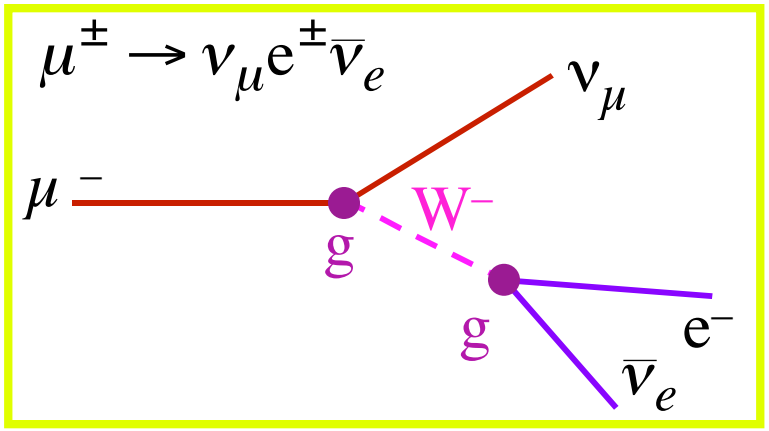
Muon Decay

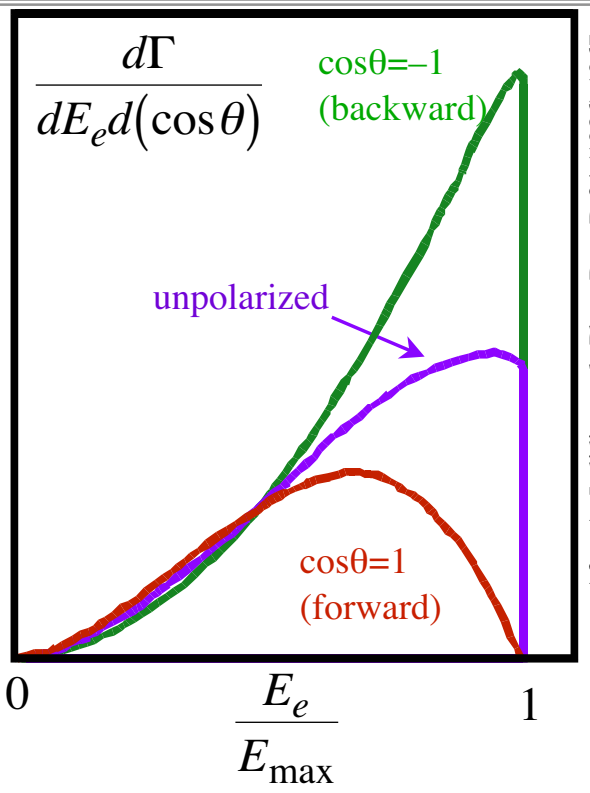


$$\Gamma_\mu = \left(\frac{1}{\tau_\mu}\right) = \frac{G_F^2 m_\mu^5}{192\pi^3} \times \left[1 - \frac{\alpha}{2\pi}\left(\pi^2 - \frac{25}{4}\right)\right]$$

radiative corrections

$$\times \left[1 - 8\left(\frac{m_e}{m_\mu}\right)^2 + 12\left(\frac{m_e}{m_\mu}\right)^4 \ln\frac{m_e}{m_\mu} + 8\left(\frac{m_e}{m_\mu}\right)^6 - \left(\frac{m_e}{m_\mu}\right)^8\right]$$

correction for finite electron mass



After A. Jodidio *et al.*, Phys. Rev. D 34 (1986) 1967.

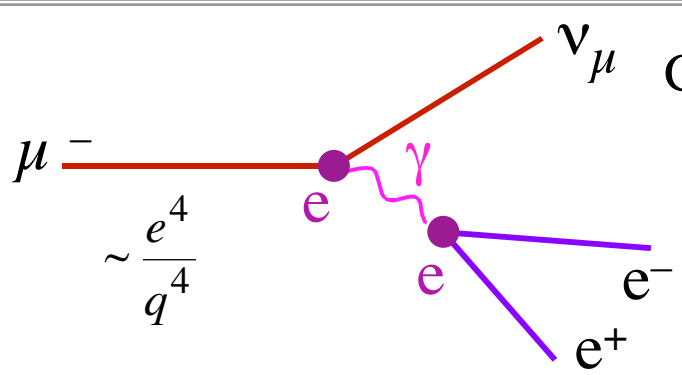
leptons left-handed,
antileptons right-handed
(polarization in decay ~ v/c)

m_μ=105.658 3568(52) MeV

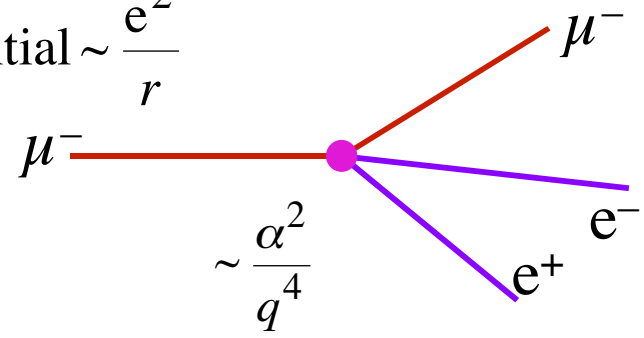
τ_μ=2.19703(4)μs

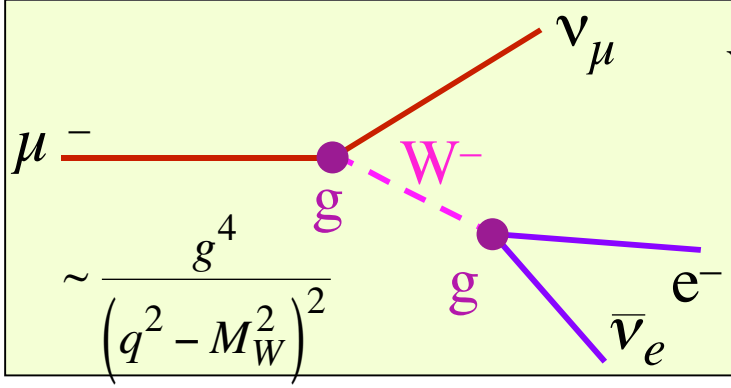
G_F=1.16639(1)×10^{-5} GeV^{-2}

4-Fermion Interactions



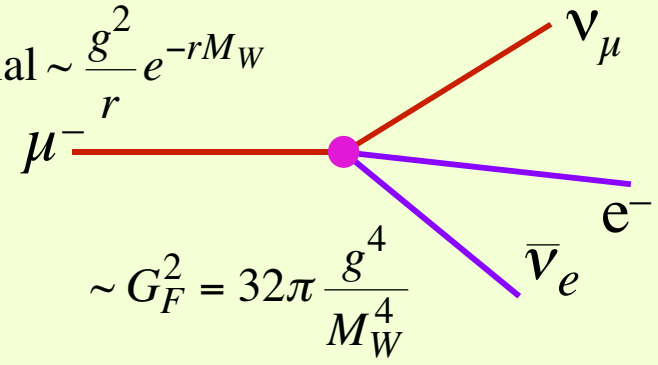
Coulomb potential $\sim \frac{e^2}{r}$





Yukawa potential $\sim \frac{g^2}{r} e^{-rM_W}$

$|q| << M_W$



$G_F = 1.16639(1) \times 10^{-5}$ GeV^{-2}
$M_W = 80.43(4)$ GeV

$\Rightarrow g^2 \approx {}^1/_{240}$, *c.f.* $\alpha = {}^1/_{137}$

The weak interaction is not so weak!

nuclear beta decays

$$ {}^{A}_{Z}N \rightarrow {}^{A}_{Z+1}N^{+} + e^{\pm} + \bar{\nu}_e \qquad (\beta^{-}) $$

$$ Q = {}^{A}_{Z}M - {}^{A+}_{Z+1}M - m_e \qquad \text{(energy release from decay of atom)} $$

$$ {}^{A}_{Z}N \rightarrow {}^{A}_{Z-1}N^{-} + e^{+} + \nu_e \qquad (\beta^{+}) $$

$$ {}^{A}_{Z}N \rightarrow {}^{A}_{Z-1}N^{*} + \nu_e \qquad \text{(electron capture)} $$

$$ d\Gamma = \frac{\overline{\left| \left\langle {}^{A}_{Z+1}N^{+} e^{\pm} \bar{\nu}_e \middle| H_{weak} \middle| {}^{A}_{Z}N \right\rangle \right|^2}}{2\pi^3 c^3 \hbar^3} P_e^2 \left(E_{\max} - E_e\right)^2 dP_e $$

$\Gamma \sim G_F^2 Q^5$

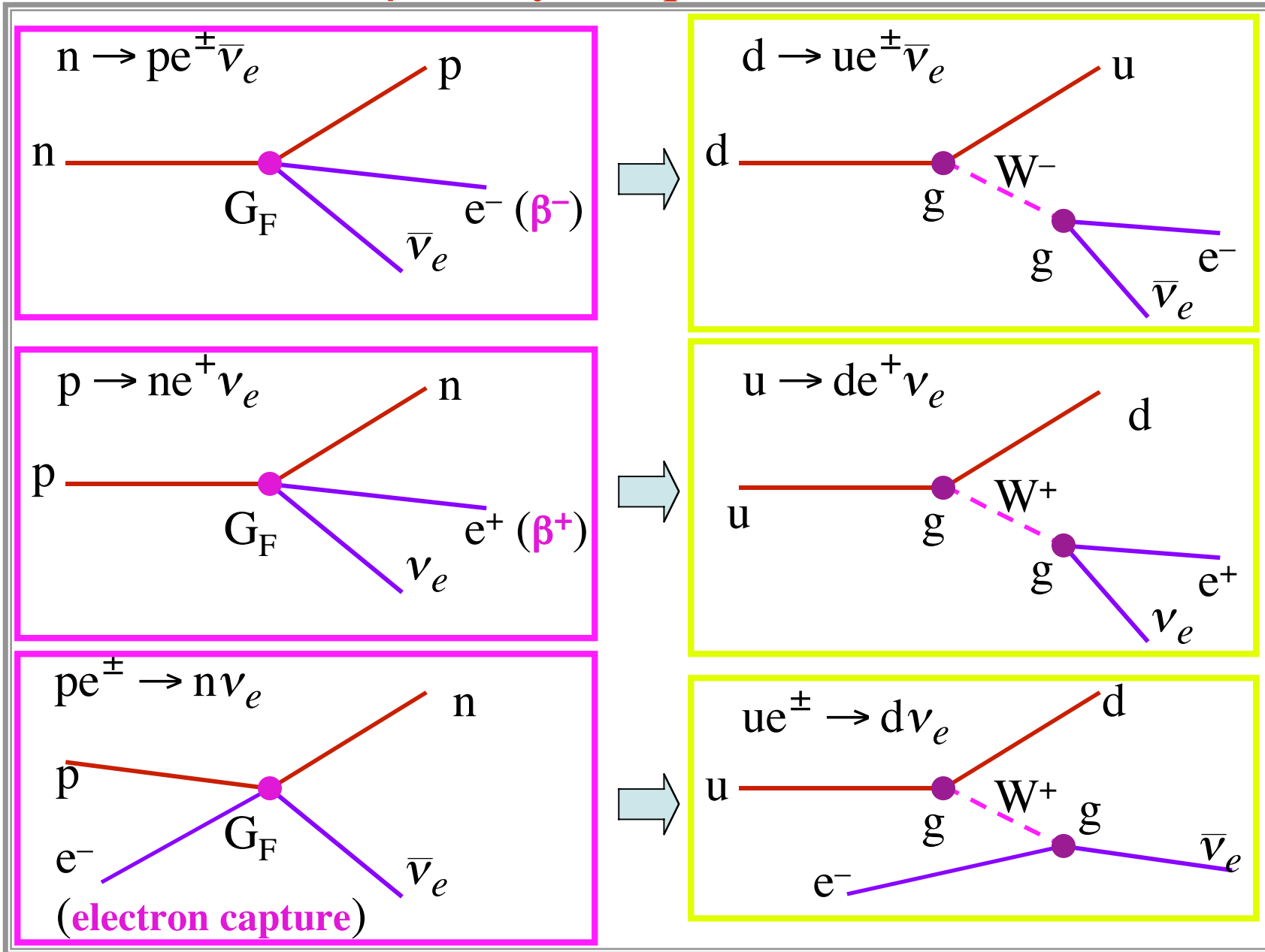
$$ \times \left[F\left(\mp Z, E_e\right)\right] \xrightarrow[\beta = \frac{v}{c} << 1]{} \frac{2\pi\eta}{1 - e^{-2\pi\eta}}, \qquad \eta = \pm \frac{Z\alpha}{\beta} $$

Fermi function
Coulomb corrections

Comparative mean-life (or half-life)

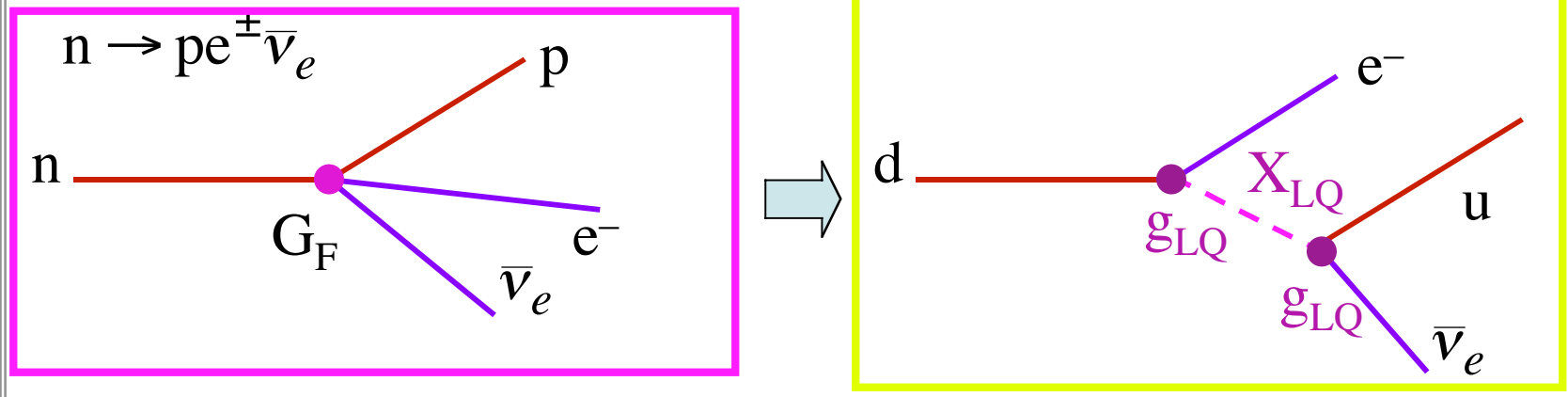
$$ f\tau \equiv \frac{2\pi^3 \hbar^7}{2\pi^3 m_e^5 c^4 \overline{\left| \left\langle {}^{A}_{Z+1}N^{+} e^{\pm} \bar{\nu}_e \middle| H_{weak} \middle| {}^{A}_{Z}N \right\rangle \right|^2}} $$

β decays at quark level

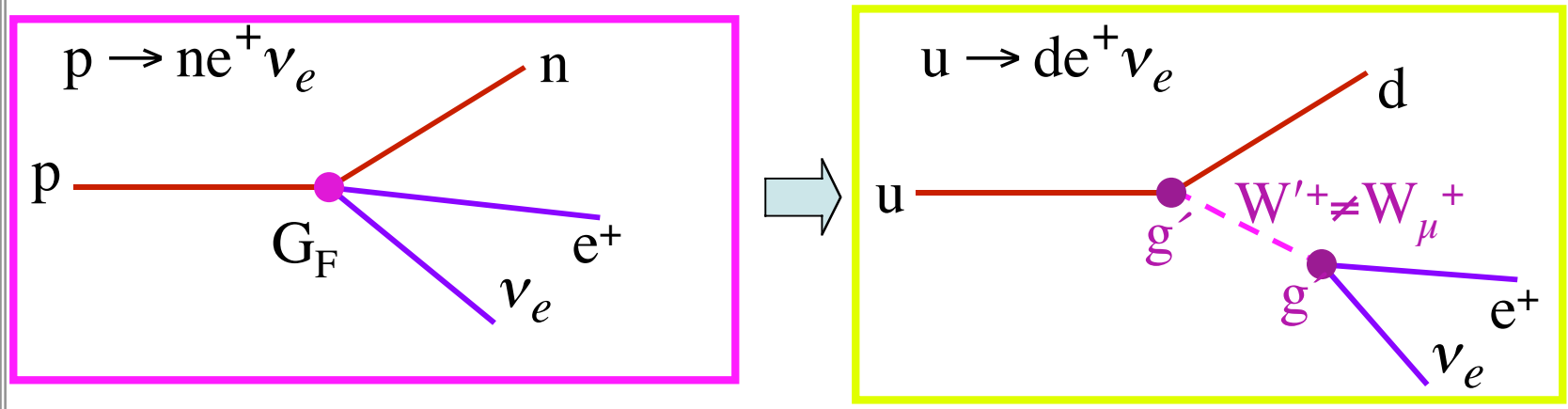


Why not?

leptoquarks?



W′ bosons?

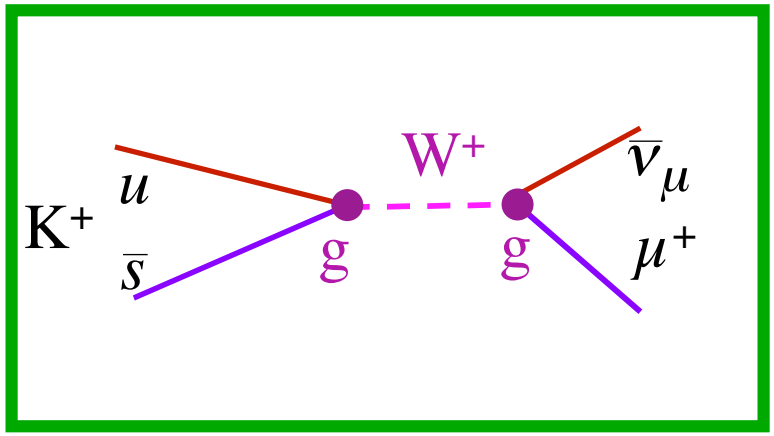


Universal Weak Interaction?

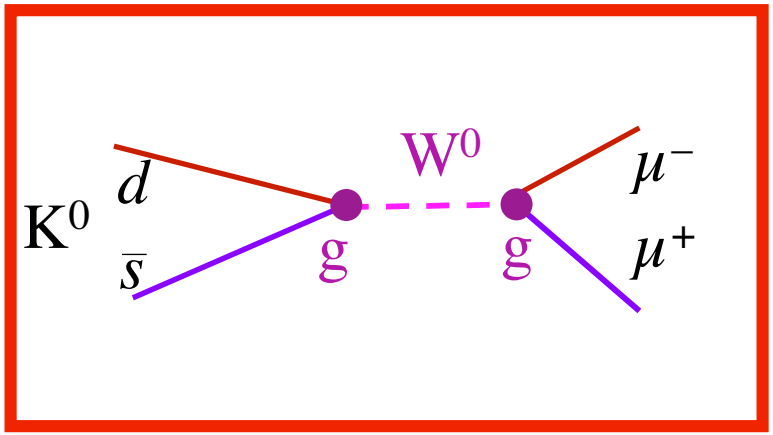
Decay ($+e\nu_e$)	$J^P \rightarrow J^P$	$t_{1/2}$ (partial)	E_{max} (MeV)	$f\,t_{1/2}$ (s)
$\mu^- \rightarrow \nu_\mu$	$^1/_2{}^- \rightarrow {}^1/_2{}^-$	1.5×10^{-6}s	53	610
$n \rightarrow p$	$^1/_2{}^+ \rightarrow {}^1/_2{}^+$	615s	0.782	1100
$\tau^- \rightarrow \nu_\tau$ (BR=18%)	$^1/_2{}^- \rightarrow {}^1/_2{}^-$	1.6×10^{-12}s	889	620
$^6He \rightarrow {}^6Li$	$0^+ \rightarrow 1^+$	0.8s	3.5	810
$^{14}O \rightarrow {}^{14}N$	$0^+ \rightarrow 0^+$	71s	1.8	3076
$\pi^+ \rightarrow \pi^0$ (BR=10^{-8})	$0^- \rightarrow 0^-$	1.8s	4.1	2×10^3
$\Sigma^- \rightarrow \Lambda^0$ (BR=6×10^{-5})	$^1/_2{}^+ \rightarrow {}^1/_2{}^+$	1.7×10^{-6}s	79	5×10^3
$K^+ \rightarrow \pi^0$ (BR=5%)	$0^- \rightarrow 0^-$	1.8×10^{-7}s	228	1×10^5
$\Sigma^- \rightarrow n$ (BR=0.1%)	$^1/_2{}^+ \rightarrow {}^1/_2{}^+$	9.5×10^{-8}s	224	7×10^4
$D^0 \rightarrow K^-$	$0^- \rightarrow 0^-$	7.9×10^{-12}s	867	4×10^3

Transitions where the initial and final wavefunctions have "maximal" overlap.

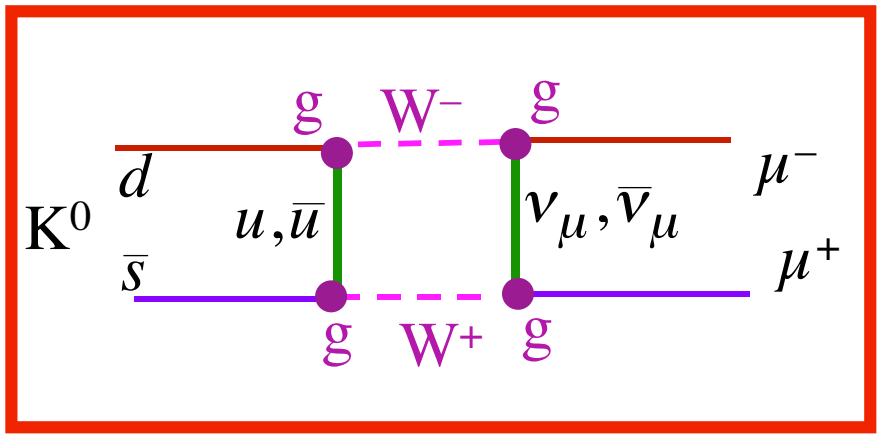
Why no flavour changing neutral currents?



$$BR\left(K^+ \to \mu^+ \nu_\mu\right) = (63.51 \pm 0.18)\%$$



$$BR\left(K_L^0 \to \mu^+ \mu^-\right) = (7.15 \pm 0.16) \times 10^{-9}$$



$$BR_{theory}\left(K_L^0 \to \mu^+ \mu^-\right) = 3 \times 10^{-4}$$