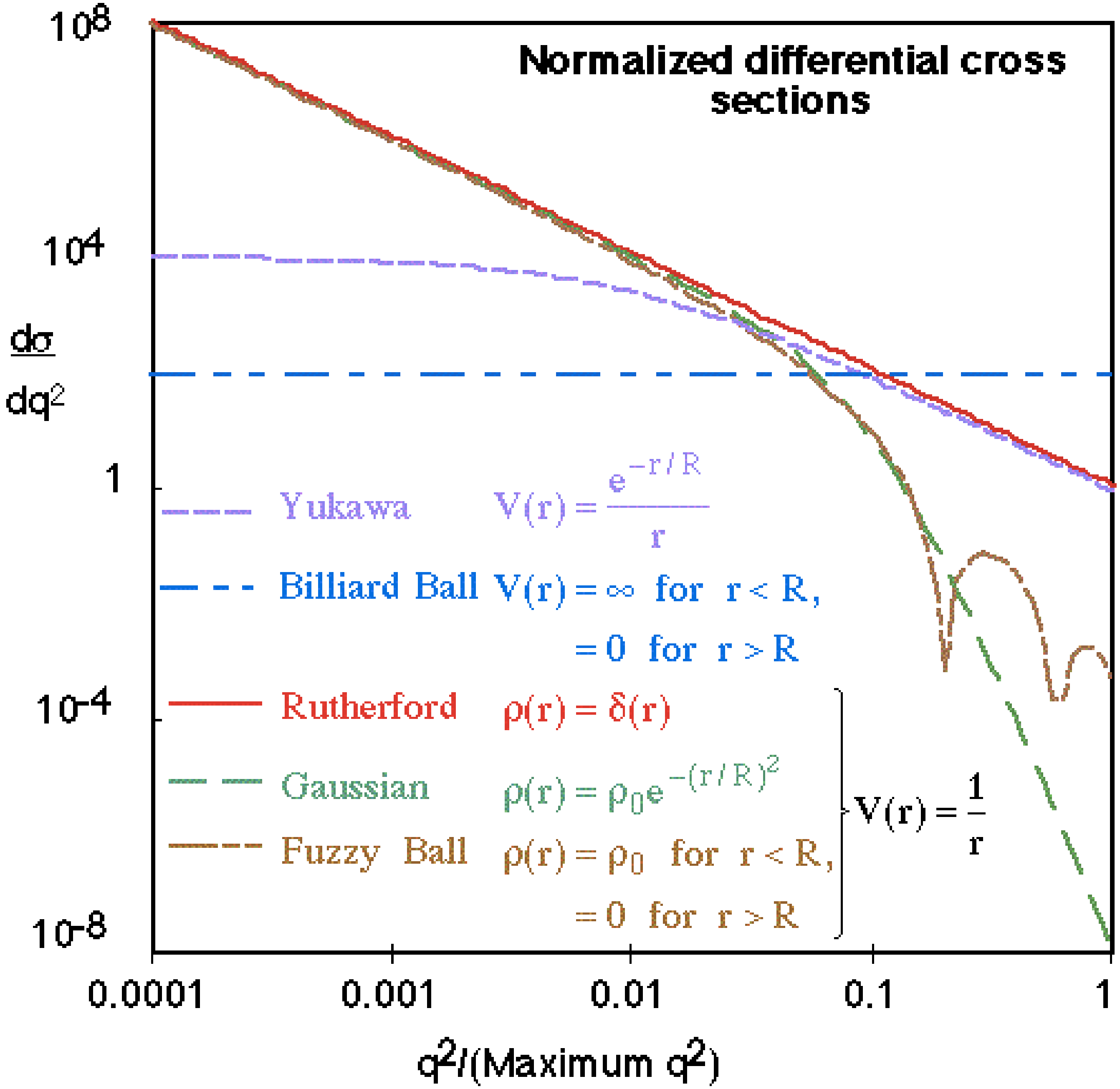


Observing structure

Comparison of differential cross sections for scattering a spinless point particle from either a Rutherford point particle or particles with a size $R=10/q_{max}$.

Form Factors

For electromagnetic scattering of incident point particles, we can consider scattering from an extended target as point-point scattering integrated over the charge distribution of the target:

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_R |F(\mathbf{q})|^2$$

The **Form Factor** is the Fourier transform of the target's charge distribution:

$$F(q^2) = \int \rho(r)\, e^{i\mathbf{q}\cdot\mathbf{r}}\, d^3r$$

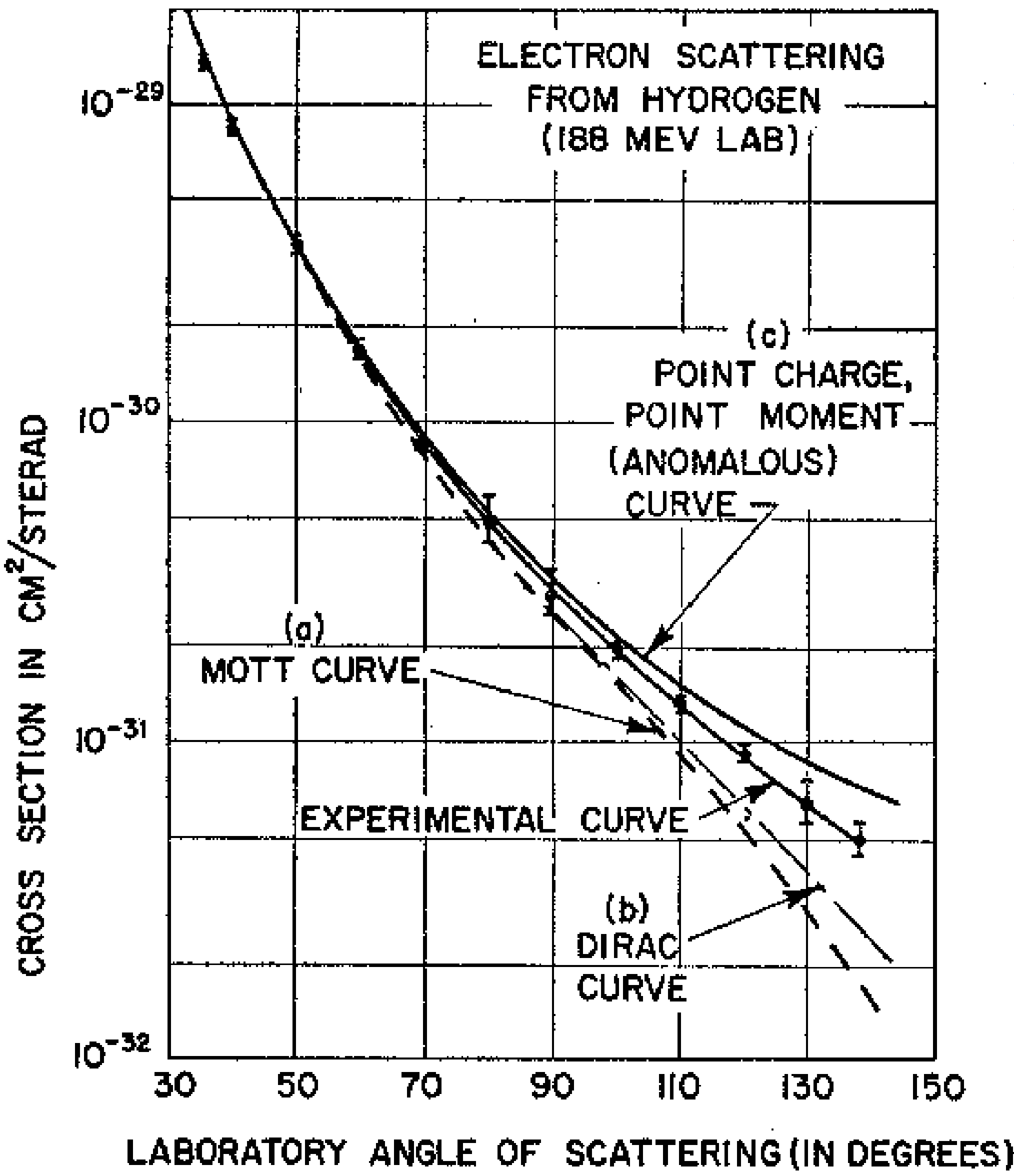
and $(d\sigma/d\Omega)_R$ is the reference cross section for point-point scattering.

For electron scattering the reference cross section is usually Mott scattering, but Rutherford scattering can be used in the low energy limit. Mott scattering is the scattering of spin 1/2 electrons by spinless, unit charge point particles:

$$\boxed{\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}\left(1-\beta^2\sin^2\frac{\theta}{2}\right) \xrightarrow[\beta_e=\frac{v_e}{c}\to 1]{} \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}\cos^2\frac{\theta}{2}}$$

The velocity dependent term is because of the point charge interacting with the magnetic dipole moment of the electron.

Proton has a size ~ 1fm



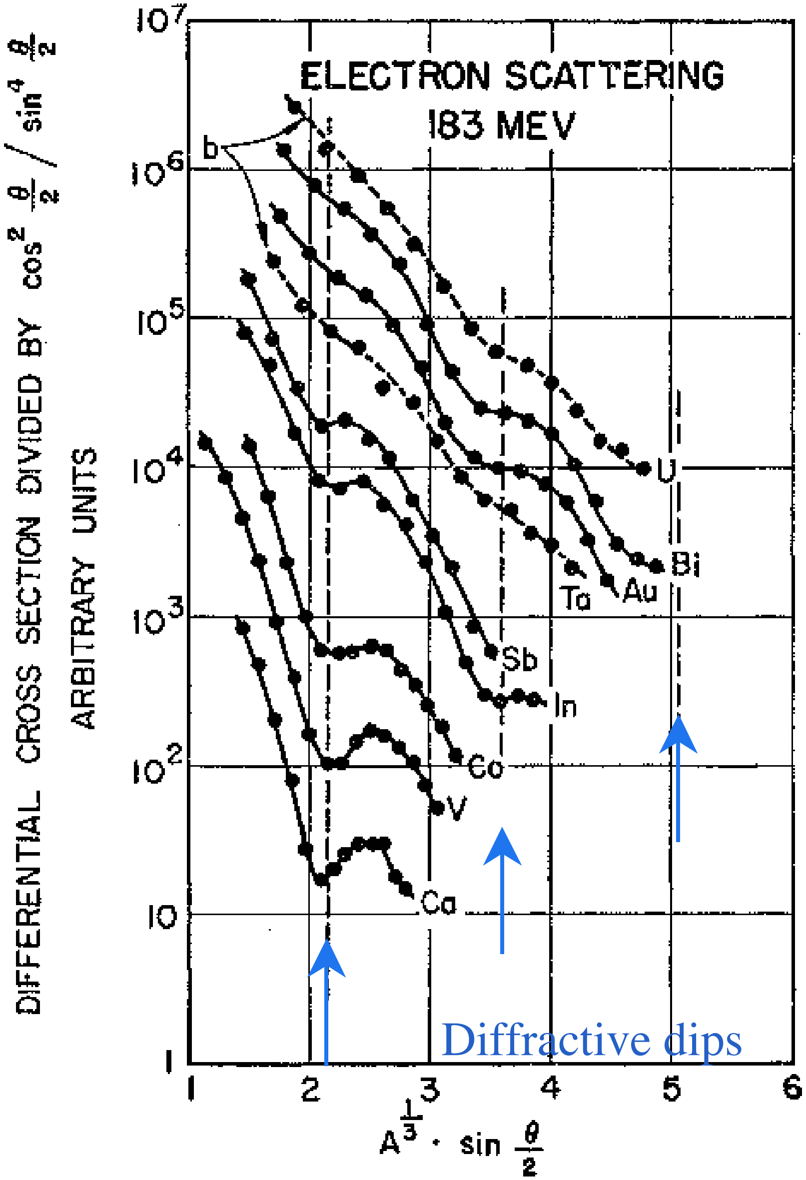
"Elastic Scattering of 188-Mev Electrons from the Proton and the Alpha Particle", R. W. McAllister and R. Hofstadter, Physical Review 102 (1956) 851–856.

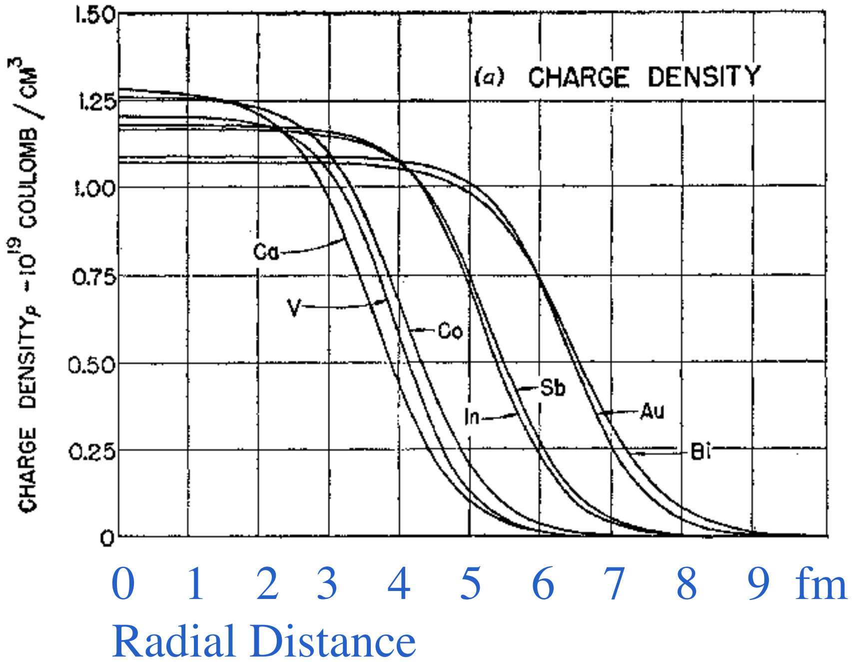
A proton's or neutron's size is reasonably well parameterized by a "dipole" form factor

$$F_{nucleon}(q^2) = 1/(1+|q^2|R^2)^2$$

corresponding to an exponential radial charge distribution with mean radius R ~ 0.85 fm.

Nuclei are fuzzy balls





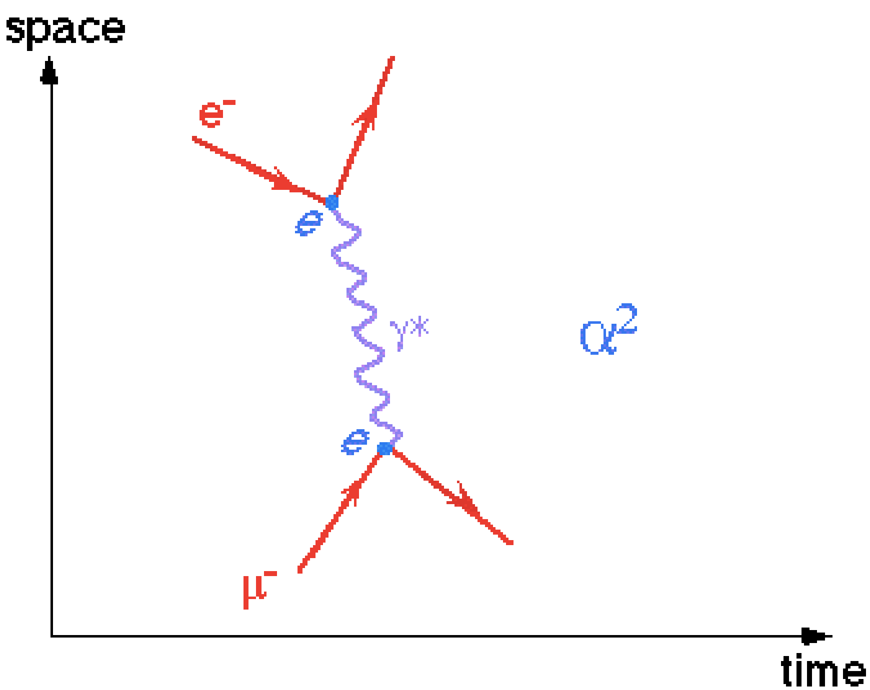
"High-Energy Electron Scattering and the Charge Distributions of Selected",
Beat Hahn, D. G. Ravenhall, and Robert Hofstadter , Physical Review 101 (1956) 1131–1142.

Muon Size?

Scattering electrons from muons at high energy is not (yet) technically feasible, but the Feynman diagrams for scattering and annihilation are just the same diagram rotated 90°. This immediately tells us that in the high energy limit is

$$d\sigma/dq^2 \sim \alpha^2/q^4 \qquad (s=q^2)$$

which is, of course, the expected and familiar result



Since the physics of annihilation is essentially the same as scattering, it is not surprising that we have the same sensitivity to form factors, i.e.

$$\sigma(e^+e^- \to \mu^+\mu^-) = (4\pi\alpha^2/3q^2)\,|F(s)|$$

where, for example, a dipole form factor would be

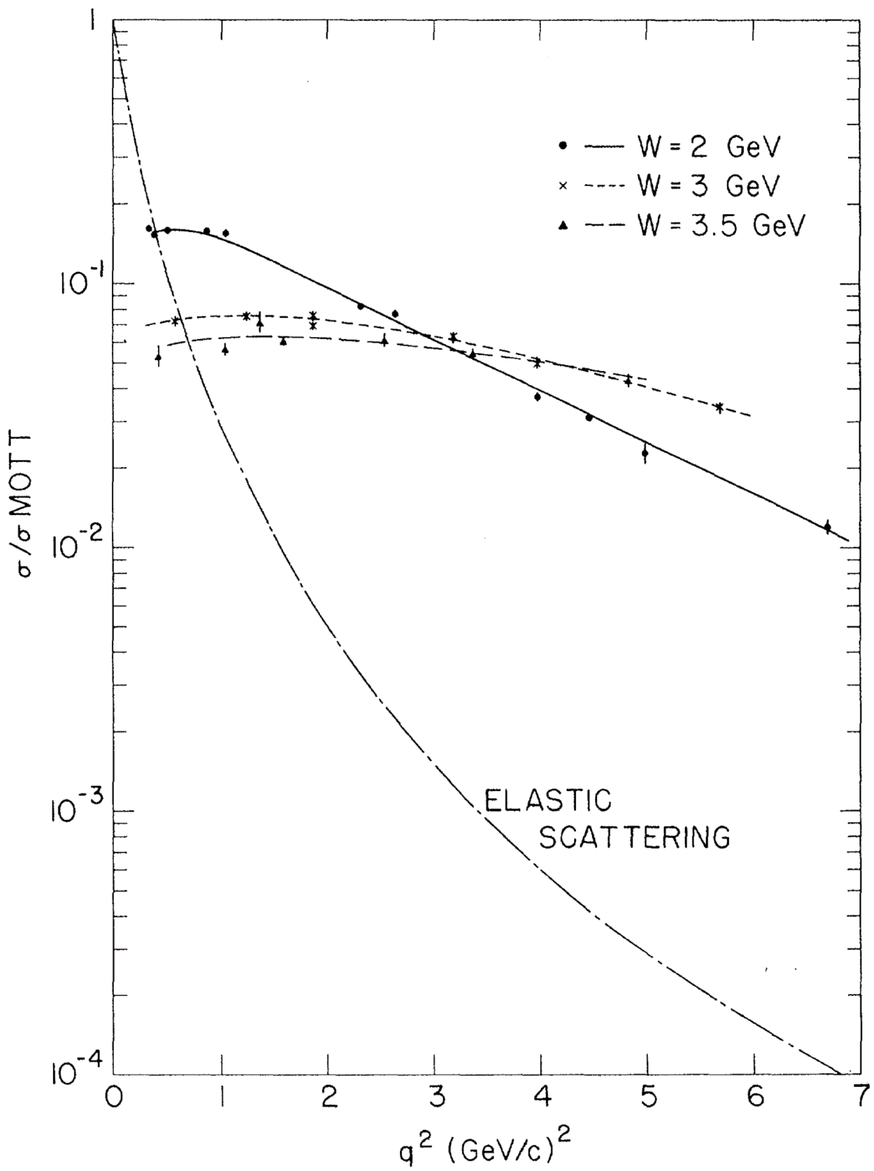
$$F_{muon}(s) = 1/(1+sR^2)^2$$

Current annihilation and magnetic moment data are consistent with R=0 ($<10^{-4}$fm, or $R^{-1} <$ 2-4 TeV).

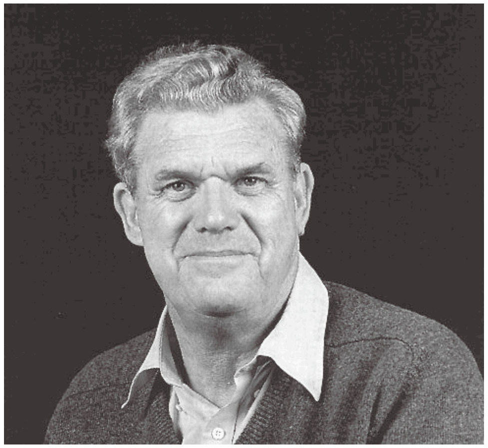
$$\left(\frac{d\sigma}{d\Omega}\right)_{e\mu} = \left(\frac{d\sigma}{d\Omega}\right)_{Rutherford}\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M_\mu^2}\sin^2\frac{\theta}{2}\right)$$

(Q^2 is squared energy-momentum transfer.)

Protons contain point constituents

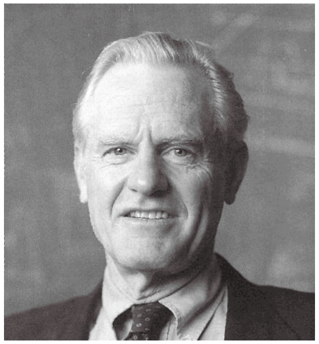


"Observed Behavior of Highly Inelastic Electron-Proton Scattering", M. Breidenbach *et al*. , Physical Review Letters 23 (1969) 935–939.

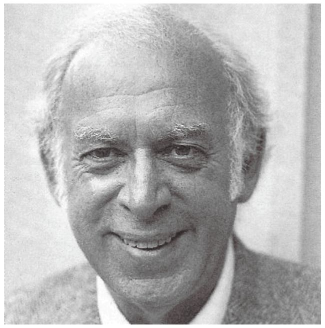


Dick Taylor

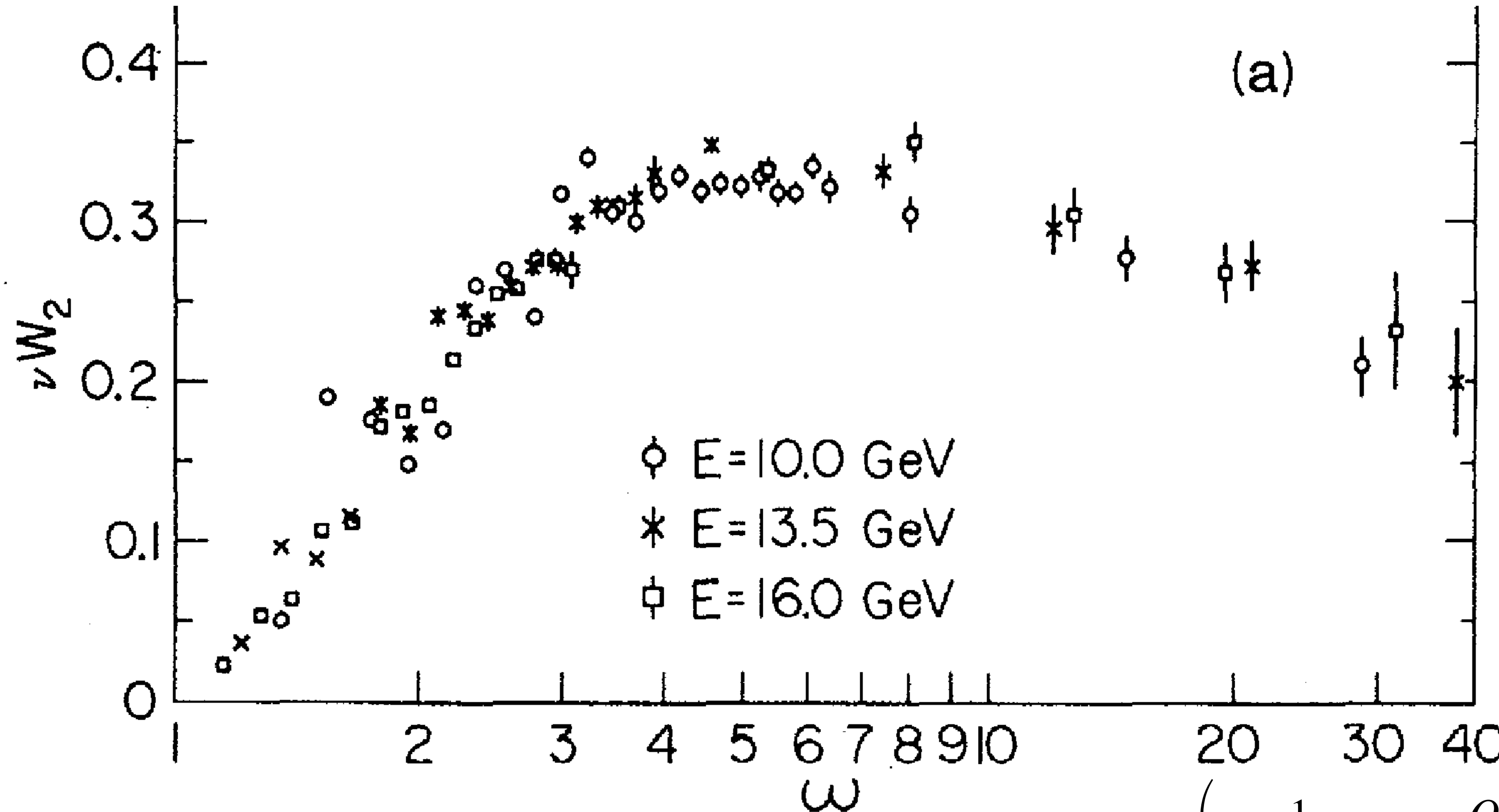
Henry Kendall



Jerome Friedman



Quark-Partons and Structure Functions



$$\left(\omega = \frac{1}{x}, x \equiv \frac{Q^2}{2M\left(E-E'\right)}\right)$$

$$\frac{d\sigma}{dE'd\Omega'} = \frac{\alpha^2}{4E^2\left(E-E'\right)\sin^4\left(\frac{\theta}{2}\right)}\left[F_2\left(x,Q^2\right)\cos^2\frac{\theta}{2} + F_1\left(x,Q^2\right)\frac{Q^2}{xM^2}\sin^2\frac{\theta}{2}\right]$$