Breit-Wigner Partial Wave Analysis

The cross section for pure elastic scattering for the l^{th} partial wave is

$$\sigma_{el}^{l} = \frac{4\pi}{k^2}(2l+1)\sin^2\delta_l = \frac{4\pi}{k^2}(2l+1)\frac{1}{1+\cot^2\delta_l}$$

This a has maximum when $\delta_l=\pi/2$. For a spinless beam and target, the phase can only depend on the invariant mass of the system, *i.e.* $\delta_l=\delta_l(E)$, where $E=s^{1/2}$, so the maximum will occur at some energy M, and we can make an expansion

$$\sigma_{el}^{l} = \frac{4\pi}{k^2}(2l+1)\frac{1}{1+\left[\underbrace{\cot\delta_l(M)}_{=0}+(E-M)\left[\frac{d\cot\delta_l(E)}{dE}\right]_{E=M}+\ldots\right]^2}$$

In lowest order we have

$$\sigma_{el}^{l} = \frac{4\pi}{k^2}(2l+1)\frac{1}{1+\left[\frac{2(E-M)}{\Gamma}\right]^2} \qquad \text{where } \frac{2}{\Gamma} \equiv -\left[\frac{d\cot\delta_l(E)}{dE}\right]_{E=M}$$

which is the Breit-Wigner resonance formula for a particle with lifetime $\tau=1/\Gamma$

$$\boxed{\sigma_{el}^{l} = \frac{4\pi}{k^2}(2l+1)\frac{\Gamma^2/4}{(E-M)^2+\Gamma^2/4}}$$

Fourier transform of exponential decay

A spinless particle with an infinite lifetime is a single plane wave with a single energy:

$$\psi = e^{i(\vec{p}\cdot\vec{x} - Et)}$$

For an unstable particle, an imaginary term is added to the energy,

$$\psi = e^{i(\vec{p}\cdot\vec{x} - [E + i\gamma\Gamma/2]t)} = e^{i(\vec{p}\cdot\vec{x} - Et)} e^{-\gamma\Gamma t/2}$$

which gives the expected exponential decay law for the rate as a function of time

$$\psi^* \psi = e^{-\gamma\Gamma t}$$

where $\gamma = E/M$ is the usual relativistic time dilation factor, M is the mass of the particle, and $\tau = \hbar/\Gamma$ is the mean lifetime of the particle.

A particle with a finite lifetime can also be described as an infinite sum of plane waves with different energies. The correct sum is just the Fourier transform of the exponential decay law

$$\phi(E) = \int_0^\infty \psi(t) e^{iEt} dt = \int_0^\infty e^{i(\vec{p}\cdot\vec{x} - Et)} e^{-\gamma\Gamma t/2} e^{iEt} dt$$

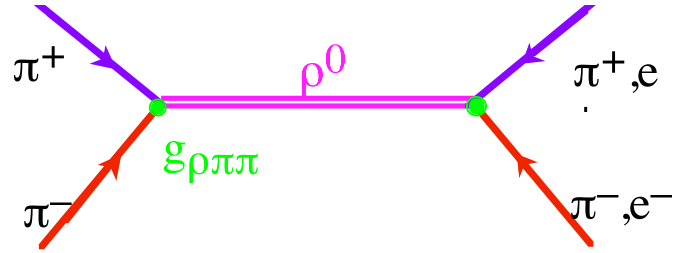
$$= e^{i\vec{p}\cdot\vec{x}} \int_0^\infty e^{-\gamma\Gamma t/2} e^{i(E - \gamma M)t} dt$$

$$= \frac{e^{i\vec{p}\cdot\vec{x}}}{i(E - \gamma M) + \Gamma/2}$$

So the rate as a function of energy in the non-relativistic ($\gamma \approx 1$) limit is

$$\boxed{|\phi(E)|^2 = \frac{1}{(E - M)^2 + \Gamma^2/4}}$$

Resonances



Scattering particles can temporarily form a particle (in what is known as the "s–channel") if they have the correct quantum numbers, for example $\pi^+\pi^-$ scattering can be mediated by a ρ^0. This will cause a resonant maximum in the scattering amplitude (*i.e.* $\delta_l = \pi/2$).

Pions are spinless particles, so they can only couple to particles with spin J=l, but in general the production cross section for a resonance, R, of total angular momentum J and mass M produced by two particles a & b (with spins s_a & s_b)

$$\boxed{\sigma_{ab\to R}(E) = \frac{4\pi\lambda\!\!\!^{-2}(2J+1)\Gamma_{ab}\Gamma/4}{(2s_a+1)(2s_b+1)\left[(E-M)^2+\Gamma^2/4\right]}}$$

where we have averaged over the possible incoming spin states (since only one such state will match each polarization state of the resonant particle) and

- Γ = total width of resonance ($\times\Gamma/\Gamma_{\text{beam}}$ if the beam energy spread Γ_{beam} is greater than the intrinsic width of the resonance).
- Γ_{ab} = partial width for decay into (or production from) the ith decay channel, *i.e.* $R\to ab$.

Notes on Production and Decays

- The total width is the sum of all the partial widths $\left(\Gamma = \sum_i \Gamma_i\right)$.

- The branching ratio (more accurately but less commonly known as the branching fraction) is the probability of decay of a particle into a particular channel, with $B(R \to ab) = \Gamma_{ab}/\Gamma$.

- If a particle is produced in one channel (i) and observed in another channel (j), *e.g.*

$$\pi^- p \to \Delta^0 \to \pi^0 n$$
$$e^+ e^- \to Z^0 \to \text{hadrons}$$

then the rate at which events are observed is

$$R = L \cdot \sigma_i \cdot B_j \propto \Gamma_i B_j \propto B_i B_j \propto \Gamma_i \Gamma_j$$

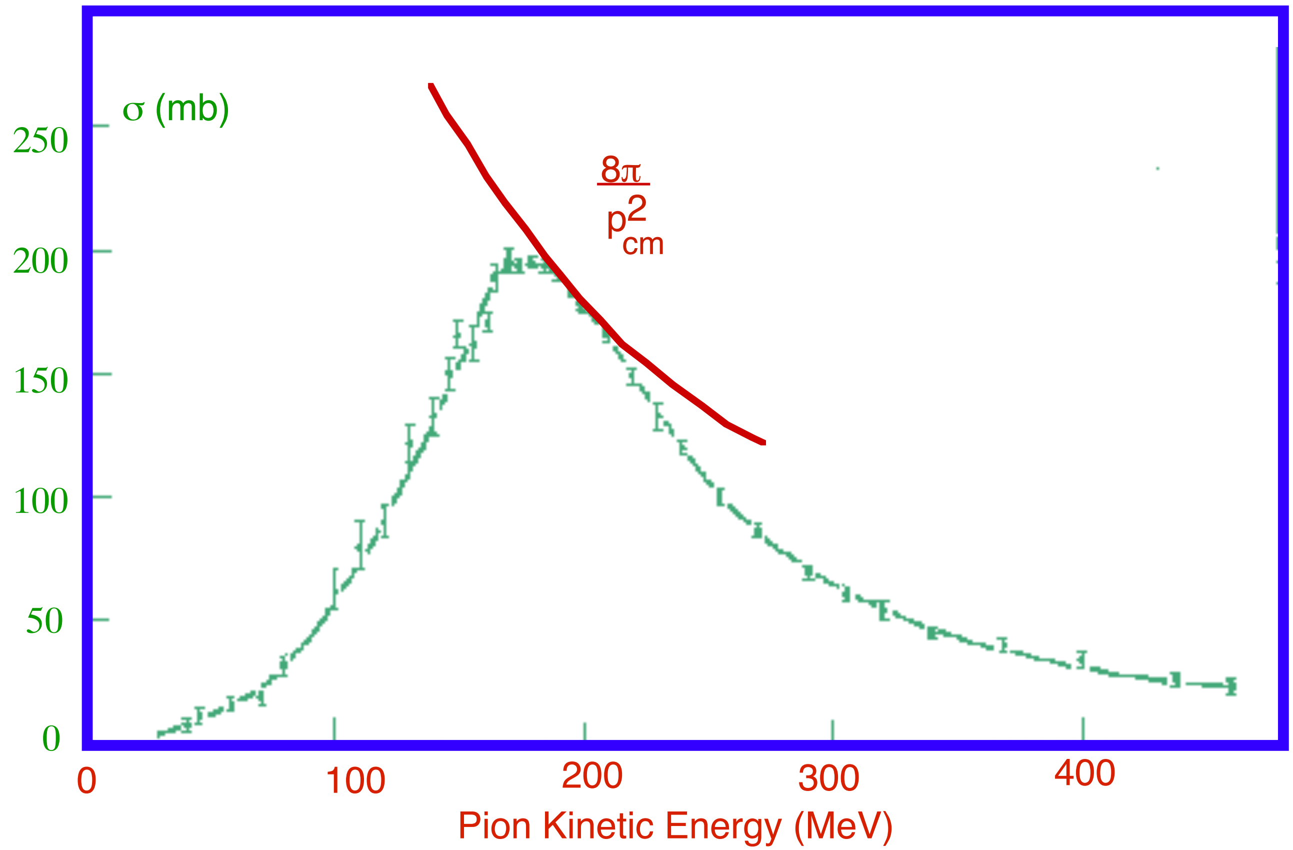
- The "partial lifetime" is defined to be $\tau_i = \hbar / \Gamma_i$.

- The "partial decay rate" is defined to be $R_i = n\Gamma_i = n_0 e^{-t/\tau} \Gamma_i$.

- The partial widths and branching ratios are determined by the underlying physics, *e.g.*

$\Gamma(\rho) = 150.2 \pm 0.8$ MeV	(Λ_{QCD})
$\tau(\mu) = 2.19703 \pm 0.00004 \mu s$	(G_F)
$BR(K_L \to \pi^+ \pi^-) = 0.2056 \pm 0.033\%$	(δ_{CP})

π^+p total cross section: $\Delta^{++}(1236)$

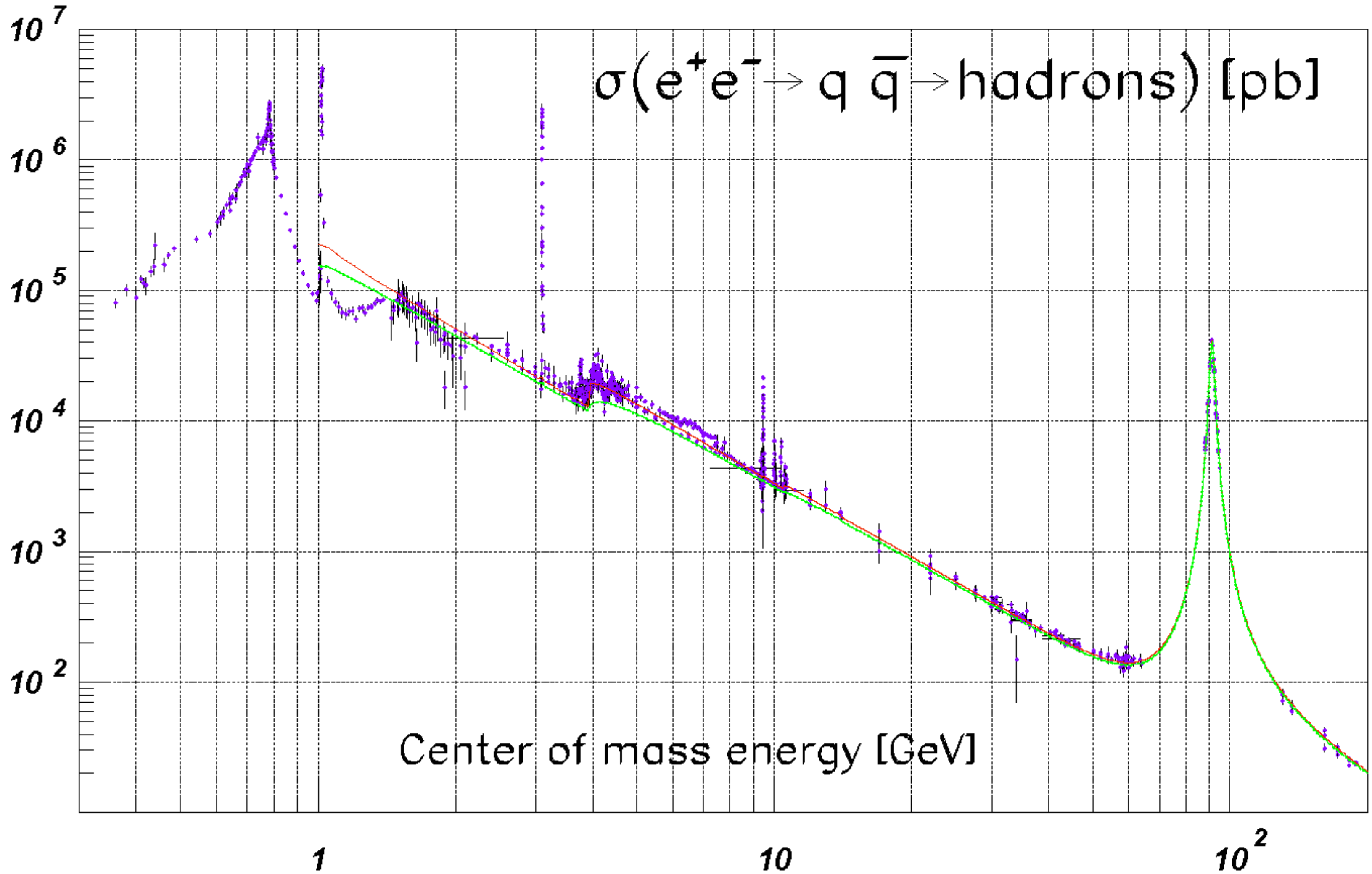


Mass: M = 1232 MeV   Full Width: Γ=120 MeV   Spin[Parity] (J^P): $^3/_2{}^+$

Branching Ratio : BR($\Delta^{++} \rightarrow \pi^+ p$)=99.5%

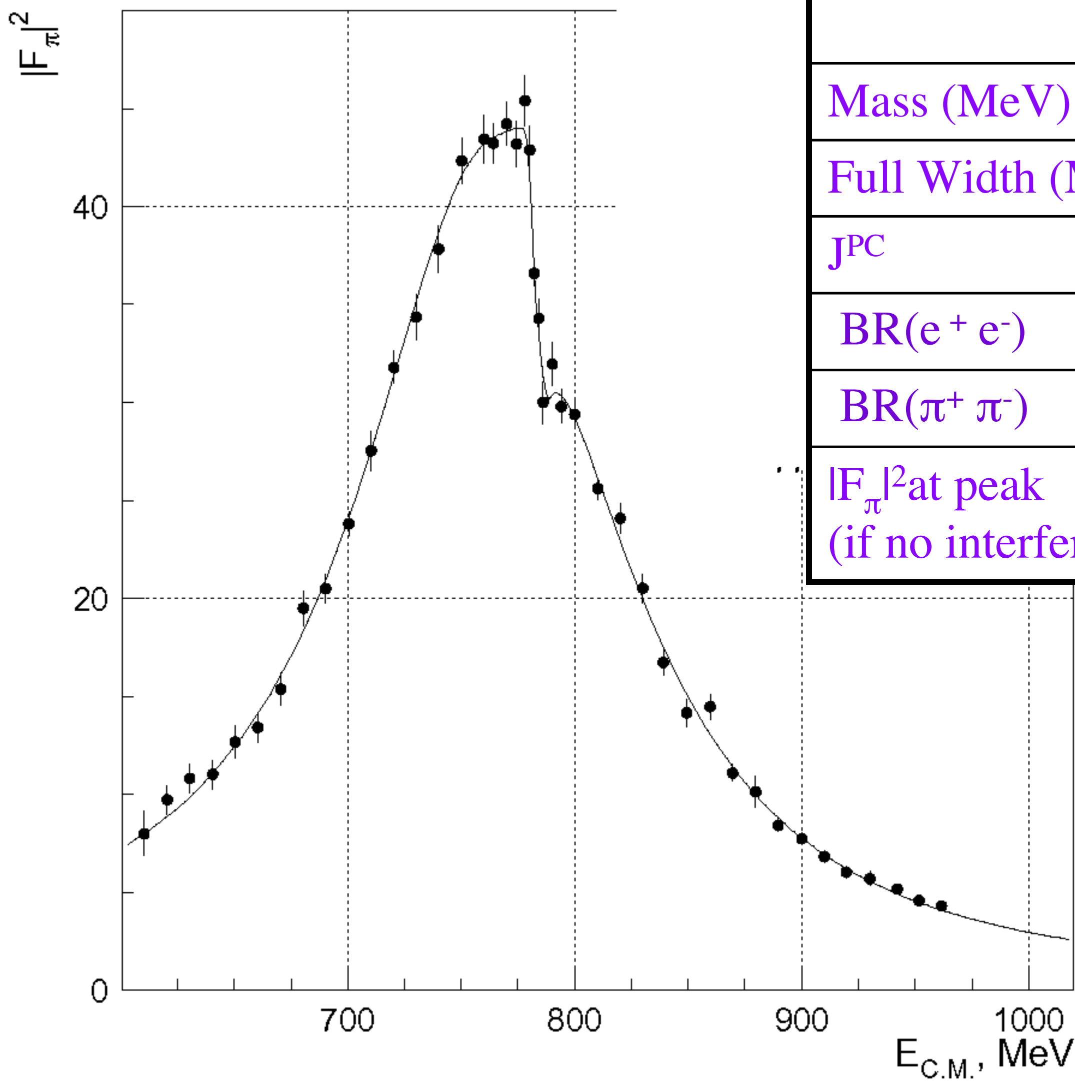
$\pi^+ p \rightarrow \pi^+ p$ cross section at peak of resonance: $$\sigma_{peak} = \frac{4\pi(\hbar c)^2(2J_{\Delta^{++}}+1)}{p^2_{cm}(2s_{\pi^+}+1)(2s_p+1)} = \frac{8\pi(\hbar c)^2}{p^2_{cm}}$$

$e^+e^- \to$ hadrons $\quad (J^{PC}=1^{--})$



O.V. Zenin *et al*., "A compilation of total cross-section data on $e^+e^-\to$hadrons and pQCD Tests", Submitted to J.Phys.G (May 2001); [http://arXiv.org/pdf/hep-ph/0110176].

ρ^0 and ω in $e^+e^- \to \pi^+\pi^-$



	ρ^0	ω^0
Mass (MeV)	769	782
Full Width (MeV)	151	8.4
J^{PC}	1^{--}	1^{--}
BR($e^+ e^-$)	4.5×10^{-5}	7.2×10^{-5}
BR($\pi^+ \pi^-$)	≅100%	2.2%
$\lvert F_\pi\rvert^2$at peak (if no interference)	38	1.3

R.R.Akhmetshin *et al*. [CMD-2 Collaboration], "Measurement of $e^+e^- \to \pi^+\pi^-$ cross section with CMD-2 around ρ^0 meson", Physics Letters B **527** (2002) 161. [http://arXiv.org/pdf/hep-ex/0112031].

Comments on $e^+e^- \to \pi^+\pi^-$

The non-resonant QED (Quantum Electrodynamic) cross section for the production of pions is

$$\sigma\left(e^+e^- \to \pi^+\pi^-\right) = F_\pi^2 \beta^3 \frac{\pi\alpha^2}{3s}(\hbar c)^2$$

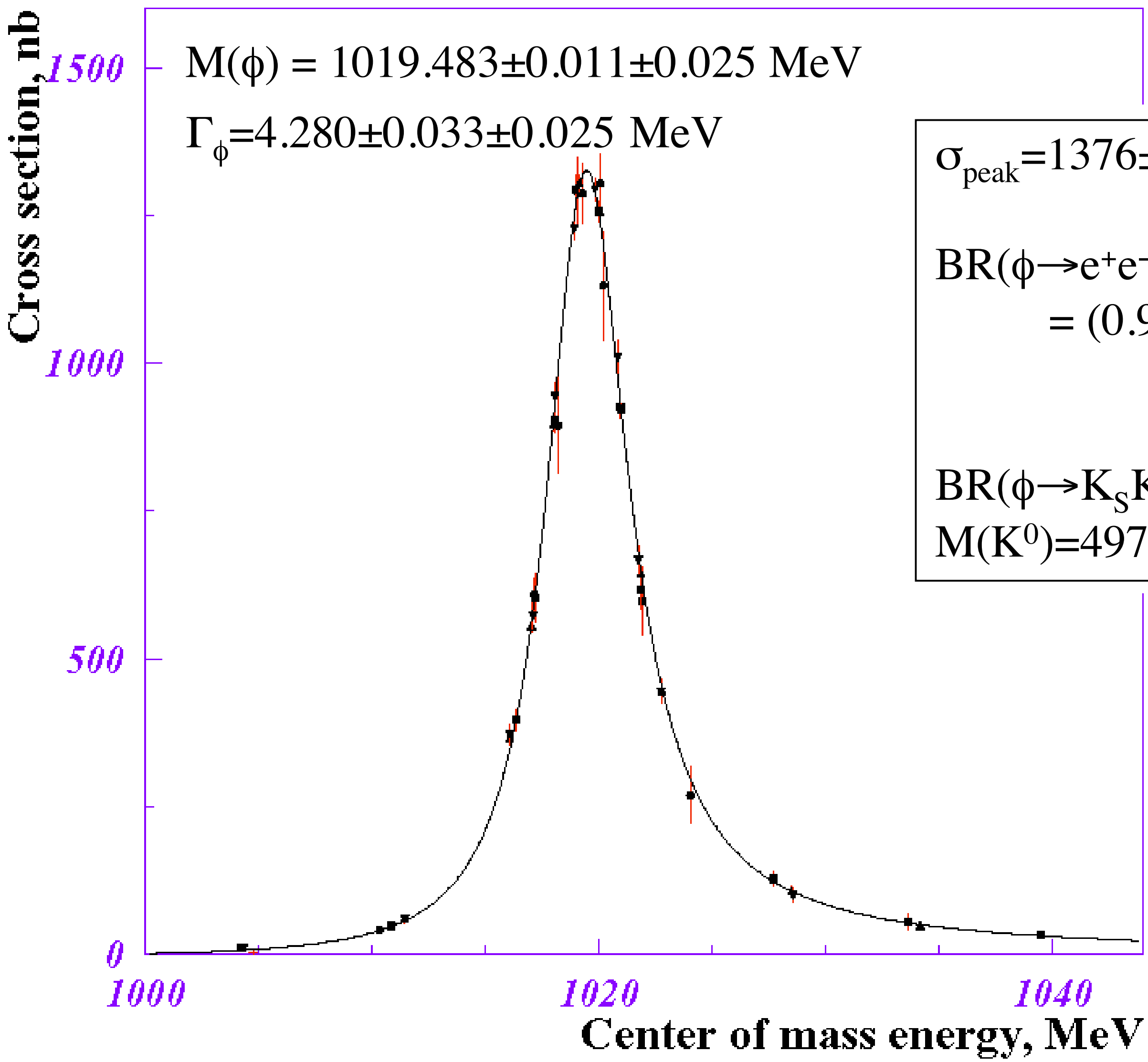
where $\frac{\pi}{3s}$ is the cross section for massless point pions, β^3 is the threshold dependence for massive pions (139.6 MeV/c^2), and F_π^2 is the form factor for real composite pions.

The Breit-Wigner resonance formula is derived for $\Gamma \ll M$, but this is not always true, *e.g.* for the ρ meson or Δ baryon, and phase space changes over the width of the resonance will make it asymmetric.

The only other neutral, flavourless meson in this mass range is the η (547MeV). It is not observed in e^+e^- because it is a pseudoscalar ($J^{PC}=0^{-+}$) meson, *i.e.*

- it cannot mix with a single virtual photon to mediate a decay into an electron-positron pair. (The theoretical branching ratio for $\eta \to e^+e^-$ is about 10^{-10}.)
- decay into $\pi^+\pi^-$ would violate CP symmetry, so it decays into $\pi^+\pi^-\pi^0$.

$e^+e^- \to \phi \to K_S K_L$ $(J^{PC}=1^{--})$



σ_{peak}=1376±6±23nb

⇓

BR($\phi \to e^+e^-$)·BR($\phi \to K_S K_L$)

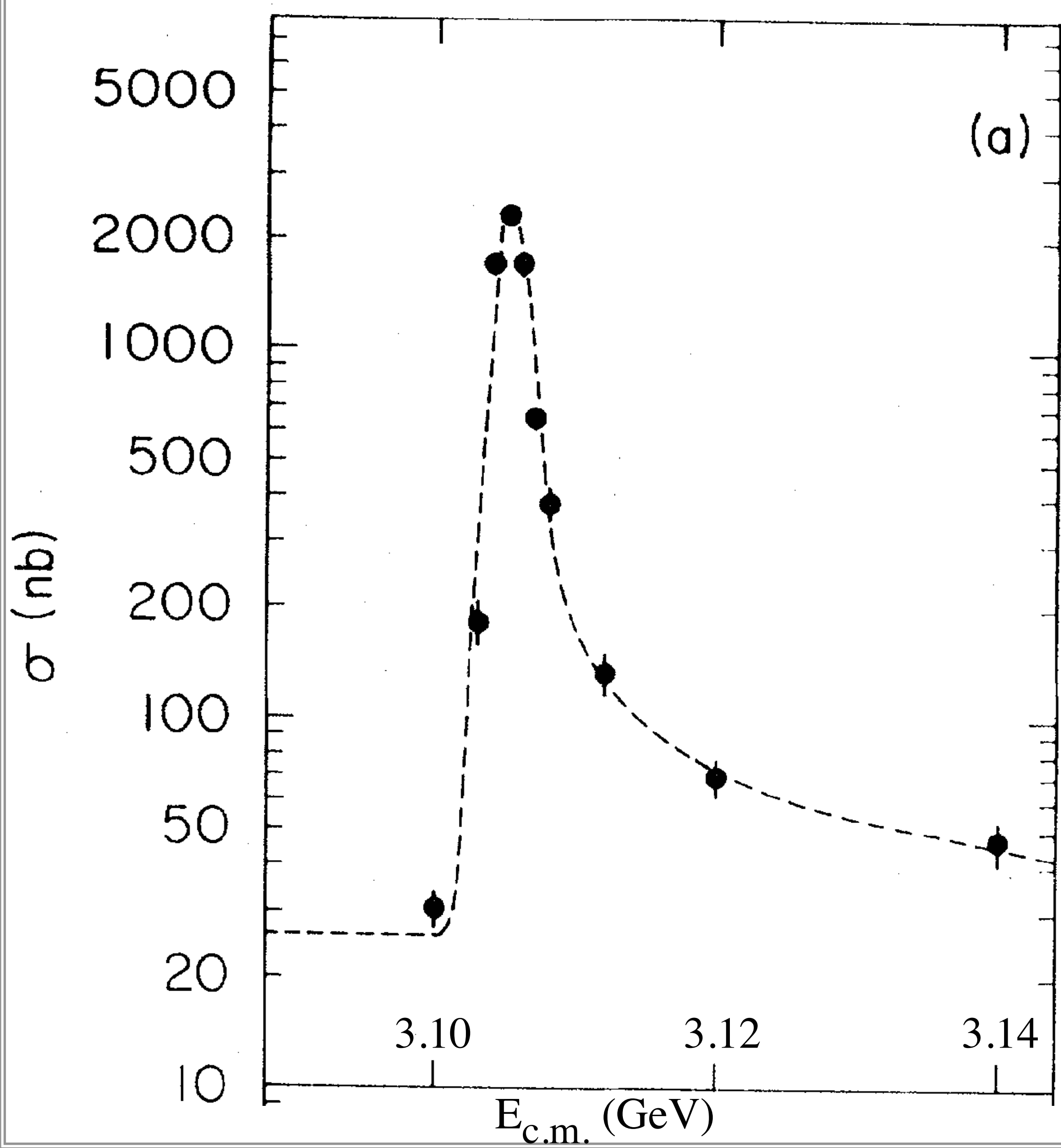
= (0.975±0.004±0017)×10^{-4}

BR($\phi \to K_S K_L$) = 33.8±0.6%

M(K^0)=497.7 MeV

R.R.Akhmetshin *et al*. [CMD-2 Collaboration],"Measurement of the ϕ meson parameters with CMD-2 detector at VEPP-2M Collider", Physics Letters B 466 (1999) 385; Erratum-ibid. B508 (2001) 217-218. [http://arXiv.org/abs/hep-ex/9906032].

$e^+e^- \rightarrow J/\psi \rightarrow$ hadrons



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J.-E. Augustin *et al.*,"Discovery of a Narrow Resonance in e^+e^- Annihilation", Physical Review Letters 33 (1974) 385; [http://www.slac.stanford.edu/cgi-wrap/pubpage?slac-pub-1504].

$e^+e^- \rightarrow J/\psi \rightarrow$ hadrons (cont'd)

Mass: $M(J/\psi) = 3096.87 \pm 0.04$ MeV   Full Width: $\Gamma = 0.07$ MeV

Branching Ratio : $BR(J/\psi \rightarrow e^+e^-) = 5.93 \pm 0.10\%$, $BR(J/\psi \rightarrow \text{hadrons}) = 87.7 \pm 0.5\%$

Cross section: $\sigma_{peak} = 80 \mu b$

But

- beam energy spread is greater than natural width → observed peak cross section reduced by ~ $\Gamma_{resonance} / \Gamma_{beam}$
- radiative tail because of higher energy e^+e^- annihilations can radiate energy (probability ~α) to fall on resonance