

MATHEMATICS & PHYSICAL SCIENCE

FRANKLIN — MATHEMATICS IS THE GOLD
STANDARD OF KNOWLEDGE

WHY? → IN MATHEMATICS HAVE PROOFS



CERTAINTY



MUST BE TRUE

WHY DOES MATHEMATICS DESCRIBE PHYSICAL

REALITY? I DON'T THINK FRANKLIN GIVES

THIS THE ATTENTION IT DESERVES

DEEP MYSTERY

IN THINKING ABOUT THIS, I WENT BACK TO
TWO BOOKS BY ROGER PENROSE

- THE EMPEROR'S NEW MIND
- SHADOWS OF THE MIND

I'M NOT ASKING YOU TO READ THEM. I
THINK I HAVE EXTRACTED WHAT I NEED.
I JUST WANT TO ACKNOWLEDGE THE
SOURCE OF MY, UNORIGINAL, IDEAS

THERE'S ALSO "THE ROAD TO REALITY"

A 1099 PAGE "BEST SELLER"

↑ I HAVE MY
DOUBTS.

MATHEMATICS COMES INTO PHYSICAL SCIENCE
IN TWO VERY DIFFERENT WAYS.

— AS AN EVERYDAY CALCULATIONAL/SYSTEMATIZING
TOOL QUANTITIES OF THINGS

CALCULUS → AREAS -----

EQUATIONS OF MOTION

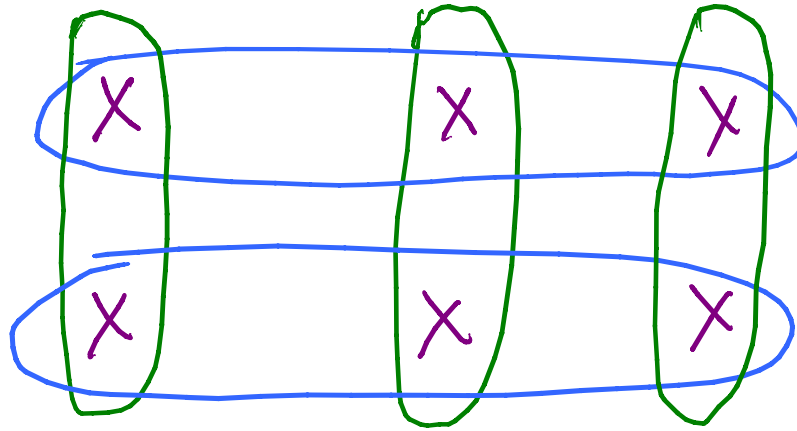
STATISTICS → MAKING JUDGEMENTS
EVIDENCE

i.e. "GETTING FROM ONE EMPIRICAL FACT
TO ANOTHER"


THE OTHER CONNECTION IS AS:-

"A MODEL OF REALITY"

FRANKLIN USES A SIMPLE EXAMPLE



$$3 \times 2 = 2 \times 3$$

- TRUE
- ABOUT THE PHYSICAL WORLD 
- NOT JUST SYMBOL MANIPULATION
- CANNOT BE INVALIDATED
BY CHOOSING A DIFFERENT
SET OF AXIOMS

MATHEMATICS IS ABOUT STRUCTURE / PATTERN

MATHEMATICAL PROOF

$$1 + 2 + 3 \text{ ----- } 98 + 99 + 100$$

$$100 + 99 + 98 \text{ ----- } 3 + 2 + 1$$

- EACH PAIR ADDS TO 101
 - THERE ARE 100 PAIRS
- $$\left. \begin{array}{l} \text{EACH PAIR ADDS TO 101} \\ \text{THERE ARE 100 PAIRS} \end{array} \right\} \Sigma = 100 \times 101$$

BUT EACH NUMBER COMES IN TWICE

$$\therefore \sum_{1}^{100} = \frac{1}{2} (100 \times 101)$$

$$\sum_{1}^n = \frac{n}{2} (n+1)$$

PROOF \rightarrow

WHAT IS IT THAT IS SPECIAL ABOUT

MATHEMATICAL
PROOF

- You UNDERSTAND THE WHOLE PROCESS COMPLETELY
- FROM THIS DETAILED UNDERSTANDING YOU ARE SURE THAT THE CONCLUSION IS CORRECT
- YOU CAN GENERALIZE THE RESULT

$$1+2+3+4 + \dots + n = \frac{n}{2} (n+1)$$

A COUPLE OF RESERVATIONS

COMPUTER PROOFS!

- KEPLER'S CONJECTURE → SPHERE STACKING
- FOUR COLOUR THEOREM
 - FORMAL PROOF OF PROGRAM VALIDITY
 - DIFFERENT LANGUAGES
 - DIFFERENT HARDWARE
 - DIFFERENT COMPILER

DO THEY PROVIDE INSIGHT, NEW CONCEPTS

DO THEY GIVE YOU THE "FEELING OF

CONFIDENCE IN CORRECTNESS

GÖDEL'S THEOREM!

ANY SYSTEM OF AXIOMS & RULES OF PROCEDURE, PROVIDED THAT IT IS BROAD ENOUGH TO ENCOMPASS SIMPLE ARITHMETIC, AND PROVIDED THAT IT IS FREE OF CONTRADICTIONS MUST CONTAIN STATEMENTS THAT ARE NEITHER PROVABLE NOR UNPROVABLE BY THE MEANS ALLOWED WITHIN THE SYSTEM

- THE STATEMENT OF THE CONSISTENCY OF THE AXIOMS OF THE SYSTEM MUST BE ONE SUCH UNDECIDABLE PROPOSITION

WE WILL COME BACK TO GÖDEL, FOR
NOW LOOK AT TWO FEATURES OF THE
INTERACTION OF PHYSICS & MATHEMATICS

- A MATHEMATIC UNDERSTANDING OF A
PROCESS HAS UNEXPECTED MATHEMATICAL
FEATURES → CORRESPOND TO UNEXPECTED
PHYSICS → YOU GET MORE
OUT THAN YOU PUT IN
- MATHEMATICAL SYMMETRIES REALIZED IN
NATURE LEAD TO PROFOUND UNDERSTANDING

GETTING OUT MORE THAN YOU PUT IN

GENERAL RELATIVITY IS AN INTERESTING EXAMPLE

- NO NEED ON OBSERVATIONAL GROUNDS
- NEWTON WORKED WELL FOR ~ 250 YEARS
- EINSTEIN THOUGHT A BETTER THEORY WOULD BE BASED ON PRINCIPLE OF EQUIVALENCE
- AT FIRST NOT MUCH OBSERVATIONAL SUPPORT
- DECADES LATER IT MAKES AMAZINGLY PRECISE PREDICTIONS
- OPENED UP A WHOLE NEW UNSUSPECTED WORLD — BLACK HOLES, BIG BANG, GRAVITATIONAL RADIATION

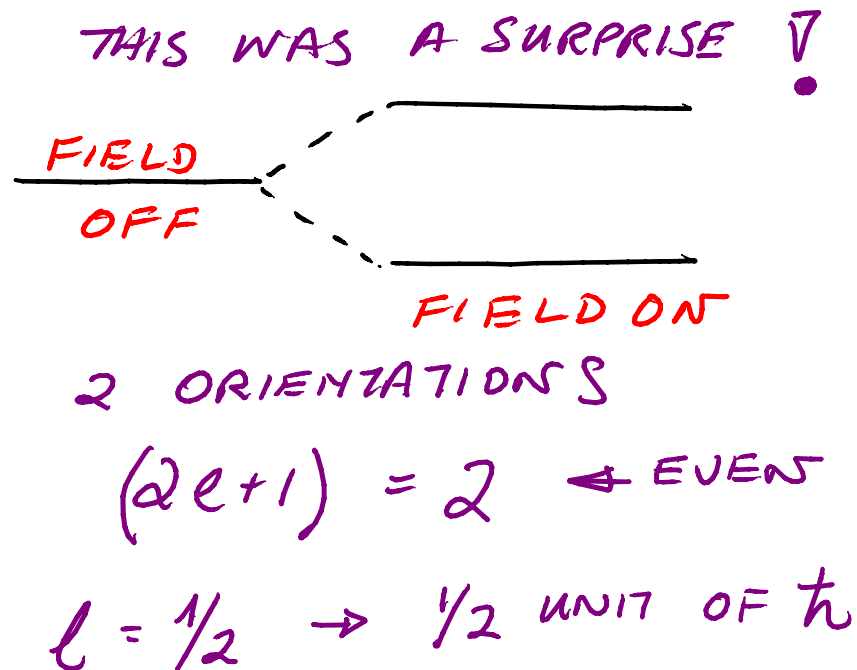
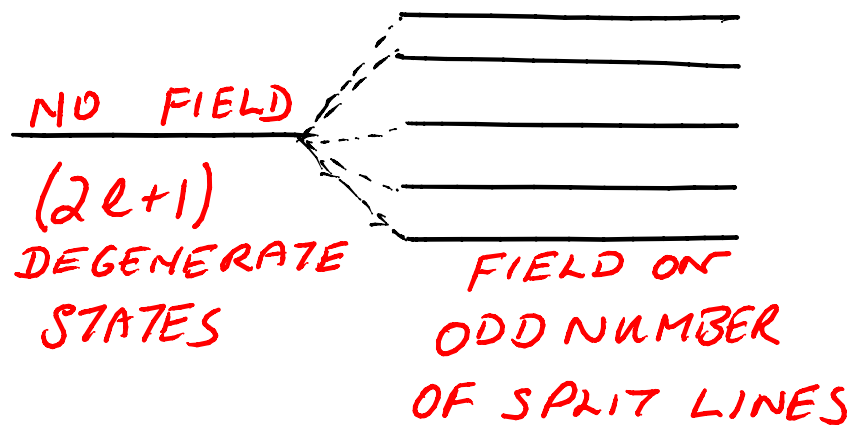
ATOMIC SPECTRAL LINES - ANGULAR MOMENTUM

$$L^2 |\psi_{lm}\rangle = l(l+1) \hbar^2 |\psi_{lm}\rangle \quad l = 0, 1, 2, \dots +1$$

$$L_z |\psi_{lm}\rangle = m \hbar |\psi_{lm}\rangle \quad m_l = -l, -l+1, \dots, -1, 0, \dots, l-1, l$$

- FOR A GIVEN l THERE ARE $(2l+1)$ ORIENTATIONS IN A MAGNETIC FIELD \rightarrow l INTEGER \leftarrow ODD

ATOMIC SPECTRAL LINES IN A MAGNETIC FIELD



- CLASSICAL INTERPRETATION — ELECTRON SPINNING ON ITS AXIS
 - HOW COULD A POINT PARTICLE SPIN?
 - PROTON HAS SPIN $\hbar/2$. CALCULATE THE VELOCITY OF ITS PERIPHERY.

• BUT SPIN $\frac{1}{2}$ BEHAVES LIKE ANGULAR MOMENTUM
 WHAT DOES THIS MEAN?

QUANTUM OPERATORS
 OBEY COMMUTATION RELATIONS

$$[L_x, L_y] = L_x L_y - L_y L_x = i \hbar L_z$$

$$[L_y, L_z] = [L_z, L_x] = i \hbar L_x, y$$

$$\left[\begin{array}{c} L^2 \\ L_x \\ L_y \\ L_z \end{array} \right]$$

← ANY SET OF OPERATORS
 OBEYING THIS "ALGEBRA"

"BEHAVES LIKE
 ANGULAR MOMENTUM"

← "GROUP ALGEBRA"
 REFLECTION OF SYMMETRY

↓
 ROTATIONAL INVARIANCE

↙
 ORBITAL ANGULAR MOMENTUM, SPIN

QUANTUM MECHANICAL WAVE EQUATIONS.

SCHRÖDINGER — NON RELATIVISTIC

$$E = p^2/2m - V$$

$$p \rightarrow +i\hbar\nabla \rightarrow E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - V\psi$$

$$E \rightarrow -i\hbar\partial/\partial t \rightarrow +i\hbar\partial\psi/\partial t + \frac{\hbar^2}{2m}\nabla^2\psi = -V\psi = 0 \quad (\text{FREE})$$

KLEIN - GORDON — RELATIVISTIC

$$p^\mu p_\mu = m^2 \rightarrow p^\mu \rightarrow i\partial^\mu \quad \text{cf NON RELATIVISTIC } p \rightarrow i\hbar\nabla$$

$$(i\partial^\mu i\partial_\mu + m^2)\phi = -V\phi = 0 \quad (\text{FREE})$$

$$E^2 = p^2 + m^2 \rightarrow E = \pm \sqrt{p^2 + m^2} \quad \text{-VE ENERGY}$$

$$\text{DIRAC TRIED TO AVOID } \sqrt{} \rightarrow i\partial^\mu \partial_\mu \rightarrow i\partial^\mu$$

KLEIN-GORDON

SCALAR

$$(\partial^\mu \partial_\mu + m^2) \phi = 0$$

"SCALAR" PARTICLE
ONLY E, \vec{p}

SPACE-TIME
ENERGY-MOMENTUM

DIRAC

NOT A
SCALAR

$$(i \gamma^\mu \partial_\mu - m) \psi = 0$$

NO LONGER
2ND ORDER

E, \vec{p} }
"SPINS"
SOME THING
EXTRA

γ^μ →
DIRAC
 γ
MATRICES

DIRAC EQUATION \rightarrow FOUR DIFFERENTIAL EQUATIONS

$$\sum_{k=1}^4 \left\{ \sum_{\mu} i(\gamma^{\mu})_{jk} \partial_{\mu} - m \delta_{jk} \right\} \psi_k = 0$$

EIGENVALUES

$$E = m, m, -m, -m$$

$$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

SPIN

$$+\frac{1}{2}$$

$$-\frac{1}{2}$$

ELECTRON

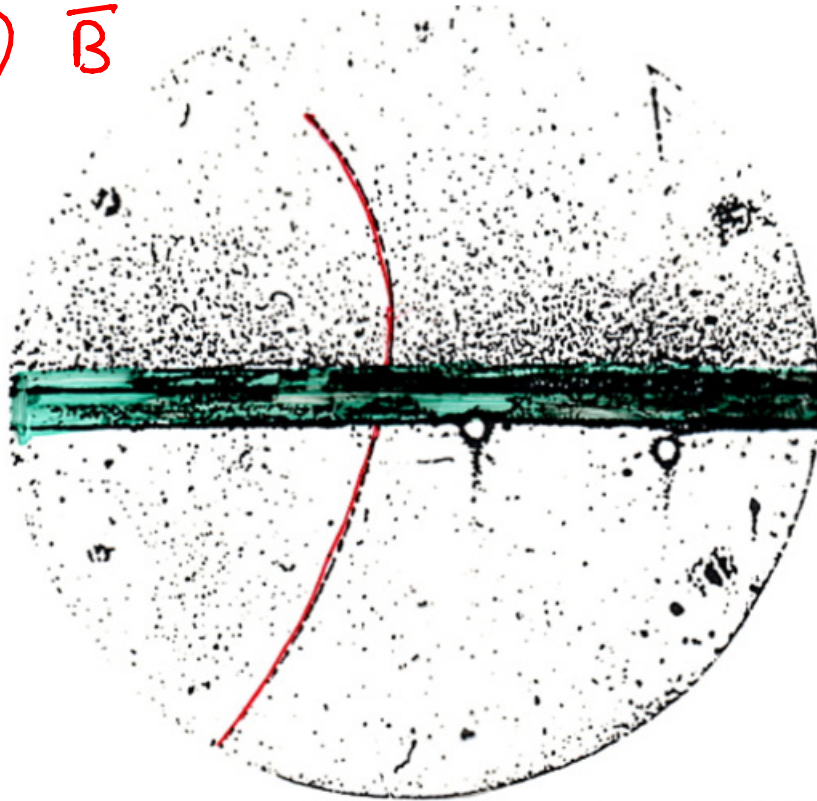
$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix} \quad \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$+\frac{1}{2}$$

$$-\frac{1}{2}$$

POSITRON

(X) \vec{B}



Pb
PLATE

• DISCOVERY OF POSITRON
BY CARL ANDERSON 1933

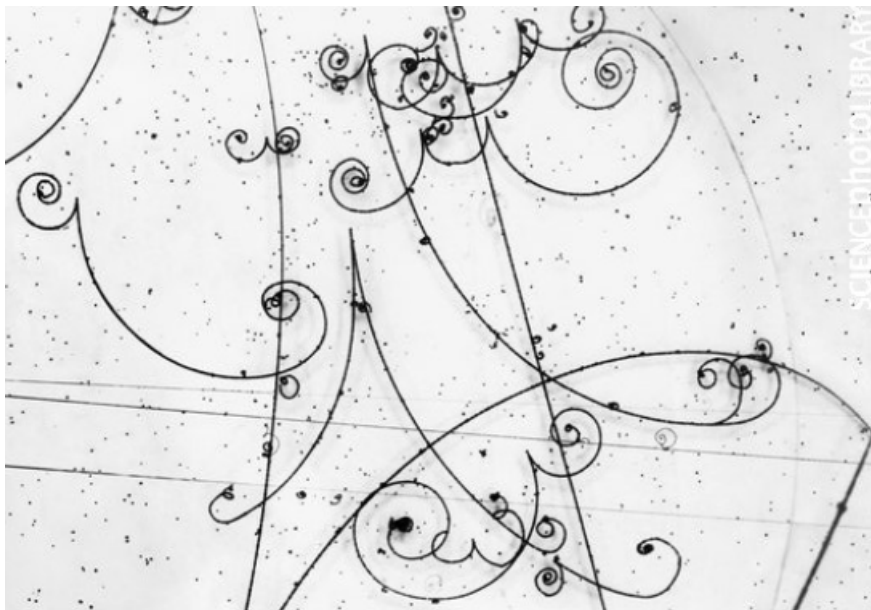
• PARTICLE SLOWS DOWN IN
Pb PLATE \therefore COMING FROM
BELOW

• FROM SIGN OF MAGNETIC
FIELD MUST B +VE

• 10X RANGE OF PROTON
(ONLY +VE PARTICLE KNOWN)

A 63 million volt positron ($H\beta = 2.1 \times 10^6$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\beta = 7.5 \times 10^6$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

$\gamma \rightarrow e^+e^-$ IN A LARGE
MODERN BUBBLE
CHAMBER



← INVISIBLE

← CURVING
IN MAGNETIC
FIELD

SYMMETRIES → CONSERVATION LAWS

NÖTHER'S THEOREM:

INVARIANCE OF SYSTEM UNDER TRANSFORMATION → SYMMETRY → CONSERVATION LAW

CONSERVED QUANTITY ↔ SYMMETRY

ANY OPERATOR WHICH COMMUTES WITH THE HAMILTONIAN CORRESPONDS TO A CONSERVED QUANTITY

$$[F, H] = 0$$

↑ CONSERVED
↓ HAMILTONIAN

SYMMETRY

CONSERVED QUANTITY

SPACE TRANSLATIONS

MOMENTUM

ROTATIONS

ANGULAR MOMENTUM

TIME

ENERGY

ROTATION IN ISOSPACE

ISOSPIN

?

LEPTON FLAVOR

QUARK FLAVOR

BARYON NUMBER

COLOUR CHARGE

GLOBAL $SU(3)$ GAUGE

WEAK CHARGE

GLOBAL $SU(2)$ GAUGE

ELECTRIC CHARGE

GLOBAL $U(1)$ GAUGE

LOCAL $U(1)$ GAUGE

EM FORCE

LOCAL $SU(2)$ GAUGE

WEAK FORCE

LOCAL $SU(3)$ GAUGE

COLOUR FORCE

TWO DISTINCT KINDS OF TRANSFORMATIONS

- **CONTINUOUS** → DEPEND ON SOME CONTINUOUS PARAMETER. CAN DIFFER FROM UNITY (= DO NOTHING) BY AN ARBITRARILY SMALL AMOUNT
EXAMPLE → ROTATION IN SPACE
- **DISCRETE** → EITHER HAPPEN OR NOT
REFLECTION, TIME REVERSAL, PARTICLE
↓
ANTIPARTICLE
SOME DISCRETE TRANSFORMATIONS → OBSERVABLES

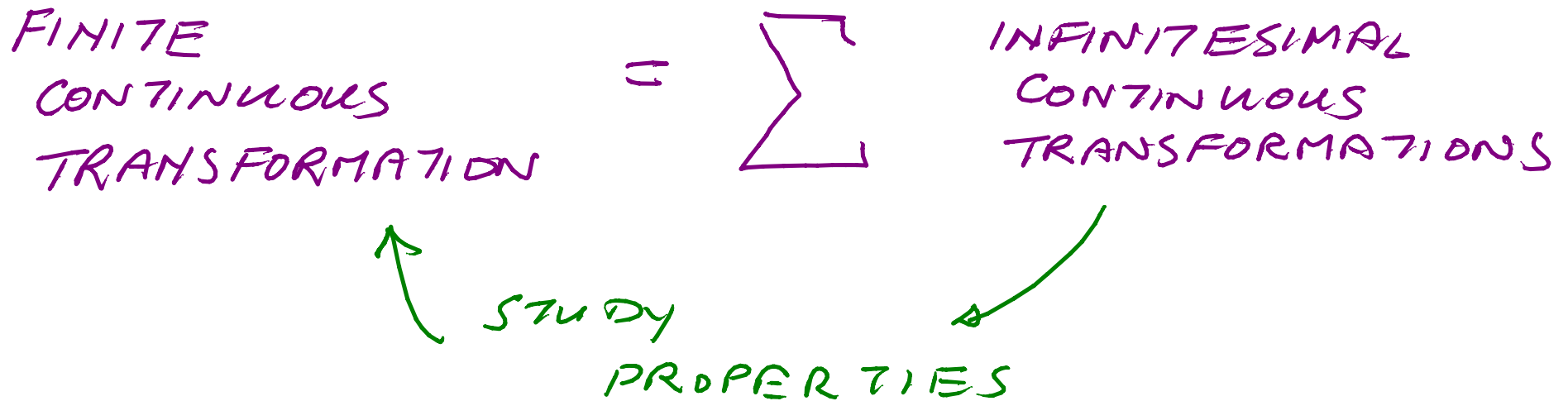
SPATIAL REFLECTION → PARITY = U_p

$$\psi(\bar{x}) \rightarrow \psi(-\bar{x}) \quad ; \quad \psi(-\bar{x}) = U_p \psi(\bar{x})$$

$U_p U_p = 1$ → HERMITIAN → MUST CORRESPOND
UNITARY → TO AN OBSERVABLE
 $U^\dagger U = 1$ ← $U^\dagger = U$

CONTINUOUS TRANSFORMATIONS

- MATHEMATICALLY USUALLY EASIER TO CONSIDER INFINITESIMAL TRANSFORMATIONS
- EXAMPLE \rightarrow BUILD FINITE ROTATION FROM MANY INFINITESIMAL ROTATIONS



UNITARY TRANSFORMATIONS & SYMMETRIES

A MATHEMATICAL REPRESENTATION OF A
UNITARY TRANSFORMATION; $U^\dagger U = 1$

REAL PARAMETER

$$U = e^{i\theta F}$$

GENERATOR OF
TRANSFORMATION

IF U IS A UNITARY OPERATOR
 F GENERATES A GROUP OF TRANSFORMATIONS

GROUP MEMBER \rightarrow

$$g_1 \cdot (g_2 \cdot g_3) = (g_1 \cdot g_2) \cdot g_3 \in G$$

$$g \cdot I = g = I \cdot g$$

$$g \cdot g^{-1} = I = g^{-1} \cdot g$$

$$\text{IF } g_i \text{ AND } g_j \in G \rightarrow g_i \cdot g_j \in G$$

GROUPS

ABSTRACT ENTITIES \rightarrow PHYSICAL SYMMETRIES

$SO(3)$ \rightarrow GROUP OF ROTATIONS ABOUT
 x, y, z \rightarrow CONSERVATION OF
ANGULAR MOMENTUM

$U(1)$ \rightarrow GROUP OF 1-D ROTATIONS $e^{i\alpha}$

$SU(2)$ \rightarrow GROUP OF ALL 2×2 MATRICES
(WITH UNIT DETERMINANT)
 $\left(\begin{pmatrix} e & \\ & v \end{pmatrix} \right)_2$ SYMMETRY OF WEAK FORCE

$SU(3)$ \rightarrow 3×3

SYMMETRY OF
COLOUR FORCE

GAUGE INVARIANCE $\psi \rightarrow e^{i\alpha(\vec{x}, t)} \psi$

PHYSICALLY OBSERVE $\psi\psi^* = e^{i\alpha} \cdot e^{-i\alpha}$

TO REALIZE THIS
IN FREE PARTICLE
DIRAC EQUATION
REDEFINE

$$\partial_\mu \rightarrow \mathcal{D}_\mu = \partial_\mu - ieA_\mu$$

$$A_\mu \rightarrow A_\mu + \frac{1}{e} \partial_\mu \alpha(\vec{x}, t)$$

$$(i\gamma^\mu \partial_\mu - m)\psi = 0$$

E, \vec{p}

SOME THING
EXTRA

'SPINS'

$$\mathcal{D}_\mu = \partial_\mu - \underbrace{ieA_\mu}_{\text{PHOTON}}$$

PHOTON

GET OUT MORE THAN YOU PUT IN

DIRAC JUST WANTED TO GET RID
OF -VE ENERGIES

DIRAC EQUATION GAVE!

UNDERSTANDING OF -VE ENERGIES

PLUS

ANTI PARTICLES

INTRINSIC ANGULAR MOMENTUM = SPIN

GAUGE INVARIANCE \rightarrow ELECTROMAGNETIC
FORCE

GAUGE THEORIES

GAUGE THEORIES HAVE BEEN BUILT FOR

COLOUR FORCE — $SU(3)$

WEAK FORCE — $SU(2)$

EM FORCE — $U(1)$

} NON-ABELIAN
(GENERATORS DO NOT
COMMUTE)

ABELIAN

(GENERATORS COMMUTE)

GENERALLY GAUGE INVARIANCE LEADS TO

MASSLESS GAUGE BOSONS

$W^\pm Z^0 ?? \rightarrow$ HIGGS MECHANISM

I WENT THROUGH THIS IN SOME DETAIL
GAUGE INVARIANCE IS THE STANDARD MODEL

I TALKED ABOUT THIS TOPIC ASSUMING
THE ELECTRONS, PHOTONS ETC. REAL.

ACTUALLY I ALSO TALKED ABOUT ∂_μ , \mathcal{D}_μ ,
 A_μ , $e^{i\alpha}$ AS IF THEY WERE REAL

REAL \rightarrow OBJECTIVE EXISTENCE

WELL HOW REAL ARE THE OBJECTS
THAT MATHEMATICS DEALS WITH?

MATHEMATICS \rightarrow PHYSICS \rightarrow OBJECTIVE
REALITY

HOW REAL ARE THE OBJECTS OF MATHEMATICS?

ONE POINT OF VIEW IS THAT THEY ARE JUST SYMBOLS MANIPULATED ACCORDING TO ARBITRARILY AGREED RULES.

CAN THEY BE OTHER THAN ARBITRARY CONSTRUCTS OF THE HUMAN MIND?

MATHEMATICAL TRUTHS SEEM TO BE INDEPENDENT OF INDIVIDUAL MATHEMATICIANS THERE SEEMS TO BE A PROFOUND MATHEMATICAL REALITY → AND IT SEEMS TO OVERLAP WITH PHYSICS

AXIOMATIC MATH, HILBERT & GÖDEL

IN 1920 HILBERT PROPOSED A PROGRAM TO PUT MATHEMATICS ON A SOLID AXIOMATIC FOUNDATION:

- ALL MATHEMATICS FOLLOWS FROM FINITE SET OF CHOSEN AXIOMS
 - AXIOMS SHOULD BE PROVEN TO BE CONSISTENT
- IF THIS COULD BE DONE, YOU MIGHT ARGUE THAT MATHEMATICS WAS A PRODUCT OF THE HUMAN MIND.

GÖDEL DEMONSTRATED THAT THIS WAS IMPOSSIBLE

MATHEMATICAL INSIGHT à la PENROSE

WHILE PROPOSITIONS MAY BE UNDECIDABLE

WITHIN A FORMAL SYSTEM A HUMAN MATHEMATICIAN

CAN HAVE INSIGHTS OUTSIDE FORMAL SYSTEM

THE MENTAL PROCEDURES BY WHICH MATHEMATICIANS

ARRIVE AT JUDGEMENTS OF TRUTH ARE NOT

SIMPLY ROOTED IN THE PROCEDURES OF A

SPECIFIC FORMAL SYSTEM

HUMAN INSIGHTS ARE NOT ALGORITHMS

GÖDEL → THE CONCEPT OF MATHEMATICAL
TRUTH CANNOT BE ENCAPSULATED
IN ANY FORMALISM

THERE IS A "SIMPLE" WAY TO GRASP THIS

HOW DO WE SET UP FORMAL SYSTEMS?

AXIOMS \rightarrow PROCEDURES \rightarrow PROPOSITIONS

WE USE OUR INTUITIVE (INDUCTIVE)

UNDERSTANDING TO CHOOSE A SET

OF AXIOMS WHICH ARE SELF-EVIDENT

REAL MATHEMATICAL TRUTH GOES BEYOND

MAN-MADE CONSTRUCTS.

MATHEMATICS INDEPENDENT OF MATHEMATICIANS

PENROSE USES THE MANDEL BROT SET
TO ILLUSTRATE A MATHEMATICAL OBJECT
DISCOVERED \rightarrow EXISTS OUTSIDE
HUMAN MIND

MANDEL BROT SET

ITERATIVE $Z \rightarrow Z^2 + C$ ↓ FIXED COMPLEX
NUMBER

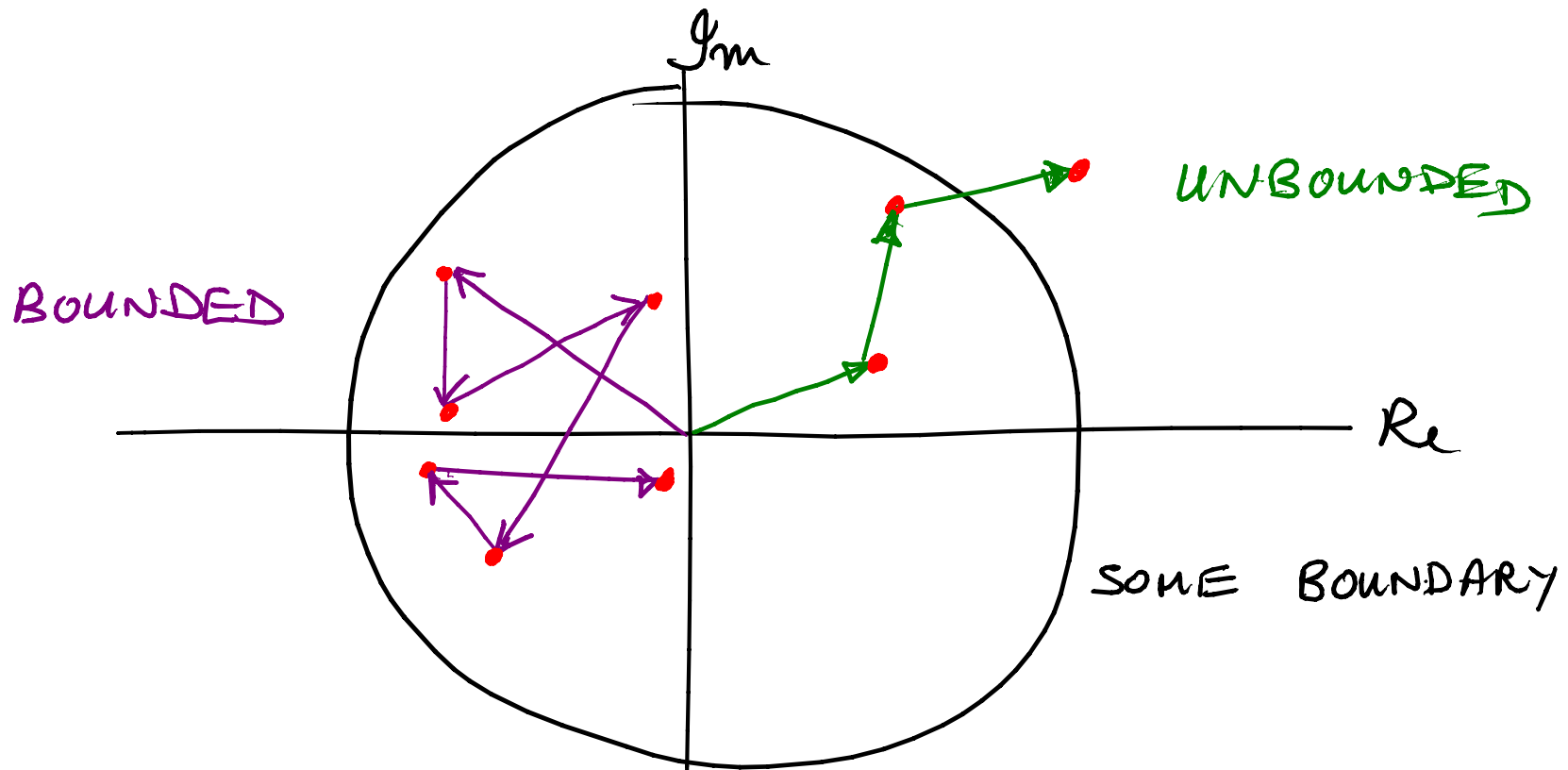
eg $Z \rightarrow Z^2 + 1.63 + i1.42$

$$3^2 \rightarrow 3^2 + 1.63 + i1.43 = 10.63 + i4.2$$

SAY $0, c; c \rightarrow c^2 + c; c^2 + c \rightarrow (c + c^2)^2 + c$

GET A SEQUENCE OF COMPLEX NUMBERS
IN THE ARGAND PLANE

$0, c, c^2 + c, c^4 + 2c^3 + c^2 + c, \dots$



$C = 0 \rightarrow$ ALL MEMBERS 0 \rightarrow BOUNDED

$C = -1 \rightarrow 0, -1, 0, -1, 0, -1, 0$ BOUNDED

$C = i \rightarrow 0, i, i-1, -i, i-1$ BOUNDED

$C = i-1 \rightarrow 0, i-1, -i-1, -1+3i, -9-i5$ UNBOUNDED

MANDELBROT SET \rightarrow REGION OF ARGAND
PLANE WHERE SERIES
BOUND

COMPUTER CHOOSES C

\rightarrow BOUNDED \rightarrow BLACK CROSS

\rightarrow UNBOUNDED \rightarrow LEAVE
WHITE SPACE

MANDELBROT'S

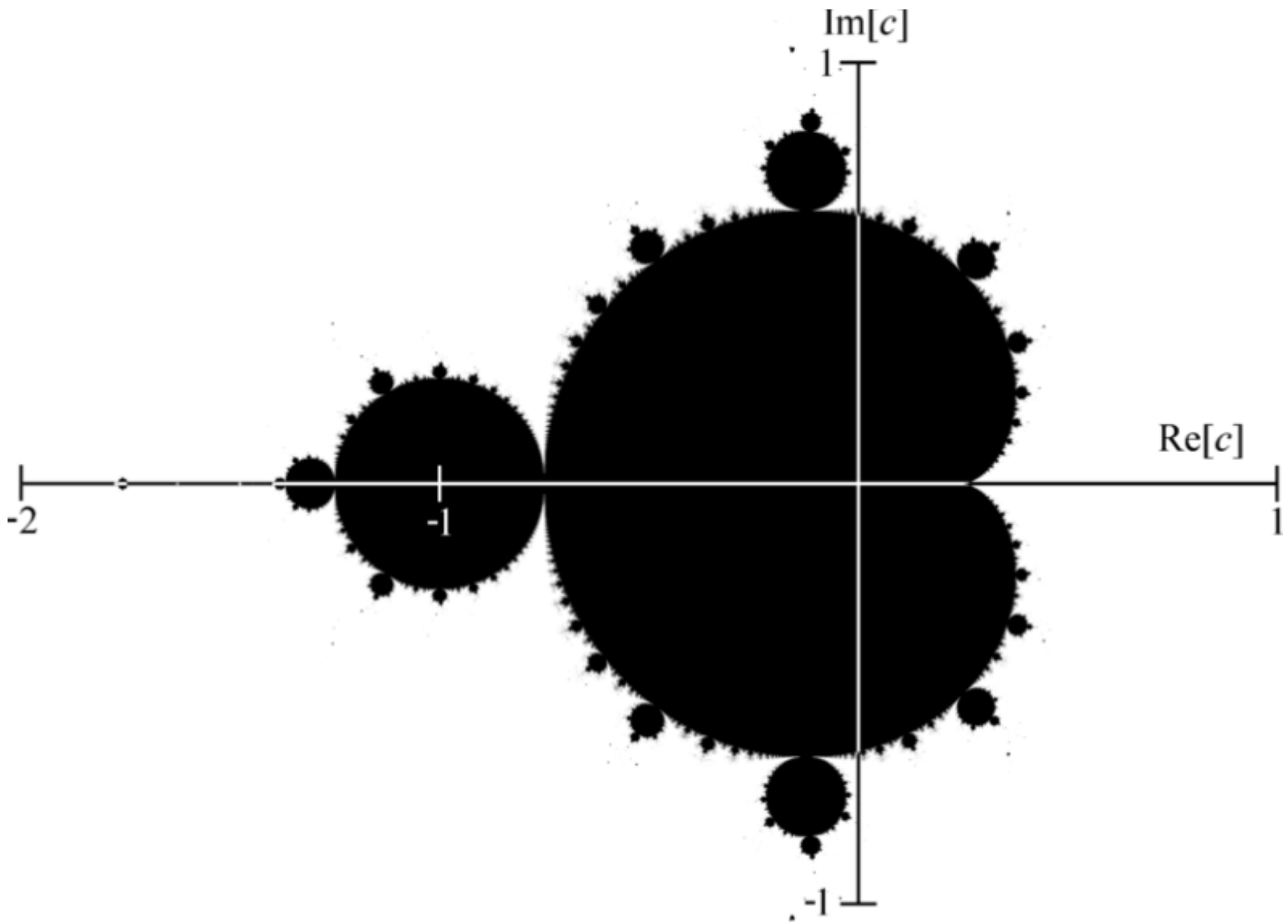
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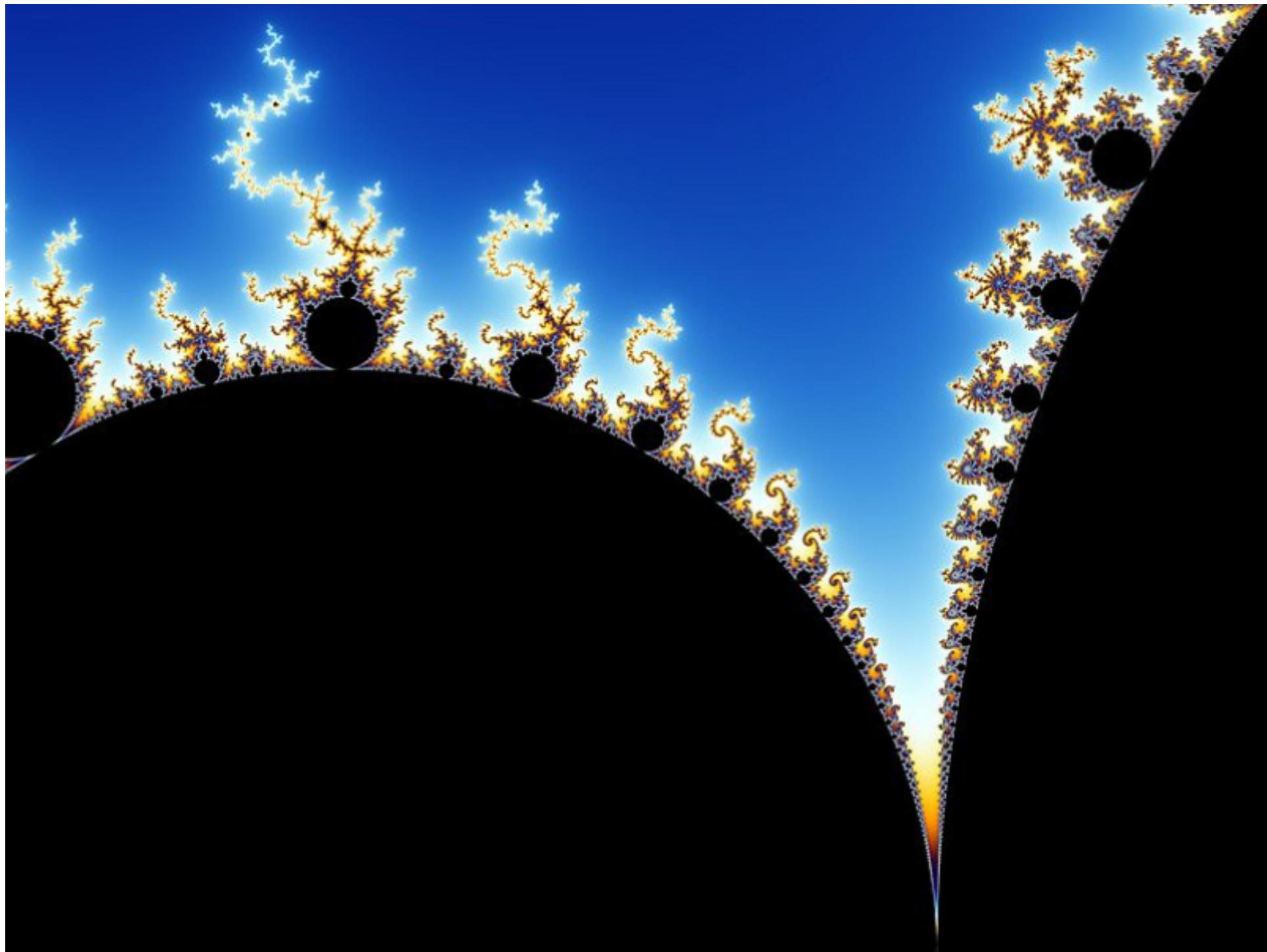
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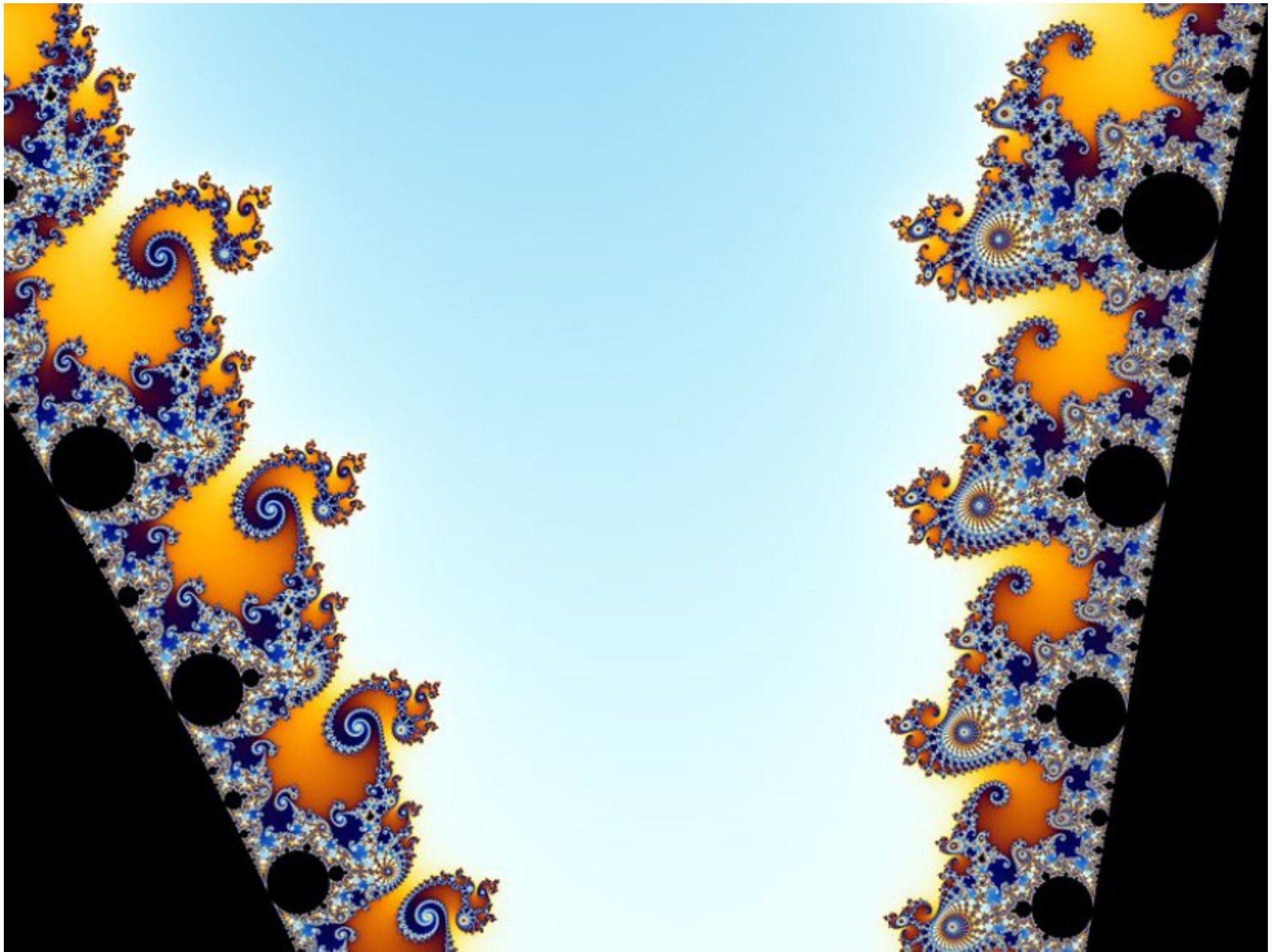
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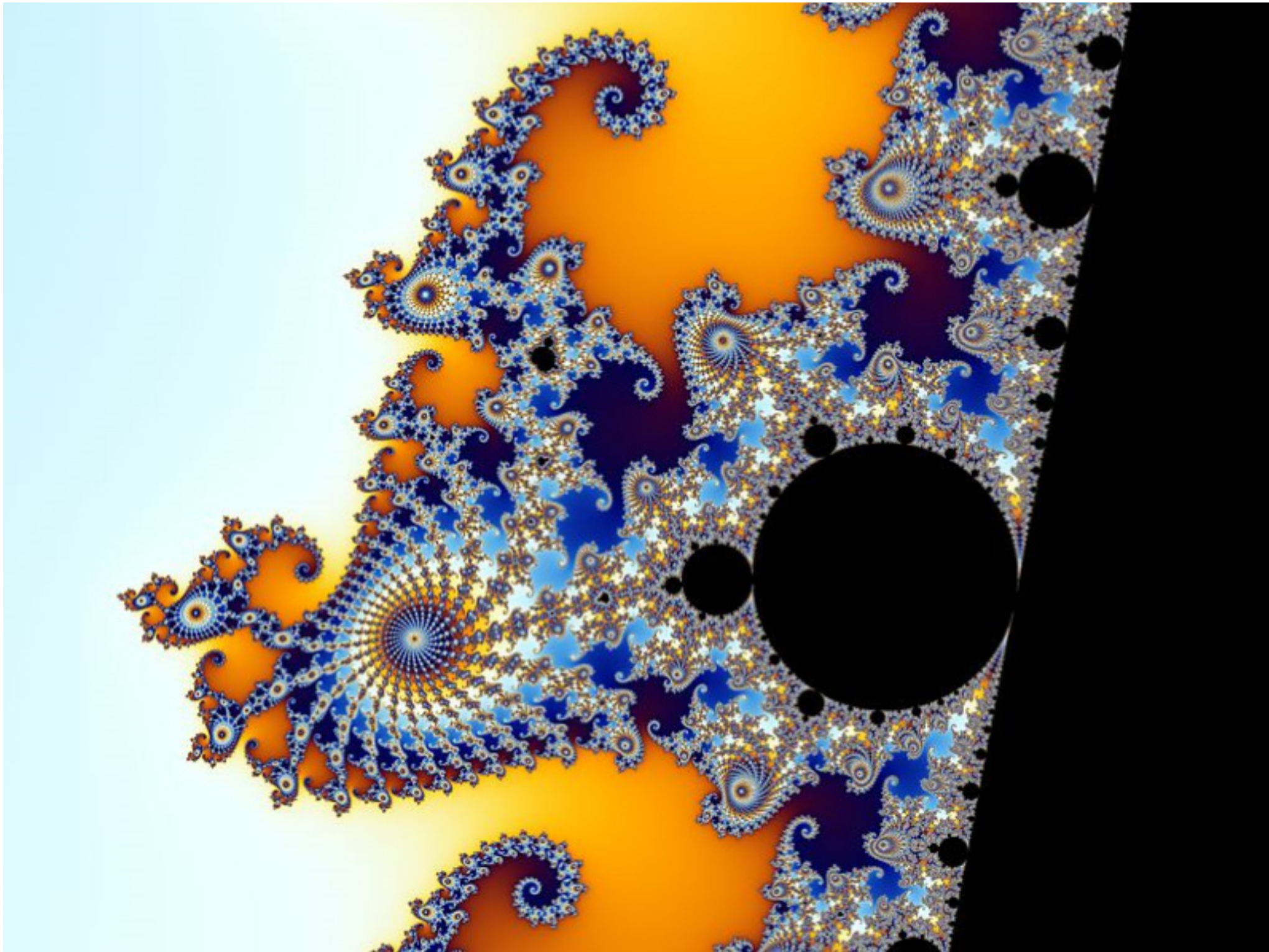
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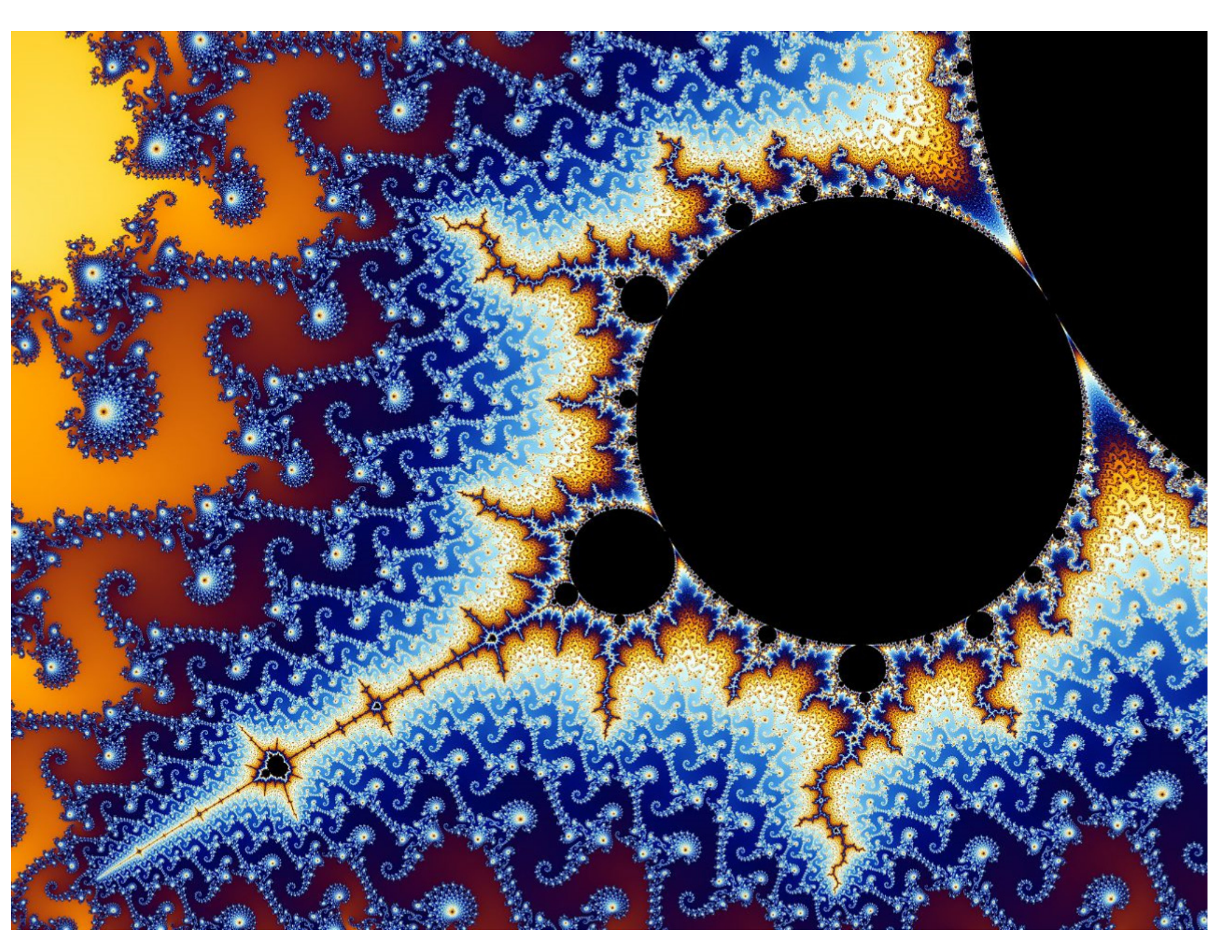
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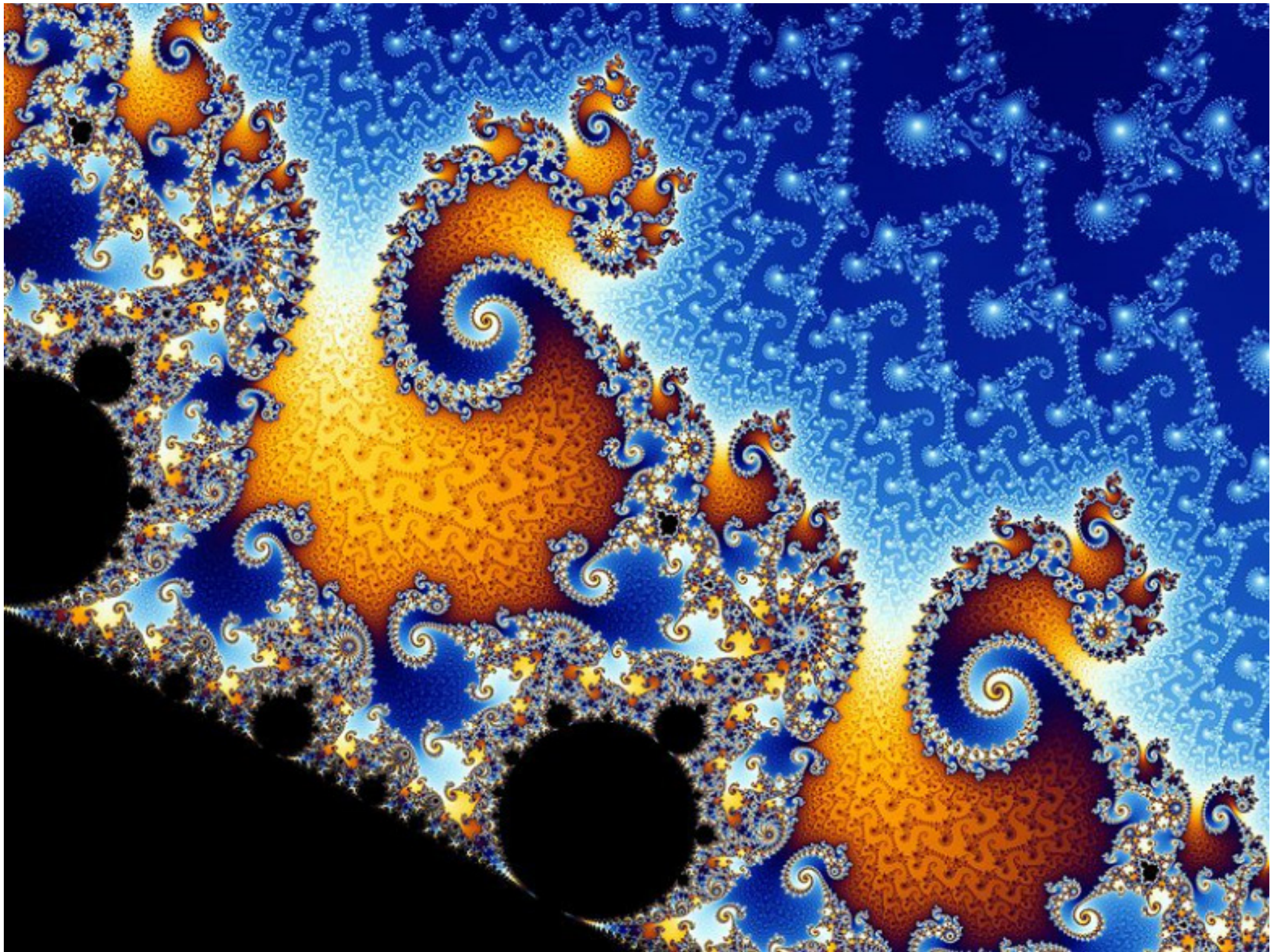












- MANDELBROT SET NOT AN INVENTION OF ONE PERSON
 - MANDELBROT HAD NO A PRIORI IDEA THAT IT WOULD HAVE THIS INFINITE COMPLEXITY
 - FULL COMPLEXITY CANNOT BE COMPREHENDED BY ONE PERSON
 - FULL COMPLEXITY CANNOT EVEN BE REVEALED
 - IT HAS INDEPENDENT EXISTENCE OUTSIDE THE MIND OF ANY MATHEMATICIANS
- NOT AN INVENTION → A DISCOVERY

EXAMPLE OF COMPLEX NUMBERS

GEROLAMO CARDANO "ARS MAGNA" 1545

FIRST COMPLETE SOLUTION OF GENERAL CUBIC EQUATION $\sqrt{-ve} \rightarrow$ JUST DID IT \rightarrow REAL SOLUTIONS

COMPLEX NUMBERS

CAUCHY INTEGRAL

REIMAN ZETA-FN \rightarrow CONJECTURE ABOUT ZEROS

} MATHEMATICS

CONVENIENT REPRESENTATIONS

QUANTUM WAVE FUNCTION

} PHYSICS

 HAS TO BE COMPLEX

\rightarrow INTERFERENCE OF AMPLITUDES

PLATONIC VIEW OF PENROSE

THERE SEEMS TO BE A COMPELLING UNKNOWNESS
TO MATHEMATICS

THE WORLD WE KNOW MOST DIRECTLY IS

① THE WORLD OF OUR CONSCIOUS PERCEPTIONS
IT IS THE WORLD OF PAIN, HAPPINESS,
COLOURS, MEMORIES

ODDLY ENOUGH (TO ME) IT IS THE WORLD
WE KNOW LEAST ABOUT IN SCIENTIFIC
TERMS.

THERE ARE TWO OTHER WORLDS

② THE PHYSICAL WORLD CONTAINS
ACTUAL OBJECTS CHAIRS, CATS, GALAXIES,
QUARKS -----

CLEARLY ① ↔ ② INTERACT

I HAVE NOT SEEN A CONVINCING EXPLANATION
OF HOW THIS INTERACTION WORKS

HOW DOES THE DETAILED EXPERIENCE
IN ② GET GENERATED BY INTERACTING
WITH ① ?

EACH OF THESE WORLDS SEEM TO BE
REAL.

③ THERE IS A THIRD WORLD, THE WORLD
OF MATHEMATICAL FORMS. IT CONTAINS

NATURAL NUMBERS

COMPLEX NUMBERS

EUCLIDEAN GEOMETRY

MAXWELL'S EQUATIONS

SCHRODINGER EQUATION

⋮

MATHEMATICAL SIMULATIONS

OF THE PHYSICAL WORLD

? QUANTUM GRAVITY?

IN WHAT SENSE COULD YOU SAY THAT
THIS WORLD IS REAL?

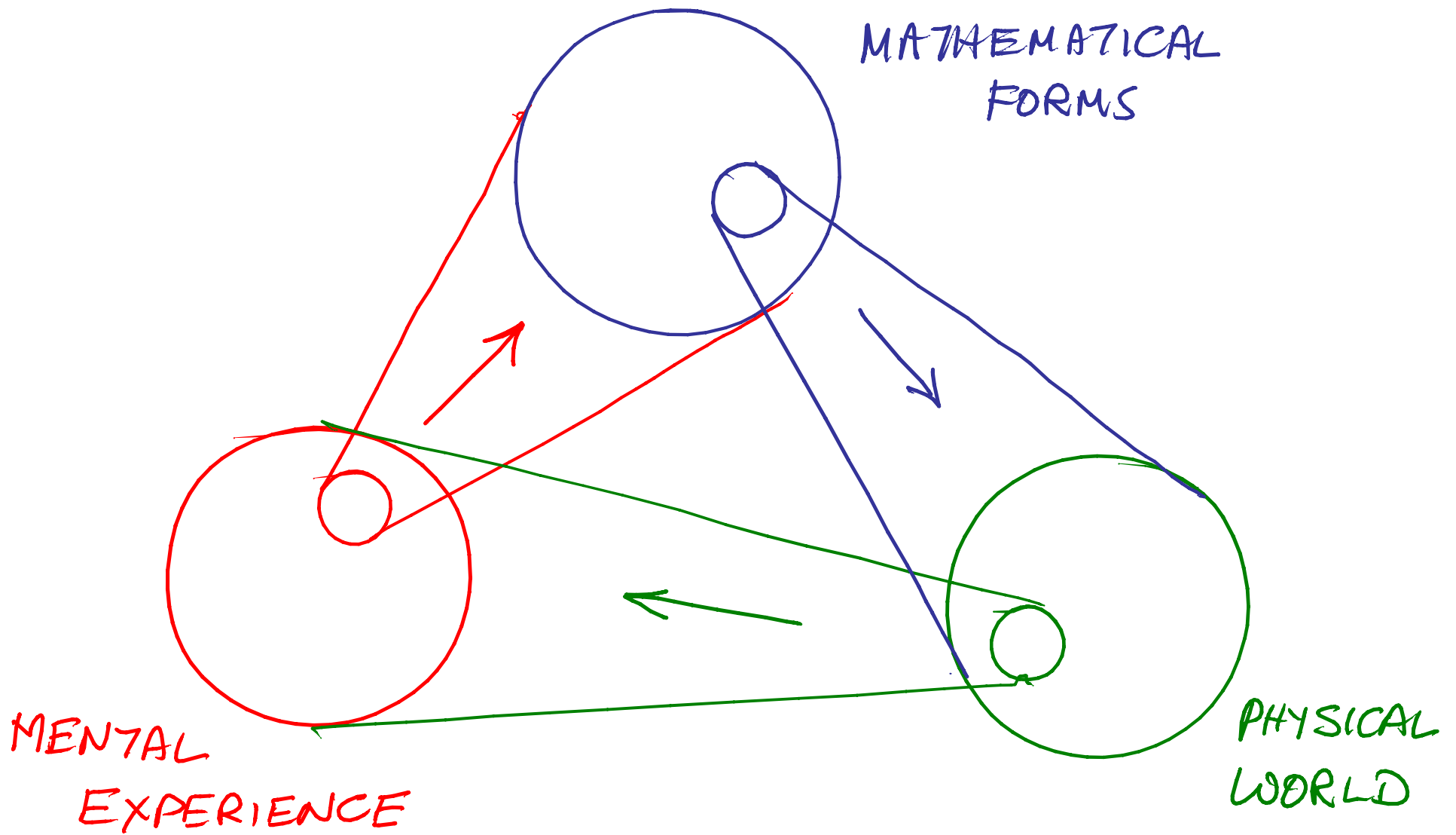
IN WHAT SENSE IS MATHEMATICAL FORM WORLD
REAL?

THE OBJECTS IN IT PRE-EXIST HUMANITY

→ THERE ARE NATURAL NUMBERS
VASTLY GREATER THAN WE
WILL EVER CALCULATE

→ THERE IS A "LAGRANGIAN" FOR
QUANTUM GRAVITY

HOW CAN A COMPLETELY ABSTRACT DOMAIN
HAVE SUCH A DIRECT INFLUENCE ON
THE PHYSICAL WORLD?



ONLY A SMALL DOMAIN IN EACH
WORLD "GENERATES" THE OTHER

COMPARE THIS PICTURE WITH

— FEYNMAN

— WIGNER ↗

THE UNREASONABLE EFFECTIVENESS
OF MATHEMATICS IN THE PHYSICAL SCIENCES

↘ SEE WEBPAGE

