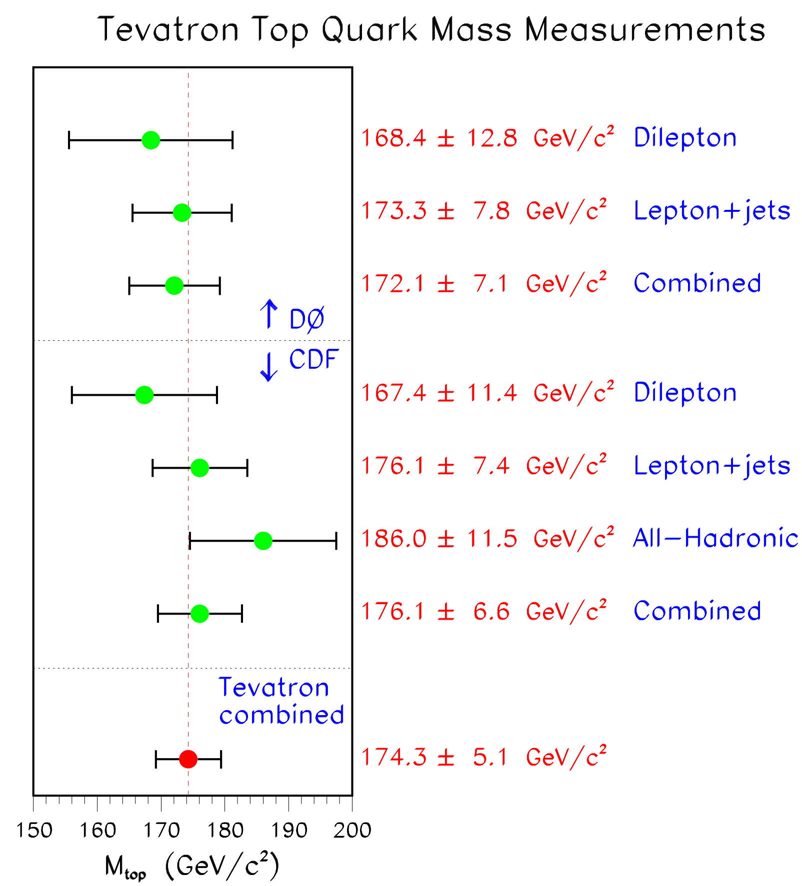
  - Weighting by best χ^2 - 2nd best χ^2
  - Weighting all χ^2 combinations
- Top mass reconstruction: maximum likelihood
  - Full $|M|^2$ integration over reduced phase space
- Conclusion

Introduction: the top quark in the Standard Model

- SU(2) isodoublet partner of the bottom quark
- Charge 2/3
- Spin 1/2
- Much heavier than other quarks

  ⇒ Decays to real W
- Short life time: no time to hadronize
- Small sample ⇒ experimental knowledge limited

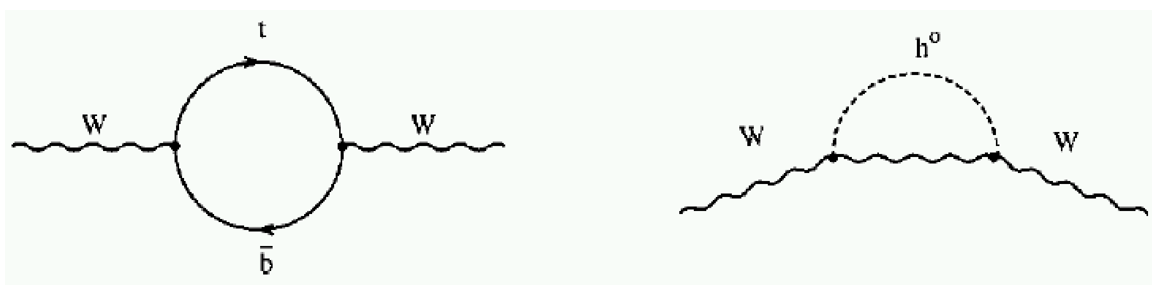


Introduction: importance of measuring the top quark mass

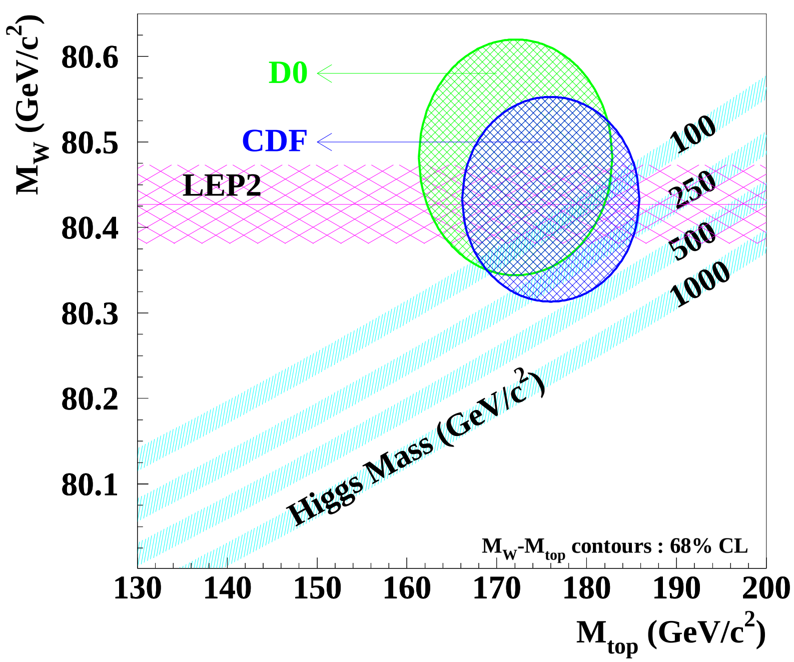
- Mass close to the scale of electroweak symmetry breaking
- Yukawa coupling: $M_{top} = \frac{v}{\sqrt{2}}$ within 3%

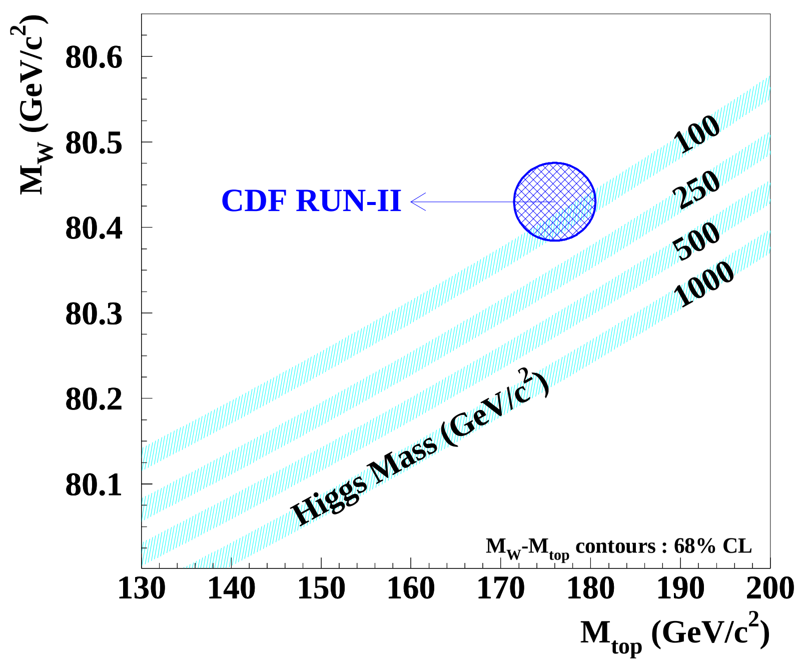
  $\Rightarrow$ Does the top quark play a role in ESB?

- A precise M_{top} measurement (along with a precise W mass measurement) can constrain the Higgs mass through radiative corrections of the W propagator:



$\Rightarrow$ Once (if) we have Higgs mass, M_{top} tests consistency of SM





- We can do better than has been done before (Run 1 uncertainty is $\sim 3\%$) to improve our knowledge of an important particle's properties.
    - One of the SM's 19 arbitrary parameters
    - Predicted parameter of other models

Introduction: $q\bar{q} \to t\bar{t} \to W^+bW^-\bar{b}$

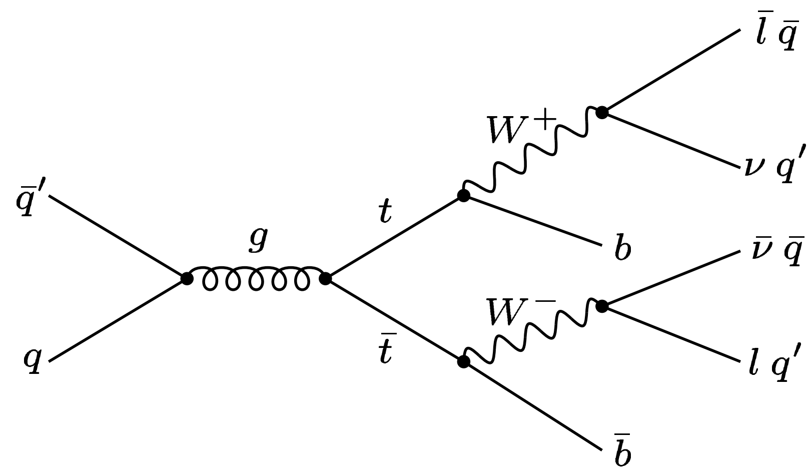
- Standard Model predicts dominant $q\bar{q}$ anihilation is dominant production process
- SM predicts branching fraction of $t \to Wb$ close to 100%
- W can decay as $q\bar{q}$ or $l\nu$

⇒ 2 leptons + ≥ 2 j: $\sim 5\%$

⇒ ≥ 6 j: $\sim 44\%$

⇒ Lepton + ≥ 4j: $\sim 30\%$

- Possible contribution from hard ISR or FSR



- One or two b jets can be identified with two b-tagging methods:
  - SVX: secondary vertex reconstruction
  - Soft lepton tagging: semileptonic B meson decay

Introduction: Systematics, statistics and combinatoric issues

Systematics: Jet energy corrections, ISR and FSR, background shape, etc. Could be reduced from $5.3\,GeV/c^2$ to $3.1\,GeV/c^2$ in Run II. See Jean-François' talk!

Statistics: Could be reduced from $4.8\,GeV/c^2$ to $1\,GeV/c^2$ in Run II

Combinatorics issues:

- No b tag: 24 top reconstruction combinations
- One b tag: 12 top reconstruction combinations
- Two b tag: 4 top reconstruction combinations

$\Rightarrow$ Uncertainty can be further decreased by improving how we deal with combinatorics

Top mass reconstruction: the χ^2 method

- We assume four highest E_T jets associated with 4 quarks
- ν reconstructed with $\not\!\!E_T$ and by constraining $M_{l\nu} = M_W$
- The following χ^2 function is minimized for each of the 24 combination:

$$\chi^2 = \sum_{l,jets} \frac{(\hat{P}_T - P_T)^2}{\sigma^2_{P_T}} + \sum_{i=x,y} \frac{(\hat{U}'_i - U'_i)^2}{\sigma^2_{U'_i}} + \frac{(M_{l\nu} - M_W)^2}{\sigma^2_{M_W}} + \frac{(M_{jj} - M_W)^2}{\sigma^2_{M_W}} + \frac{(M_{l\nu j} - M_t)^2}{\sigma^2_{M_t}} + \frac{(M_{jjj} - M_t)^2}{\sigma^2_{M_t}}$$

- M_t distributions compared to MC template for each input M_t
- Goal: Optimize the use of the information taken from the sample to improve the M_t measurement

  $\Rightarrow$ 4 strategies are explored

1: Take best χ^2

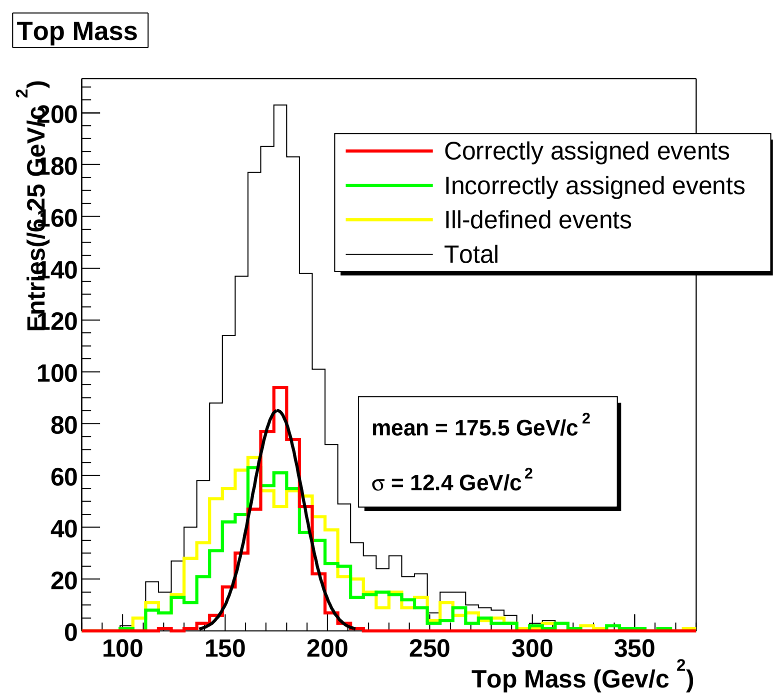
Only best χ^2 combination of each event is considered. Event cut if $\chi^2 > 10$

- 5th jet complicates problem:

  $\Rightarrow$ Event cut if 5th jet $E_t > 15GeV$

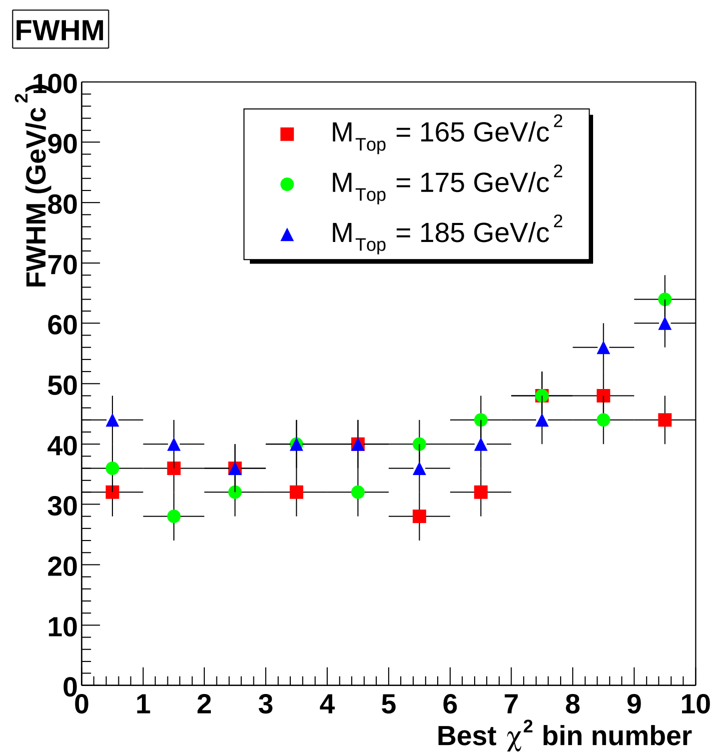
  $\Rightarrow$ 5th jet ignored if $E_t < 15GeV$

- The fitter choosed the right combinations: 23%
- The fitter failed to choose the right combination: 35%
- The fitter's entry requirement is not met: 42%



2: Weight each event according to best χ^2

- Consider only best χ^2 of each event
- Separate the sample in terms of best χ^2 bins
- Look if the generated input mass influences the mass distribution
- For a given best χ^2 in a given bin, the mass is adjusted according to the mean difference with the input mass (ΔM)
- The width of the distribution determines the uncertainty in a given bin (FWHM)

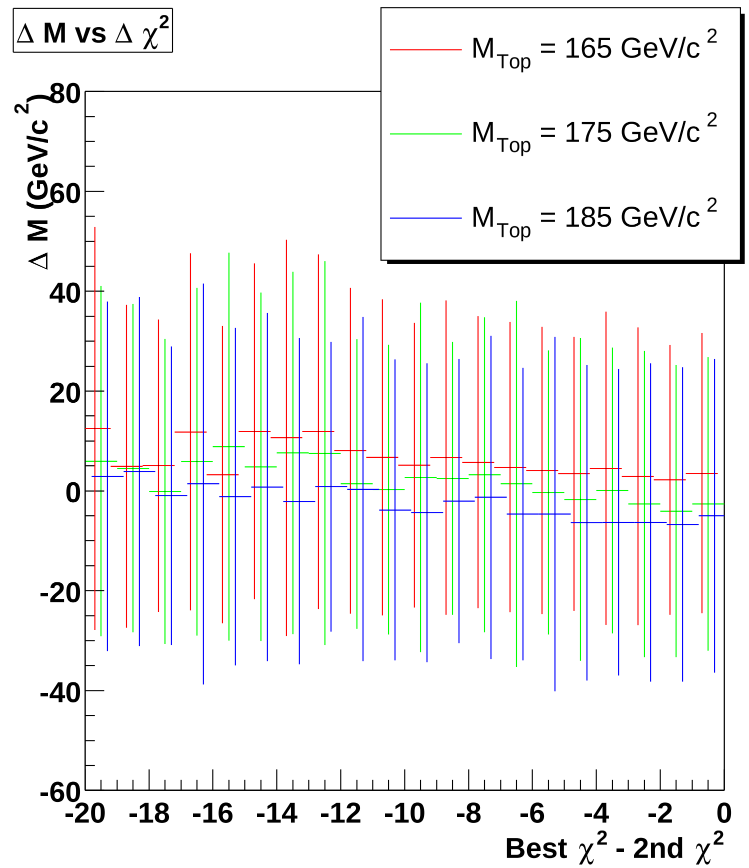


3: Weight each event according to difference between best χ^2 combination and other χ^2 combination

- Consider how much better is the best χ^2 compared to other χ^2 in a each event

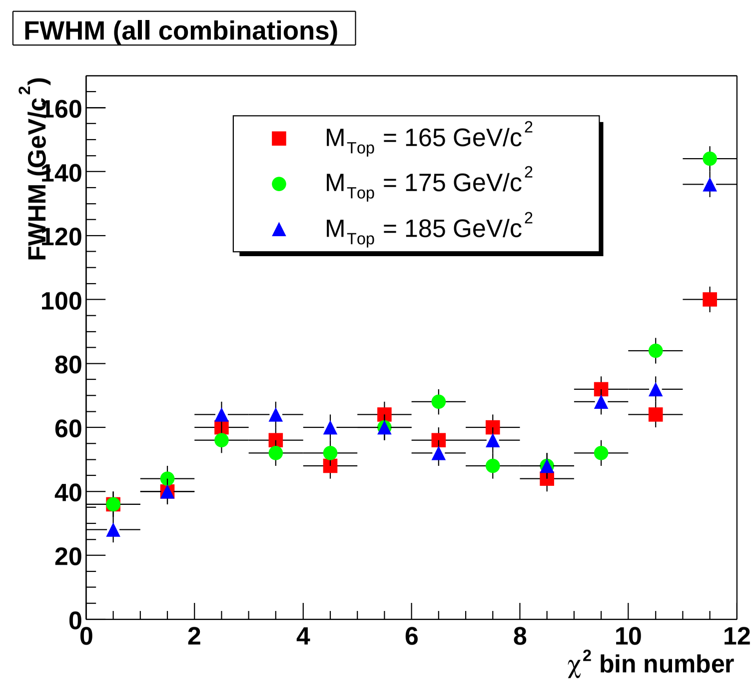
- Look if best χ^2 - 2nd best χ^2 influences ΔM

- Weight of each event would be computed according to how much better is the best χ^2 combined with the width of distribution of best χ^2 bin (method 1)



4: Weight each combination of each event according to χ^2

- Consider all combination of all events
- Separate the sample in terms of all χ^2 bins of all combinations
- Look if the generated input mass influences the mass distribution
- For a given χ^2 in a given bin, the mass is adjusted according to the mean difference with the input mass (ΔM)
- The width of the distribution determines the uncertainty in a given bin (FWHM)



Top mass reconstruction: maximum likelihood

Full $|M|^2$ integration over reduced phase space

- N events measured in phase space x_1, x_2, ... x_N
- Likelihood $L(M_t)$ of measuring x_1, ... x_N for each M_t hypothesis:

$$-\log\,L(M_t) = -\sum_{i=1}^{N} \log P(x_i; M_t)\,+\,N\int Acc(x)P(x;M_t)dx$$

$P(x_i; M_t)$: probability of observing x_i if M_t $Acc(x)$: detector acceptance (0 or 1)

- Minimizing $-\log\,L(M_t)$ gives which M_t parameter is most likely to have produced x_1, ... x_N
- Probability function of signal and modeled background

$$P(x;M_t) = c_1 P_{t\bar{t}}(x;M_t) + c_2 P_{background}(x)$$

$$P_{t\bar{t}}(x; M_t) = \int d^n\sigma dq_1 dq_2 f(q_1) f(q_2) W(y, x)$$

$f(q_1)f(q_2)$ PDF of initial parton momentum q_1 and q_2 W(y,x) transfer function

Problem: numerical integration is way too long

CDF detector		
ID	$\sqrt{\eta^2+\phi^2}$ resolution	E (or P) resolution
Electrons	~ 0.01	$\delta P_t/P_t = \sim 0.001 GeV^{-1} P_t$ or $\sim \frac{13.5\%}{\sqrt{E}}$
Muons	~ 0.01	$\delta P_t/P_t = \sim 0.001 GeV^{-1} P_t$
Jets	~ 0.05	$\sim \frac{130\%}{\sqrt{E}}$

Solution: integrate over reduced phase space for each event

$$W(y, x) = \delta^3(\vec{p_l} - \vec{p_l^x}) \prod_{4\,jets} \frac{1}{\sigma_i\sqrt{2\pi}} exp[-\frac{(p_i - p_i^x)^2}{2\sigma_i^2}] \prod_{4\,jets} \delta^2(\Omega_j - \Omega_j^x)$$

- 20 unknows: $\vec{p}$ of l, ν, b, $\bar{b}$, q, $\bar{q}\prime$ + p_z of 2 incoming partons
- 15 constraints: $\vec{p}$ of l (3) + Ω of b, $\bar{b}$, q, $\bar{q}\prime$ (8) + conservation of energy and momentum (4)

$\Rightarrow$ Integration must be performed over 5 variables

$\Rightarrow$ To save time in numerical integration, integrate over Breit-Wigner peaks: M_W^2 and M_t^2

$$P_{t\bar{t}}(x, M_t) = \int dp_1 dM_{W_1}^2 dM_{W_2}^2 dM_{t_1}^2 dM_{t_2}^2 |M|^2 \frac{f(q_1)}{|q1|} \frac{f(q_2)}{|q2|} \Phi_6 \times \prod_{i=1}^{4} \frac{1}{\sigma_i \sqrt{2\pi}} exp[-\frac{(p_i - p_i^x)^2}{2\sigma_i^2}]$$

Can integration be performed with COMPHEP, MADGRAPH?

General advantages of integration over reduced phase space:

- All measured quantities contribute to probability (except unclustered energy)
- Increases computing power by focusing on simulation of low resolution measurements (jet energies)
- MC template modeled for each measured data event configuration

General disadvantages of integration over reduced phase space:

- Does not account for non neglectable contribution from initial state and final state radiation

$\Rightarrow$ Must reject all events with high E_T 5th jet

$\Rightarrow$ Effect of low E_T 5th jet unclear

Conclusion

- There is little room for improvement with the χ^2 method
- In general, likelihood method more powerfull than χ^2 method (D0 reduced statistical uncertainty from $5.6\,GeV/c^2$ to $3.6\,GeV/c^2$)
- Must investigate effect of soft ISR and FSR on full $|M|^2$ integration
- Will look into simpler weighting likelihood method that don't involve full $|M|^2$ integration (Dalitz-Goldstein)