Repetitive Acceleration: Cyclic or Linear

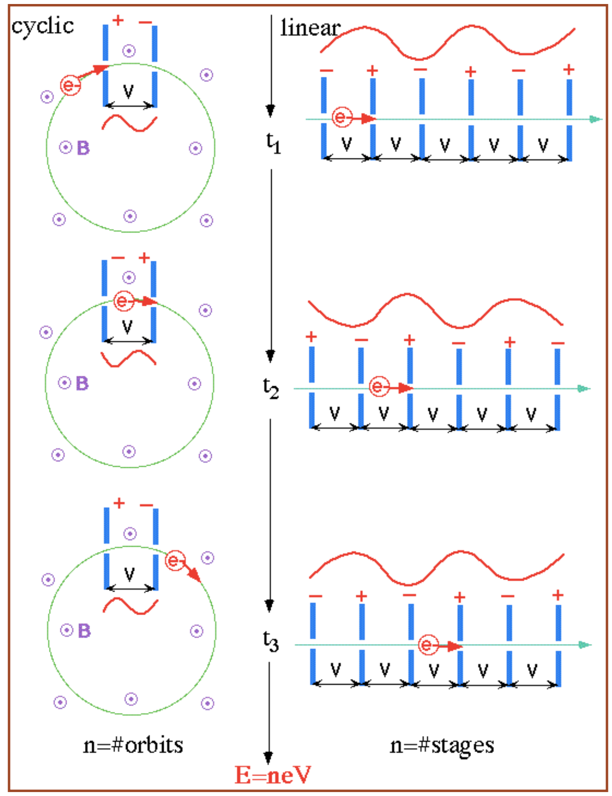
Breakdown voltage (of dry air)
≈ 3MV/m

⇓

Accelerate in Stages

⇓

E=neV



Good News and Bad News about Monopoles

In general, static magnetic fields are much stronger than static electric fields. The Lorentz electromagnetic force law

$$\mathbf{F} = q\,(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad \text{(SI)}$$

tells us that a 1 Tesla magnetic field exerts the same force on a relativistic particle ($v \cong c$) as a 3×10^8 Volt/metre electric field. This can also be seen from the units, *i.e.*

$$1 \text{ Volt/metre} = 1 \text{ (metre/second)Tesla}$$

Using the conversion constant $c = 3\times10^8$ m/s, we have 1m/s = $1/(3\times10^8)$, so

$$3\times10^8 \text{ Volt/metre} = 1 \text{ Tesla}$$

It is easy to generate a 10T (= 3×10^3 MV/m) static magnetic field, but static electric fields above about 3 MV/m are almost impossible. This is because our world is made from particles with electric charge but no magnetic charge, so high electric fields are shorted out by sparks consisting of electrically charged particles. There is no known reason why magnetic charges ("magnetic monopoles") should not exist, and they probably do but very rarely and with masses of the order of 10^{15} GeV/c^2.

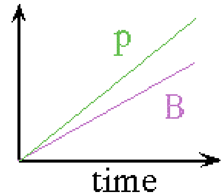
e.g. The complete Lorentz force law is

$$\mathbf{F} = q_e\,(\mathbf{E} + \mathbf{v} \times \mathbf{B}) + q_m\,(\mathbf{B} + \mathbf{v} \times \mathbf{E}) \quad \text{(signs?)}$$

Synchrotrons

- ring of magnets
- small amount of radiofrequency (r.f.) acceleration
- magnetic field increases synchronously with beam momentum

$$B = \frac{p}{|q|r}$$



Example: Fermilab Tevatron

- r = 1 km
- B = 0 - 5 Tesla
- ΔE/orbit ~ 3 MeV (10m @ 1/3 MV/m)

⇒ E_{final} = 1 TeV (protons, antiprotons)

Main Fermilab website is
http://www.fnal.gov
http://www-visualmedia.fnal.gov/VMS_Site/gallery/ph_buildings.html

Fermilab

http://www-visualmedia.fnal.gov/VMS_Site/gallery/ph_accelerator.html

Tevatron and Old Main Ring Magnets

Effective Accelerating Gradient

$$G = \frac{\text{final particle energy}}{\text{length of accelerator}}$$

for a synchrotron

$$G = \frac{E_{max}}{2\pi r} \approx \frac{p_{max}c}{2\pi r} \quad (E >> m)$$

$$\therefore \; G\ (J/m) = \frac{|q|cB_{max}}{2\pi} \quad (SI)$$

$$\therefore \; G\ (eV/m) = \frac{|q/e|cB_{max}}{2\pi}$$

$$= (48\,\frac{MeV/m}{Tesla})\cdot B_{max}$$

e.g.

Tevatron: G ~ 150 MeV/m,
LHC: G ~ 250 MeV/m

c.f. 3 MV/m typical static
high voltage breakdown

http://lhc.web.cern.ch/lhc/
high resolution picture from
http://preprints.cern.ch/cgi-bin/setlink?base=PHO&categ=photo-si&id=9105065

Large Hadron Collider at CERN

Limits to circular accelerators

protons
- size ($$)
- B (H_c, stress $\propto B^2$)
- S/N ($\sigma_{new\ physics} \sim 4\pi/E^2$, $\sigma_{total} \sim 10^2$ mb)

electrons
- synchrotron radiation
  energy loss / orbit
  $= (4\pi/3)\, e^2\, \beta^2\, \gamma^4/r$
  $= 88.5\times10^{-6}\,[E(GeV)]^4/r(m)$

e.g. 88.5 TeV/orbit if E=1TeV and r=1km

http://lhc.web.cern.ch/lhc/
high resolution picture from
http://preprints.cern.ch/cgi-bin/setlink?base=PHO&categ=photo-ac&id=9102075

LEP Copper RF

Acceleration by Electromagnetic Waves

Power per unit area of an electromagnetic wave (Poynting Vector)

$$\mathbf{P} = \mathbf{E}\mathrm{x}\mathbf{B}/\mu_0$$

In vacuum: $E_{max}=cB_{max}$, but E & B are 90° out of phase, so

$$P_{rms} = (E_{rms})^2/c\mu_0$$

$$\therefore\ E_{max} = 2^{1/2}\, E_{rms} = (2P_{rms}\mu_0 c)^{1/2}$$

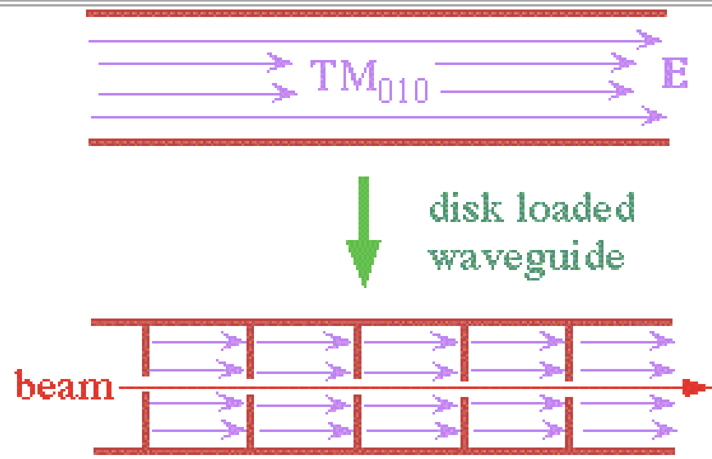
So the maximum electric field is

$$E_{max}(V/m) = 27.5[P_{rms}(W/m^2)]^{1/2}$$

e.g. 10^{20} W/m^2 $\Rightarrow E_{max}$= 275 GV/m

But in free space **E** is transverse to the direction of propagation of the wave, so there can be no continuous acceleration, just oscillation.

Acceleration by Electromagnetic Waves



$v_{phase} > c$

$v_{beam} \lesssim c$

$v_{phase} = c$

e.g. SLAC ($e^{\pm}$ linac)

λ = 10.5 cm (3Ghz)
G = 17 MeV/m
L = 3 km
E_{final} = 50 GeV

Fermilab proton linac injector

Typical limits on accelerating gradients

for copper structures (e.g. SLAC disk-loaded waveguide).