Parity

There are two obvious space transformations: translation ($\mathbf{x} \Rightarrow \mathbf{x}+\delta\mathbf{x}$) and reflection ($\mathbf{x} \Rightarrow -\mathbf{x}$). Translation is a continuous operation, but reflection is discrete - it is impossible to reflect by an infinitesimal amount.

The reflection operation is known as the parity operator, i.e.

$$\boldsymbol{P}\psi(\mathbf{x}) = \psi(-\mathbf{x})$$

In general, the parity operator reverses all 3-vectors, e.g.

$$\text{position} \Rightarrow -\text{ position } (\mathbf{x}\Rightarrow-\mathbf{x})$$

$$\text{momentum} \Rightarrow -\text{ momentum } (\mathbf{p}\Rightarrow-\mathbf{p})$$

but not axial-vectors, e.g.

$$\text{angular momentum} \Rightarrow \text{angular momentum } (\mathrm{L}\Rightarrow\mathrm{L})$$

$$(i.e.\ \mathbf{L} = \mathbf{r}\times\mathbf{p} \Rightarrow (-\mathbf{r})\times(-\mathbf{p}) = \mathbf{L})$$

Spherical harmonics are the eigenfunctions of the orbital angular momentum operators and have parity

$$\boldsymbol{P}Y_{\mathrm{lm}}(\theta,\varphi) = (-1)^{l}Y_{\mathrm{lm}}(\theta,\varphi)$$

The relative parity of particles and antiparticles can be determined from the form of their relativistic wave functions. Particles and antiparticles have the same parity if they are bosons; particles and antiparticles have opposite parity if they are fermions.

If $\psi(x)$ is an eigenfunction of parity, then

$$\boldsymbol{P}\psi(\mathbf{x}) = P_\psi \psi(\mathbf{x})$$

where the eigenvalues of the observable are $P=\pm 1$ because

$$\boldsymbol{P}^2\psi(\mathbf{x}) = \boldsymbol{P}\boldsymbol{P}\psi(-\mathbf{x}) = P_\psi{}^2\,\psi(\mathbf{x})$$

If parity is conserved, then the parity of a system is the product of the parity of its components.

$$\begin{aligned}
\Psi(\mathbf{x}) &= \psi_1(\mathbf{x})\,\psi_2(\mathbf{x}) \\
\therefore\ \boldsymbol{P}\Psi(\mathbf{x}) &= \boldsymbol{P}\,[\psi_1(\mathbf{x})\,\psi_2(\mathbf{x})] \\
&= \psi_1(-\mathbf{x})\,\psi_2(-\mathbf{x}) \\
&= P_1\psi_1(\mathbf{x})\,P_2\psi_2(\mathbf{x}) \\
&= P_1P_2\,\psi_1(\mathbf{x})\psi_2(\mathbf{x}) \\
&= P_\Psi\Psi(\mathbf{x}) \\
\therefore\ P_\Psi &= P_1P_2
\end{aligned}$$

In general, discrete symmetries lead to multiplicative conserved quantitites.

Strong and electromagnetic interactions conserve P (e.g. $\eta \Rightarrow \pi^+\pi^-$ is forbidden); weak interactions do not conserve P.

The Tau-Theta Puzzle

In the early 1950's, two strange particle decays were observed

$$\tau^+ \Rightarrow \pi^+\pi^+\pi^- \qquad \text{and} \qquad \theta^+ \Rightarrow \pi^+\pi^0$$

Pions are eigenstates of parity with intrinsic spin$^{\text{Parity}}$

$$J^P = 0^- \quad \text{(i.e. a pseudoscalar)}$$

The parity of two pions (π_1 & π_2) with relative orbital angular momentum L(1,2) is

$$P(\pi\pi) = P(\pi_1) \times P(\pi_2) \times (-1)^{L(1,2)} = (-1) \times (-1) \times (-1)^{L(1,2)} = (-1)^{L(1,2)}$$

The parity of three pions is

$$P(\pi\pi\pi) = [P(\pi_1) \times P(\pi_2) \times (-1)^{L(1,2)}] \times P(\pi_3) \times (-1)^{L(12,3)} = -\,(-1)^{L(1,2)} \times (-1)^{L(12,3)} = -\,(-1)^{L(1,2)+L(12,3)}$$

where L(12,3) is the orbital angular momentum between the third pion and the first two. If J is the spin of the parent particle, then by angular momentum conservation, we must have

$$J(\theta^+) = L(1,2) \qquad J(\tau^+) = L(1,2) + L(12,3)$$

If parity is conserved in the decay, the parity of the parent particle is the same as the parity of the final state pion system. So we expect

$$P(\theta^+) = P(\pi\pi) = (-1)^{L(1,2)} = (-1)^{J(\theta+)} \qquad P(\tau^+) = P(\pi\pi\pi) = -\,(-1)^{L(1,2)+L(12,3))} = -(-1)^{J(\tau+)}$$

So the θ^+ and τ^+ cannot have the same spin$^{\text{parity}}$ (J^P) if parity is conserved, and the θ^+ and τ^+ must be different particles.

The existence of two new particles would not be of concern except that the θ^+ and τ^+ appeared to be otherwise identical - they had the same masses, the same lifetimes, and were produced and interacted with the same lifetimes.

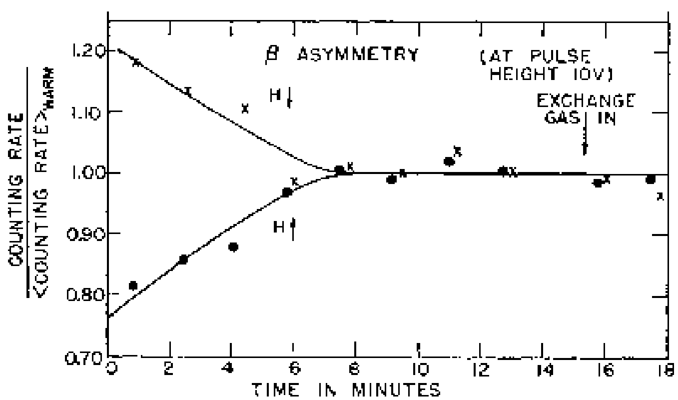
Further measurements showed that $J(\theta^+) = J(\tau^+) = 0$.

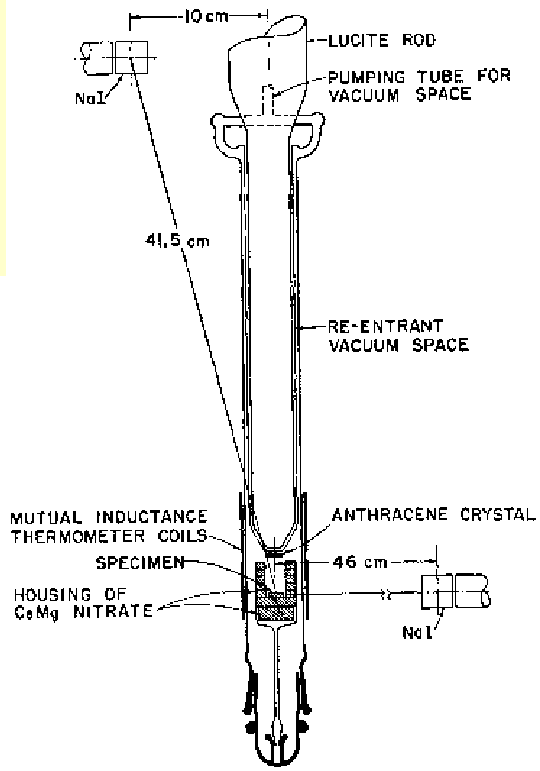
Why would there be two identical particles differing only by their parity?

Parity Violation

In 1956 T. D. Lee and C. N. Yang pointed out that parity conservation had never been tested for the weak interaction. If parity was not conserved, the θ^+ and τ^+ were simply the three and two pion decay modes of a single particle.

C.S. Wu quickly tested parity conservation in beta decays of Co^{60} and discovered that parity is 100% violated. Neutrinos are "left-handed", their spin is always anti-parallel with their momentum.





These results were immediately confirmed by other experiments. These, or similar experiments, could have been done 30 years earlier. (In fact, some experiments probably saw parity violating effects in the 1920's, but nobody realized it since nobody expected it. Part of the problem was that the idea and importance of "parity" was only recognized in the 1930's by Wigner.)

The θ^+/τ^+ is now known as the K^+.

Charge Conjugation

Charge conjugation reverses all "charges", e.g electric charge, baryon number, lepton number, strangeness, muon number, ….

$$\boldsymbol{C}\psi = \overline{\psi}$$

Positions, energy, momenta, angular momenta, and spins are not affected by the C operator.
If ψ is an eigenstate of of charge conjugation, then

$$\boldsymbol{C}\psi = \mathrm{C}\psi$$

where the eigenvalues of the observable are C=±1 because

$$\boldsymbol{C}^2\psi = \boldsymbol{C}\mathrm{C}\overline{\psi} = \mathrm{C}\boldsymbol{C}\overline{\psi} = \mathrm{C}^2\psi$$

A wavefunction may be in an eigenstate of C

$$\boldsymbol{C}\psi = \pm\psi$$

if all additive quantum numbers are zero. Electromagnetic fields are produced by charges which change sign under charge conjugation, so

$$\boldsymbol{C}\gamma = (-1)\gamma \qquad (J^{PC}=1^{--})$$

The C parity of neutral fermion-antifermion systems can be determined either by experiment, or by more detailed relativistic quantum analysis giving $C=(-1)^{L+S}$.

$$\boldsymbol{C}\pi^0 = (+1)\pi^0 \qquad (J^{PC}=0^{-+}) \qquad\qquad \boldsymbol{C}\rho^0 = (-1)\rho^0 \qquad (J^{PC}=1^{--})$$

(Where the particle symbol represents the "charge" part of the particle wavefunction.) Particles which are not their own antiparticles cannot be eigenstates of C, e.g.

$$\boldsymbol{C}\pi^+ = \pi^- \qquad (J^P=0^-) \qquad\qquad \boldsymbol{C}(\text{neutron}) = \text{antineutron} \qquad (J^P={}^1/_2{}^+)$$

If charge conjugation is a symmetry of the physics, then the charge conjugation eigenvalue is a multiplicative conserved quantity. Strong and electromagnetic interactions conserve C (e.g. $\pi^0 \Rightarrow \gamma\gamma\gamma$ is forbidden); weak interactions do not conserve C.

CP

CP is the combined charge conjugation - parity operator. The weak interaction violates C and P a 100%, but CP is almost conserved. The only place this CP violation has yet been observed is in the the neutral kaon system.

Strangeness eigenstates:

$$K^0 = d\bar{s} \ , \ \overline{K}^0 = \bar{d}s$$

CP eigenstates:

$$K_1 = \frac{(K^0 + \overline{K}^0)}{\sqrt{2}}, \ K_2 = \frac{(K^0 - \overline{K}^0)}{\sqrt{2}}$$

Mass eigenstates:

$$K_S = \frac{(K_1 + \varepsilon K_2)}{\sqrt{1+|\varepsilon|^2}}, \ K_L = \frac{(K_2 + \varepsilon K_1)}{\sqrt{1+|\varepsilon|^2}}$$

$K_2 \Rightarrow \pi\pi\pi$ is allowed, but $K_2 \Rightarrow \pi\pi$ is forbidden by CP invariance. The K_L ("K-long") is mostly K_2 and has a much longer lifetime than the K_S ("K-short") because there is so little phase space for $K \Rightarrow \pi\pi\pi$ decays relative to $K_2 \Rightarrow \pi\pi$ decays.