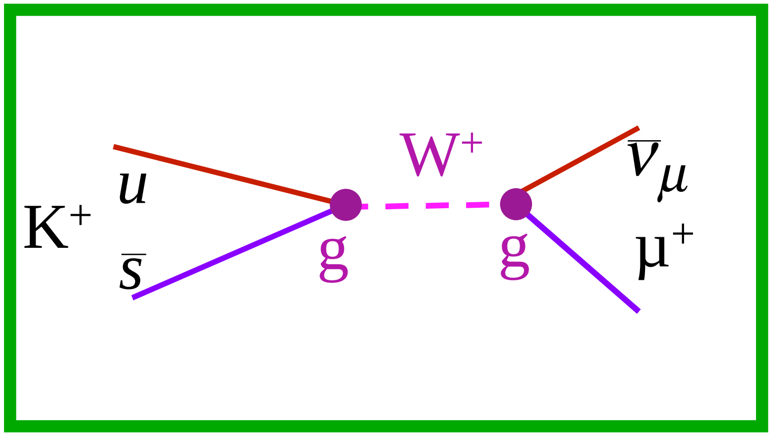
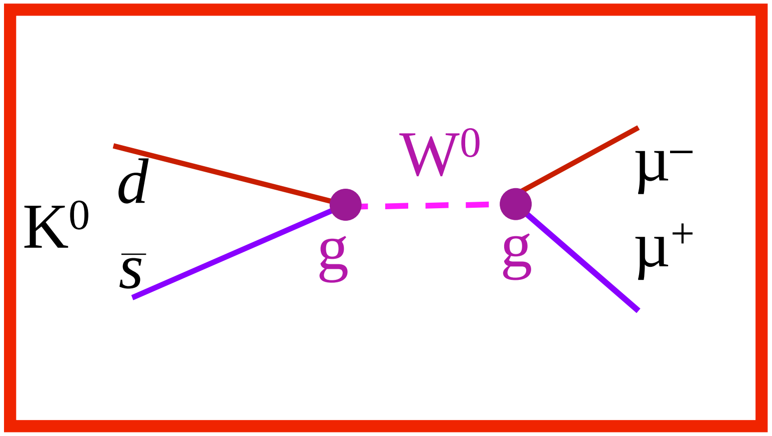
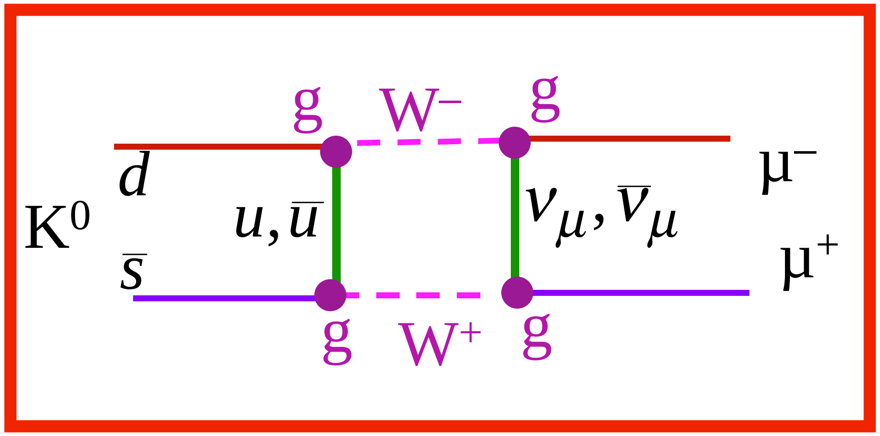
Why no flavour changing neutral currents?





$BR(K^+ \to \mu^+ \nu_\mu) = (63.51 \pm 0.18)\%$

$BR(K_L^0 \to \mu^+ \mu^-) = (7.15 \pm 0.16) \times 10^{-9}$



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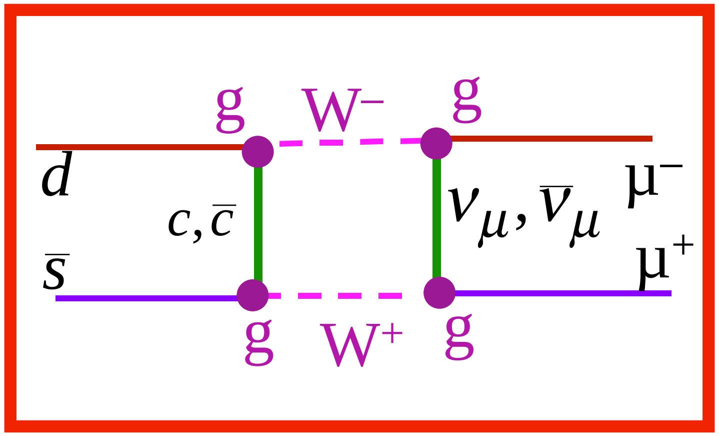


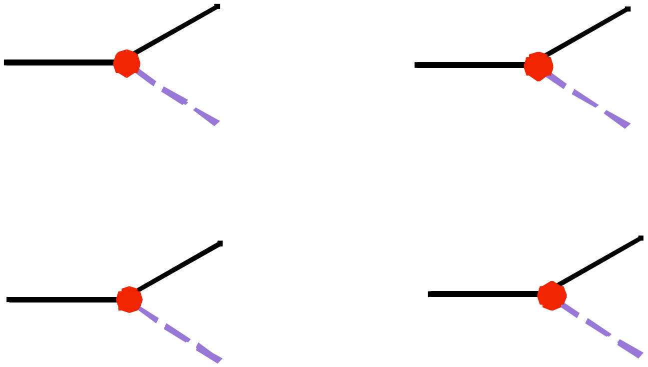
$BR_{theory}(K_L^0 \to \mu^+ \mu^-) = 3 \times 10^{-4}$

$$BR_{theory} \to 7 \times 10^{-5} \frac{m_u^2 - m_c^2}{M_W^2} \ln \frac{M_W^2}{\langle m_q^2 \rangle}$$

Charm

Strange particles live about 20 times too long; no strangeness changing neutral currents are observed. In order to save the idea of a universal weak interaction, charm was hypothesized.

Charm does not exclusively couple to strange quarks, it also couples to down quarks about 5% of the time. If we only consider u, d, s, & c quarks, then the relative strength of the coupling amplitudes is parameterized by the Cabibbo angle, θ_c.



At this point we must define up, down, strange, and charm quarks:

u ≡ charge +2/3 quark with mass ~ 5 MeV

d ≡ charge -1/3 quark with mass ~ 10 MeV

s ≡ charge -1/3 quark with mass ~ 0.2 GeV

c ≡ charge +2/3 quark with mass ~ 1.3 GeV

These definitions are often referred to as the quark mass eigenstates. We are free to define either the “up” or “down” mass eigenstates as being weak eigenstates.

Charm Couplings

If we choose “up”, then the up and charm mass eigenstates are weak eigenstates by definition; the down and strange weak eigenstates, d´ & s´, are defined to be the weak eigenstates coupling to u and c respectively:

In terms of their mass eigenstates and the Cabibbo angle we have:

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{bmatrix} \cos\,\theta_c & -\sin\,\theta_c \\ \sin\,\theta_c & \cos\,\theta_c \end{bmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Measurements of the strength of nuclear beta decays relative to muon decays gives

$$\cos\,\theta_c = 0.9735 \pm 0.0008,$$

measurements of the strength of strange particle decays give

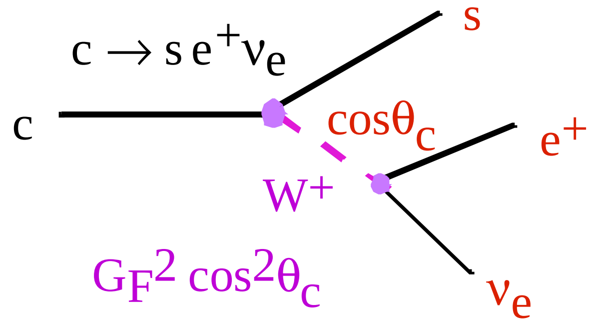
$$\sin\,\theta_c = 0.2196 \pm 0.0023,$$

and measurements of the strength of charm decays give

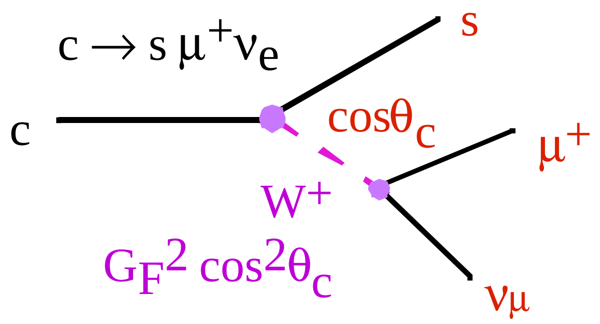
$$\sin\,\theta_c = 0.224 \pm 0.016$$

$$\cos\,\theta_c = 1.04 \pm 0.16$$

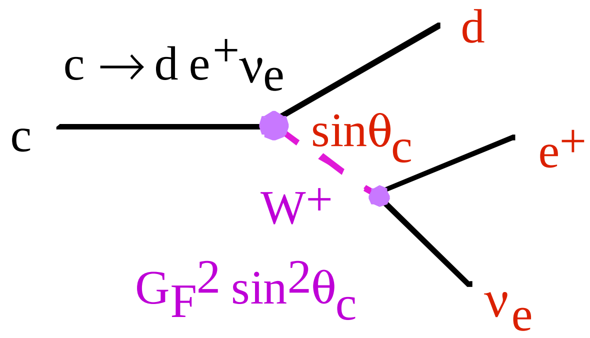
Semileptonic Charm Decays


$c \rightarrow s\, e^+ \nu_e$

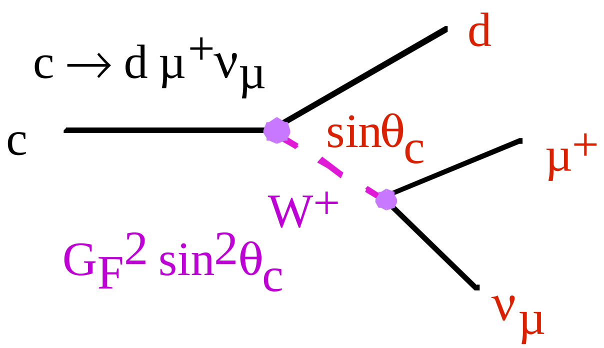
$G_F^2 \cos^2\theta_c$


$c \rightarrow s\, \mu^+ \nu_e$

$G_F^2 \cos^2\theta_c$


$c \rightarrow d\, e^+ \nu_e$

$G_F^2 \sin^2\theta_c$

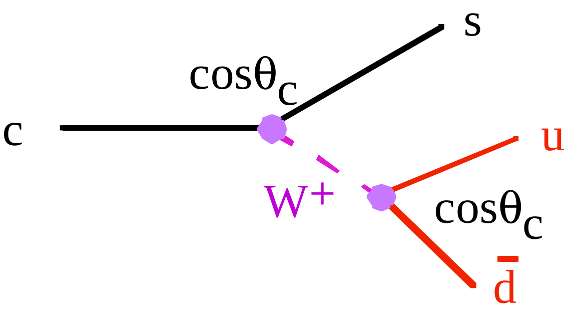

$c \rightarrow d\, \mu^+ \nu_\mu$

$G_F^2 \sin^2\theta_c$

Hadronic Charm Decays

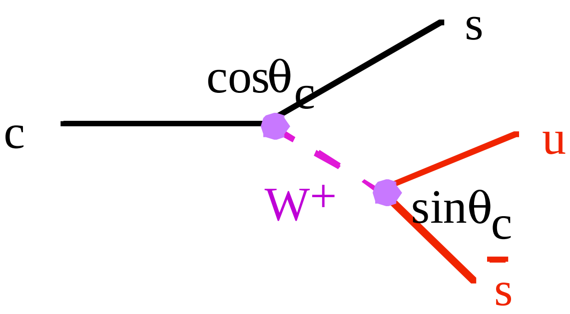
$c \rightarrow s\,u\,\bar{d}$

$3\ \boxed{G_F^2 \cos^2\theta_c \cos^2\theta_c}$



$c \rightarrow s\,u\,\bar{s}$

$3\ \boxed{G_F^2 \cos^2\theta_c \sin^2\theta_c}$



Must remember that quarks come in 3 colours

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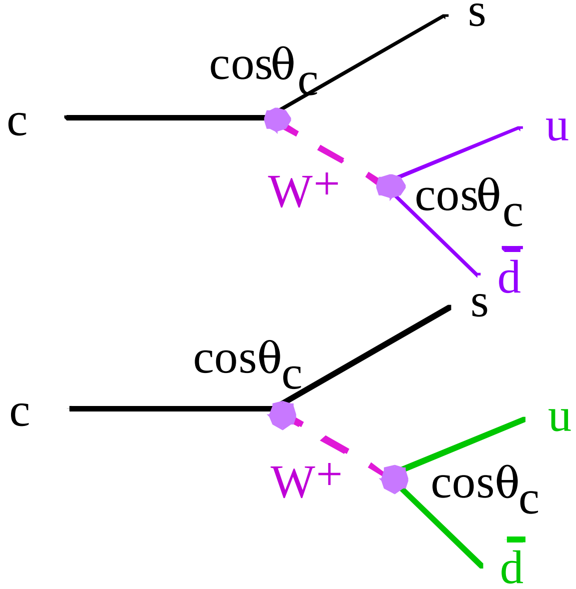

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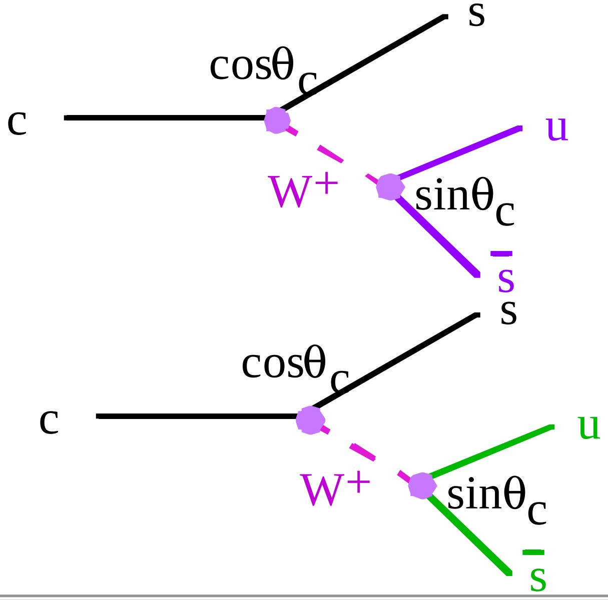
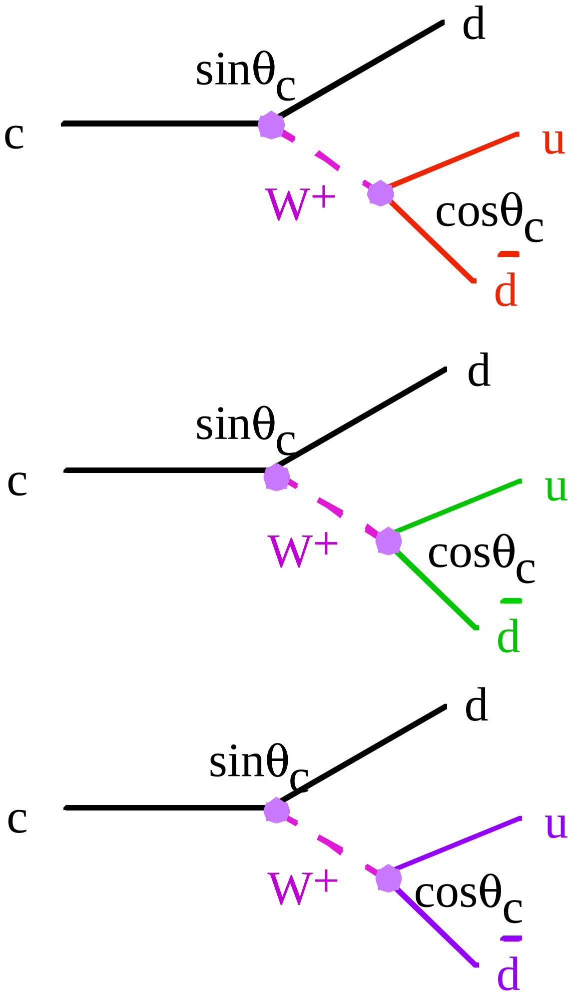


Cabibbo Unfavoured Decays

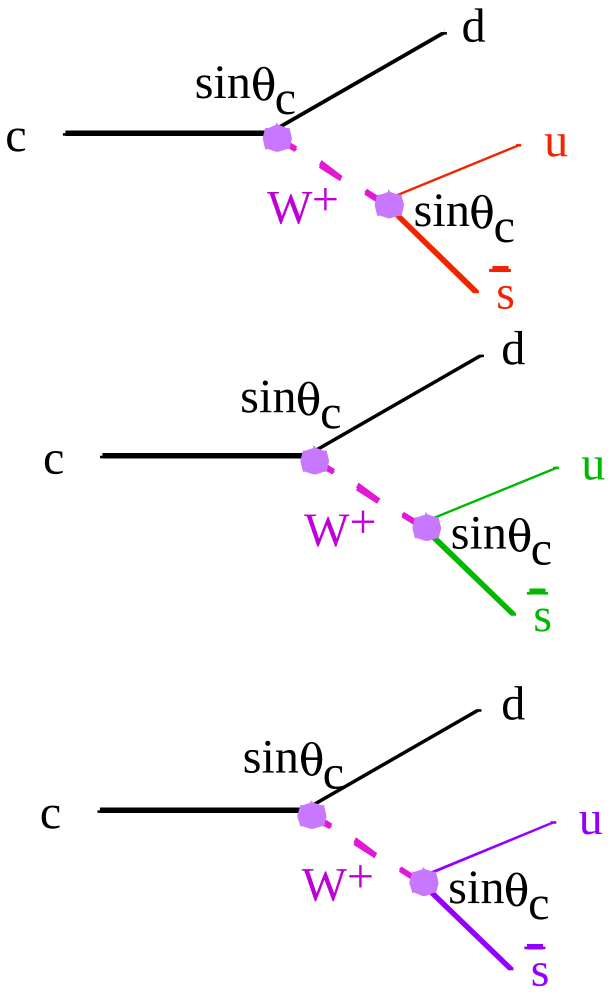
$c \rightarrow d\, u\, \bar{d}$

3 $\boxed{G_F^2 \sin^2\theta_c \cos^2\theta_c}$



$c \rightarrow d\, u\, \bar{s}$

3 $\boxed{G_F^2 \sin^2\theta_c \sin^2\theta_c}$



Spectator charm decays?

$$\Gamma_c = \Gamma\left(c \to s' e^+ \nu_e\right) + \Gamma\left(c \to s' \mu^+ \nu_e\right) + \Gamma(c \to s' u d')$$

$$\approx \left(G_F^2 + G_F^2 + 3G_F^2\right) m_c^5 \approx \left(G_F^2 + G_F^2 + 3G_F^2\right) m_\tau^5$$

$$\approx \Gamma\left(\tau^+ \to \overline{\nu}_\tau e^+ \nu_e\right) + \Gamma\left(\tau^+ \to \overline{\nu}_\tau \mu^+ \nu_e\right) + \Gamma\left(\tau^+ \to \overline{\nu}_\tau u d'\right)$$

$$m_\tau = 1777.03^{+0.30}_{-0.26}\,MeV \qquad \tau_\tau = 0.2906 \pm 0.0011\,ps$$

$$m_{D^0} = 1864.5 \pm 0.5\,MeV \qquad \tau_{D^0} = 0.4126 \pm 0.028\,ps$$

$$m_{D^+} = 1869.3 \pm 0.5\,MeV \qquad \tau_{D^+} = 1.051 \pm 0.013\,ps$$

$$m_{\Lambda_c^+} = 2284.9 \pm 0.6\,MeV \qquad \tau_{\Lambda_c^+} = 0.206 \pm 0.012\,ps$$

$$BR\left(\tau^\pm \to \nu_\tau e^\pm \overline{\nu}_e\right) = 17.83 \pm 0.06\%$$

$$BR\left(\tau^\pm \to \nu_\tau \mu^\pm \overline{\nu}_\mu\right) = 17.37 \pm 0.07\%$$

$$BR\left(D^0 \to e^+ + X\right) = 6.75 \pm 0.29\%$$

$$BR\left(D^+ \to e^+ + X\right) = 17.2 \pm 1.9\%$$

$$BR\left(\Lambda_c^+ \to e^+ + X\right) = 4.5 \pm 1.7\%$$

Hadronic corrections

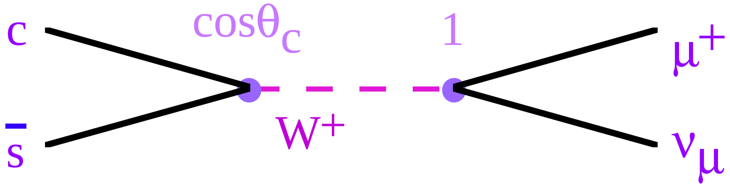
BUT quarks do not exist in isolation. When they weakly decay they are part of a hadron (except for top quarks). This leads to two effects:

(1) Strong interactions affect the decay rates. These affects are similar to the Coulomb corrections and nuclear wavefunction corrections for beta decays.

(2) Annihilation and exchange decays are possible:

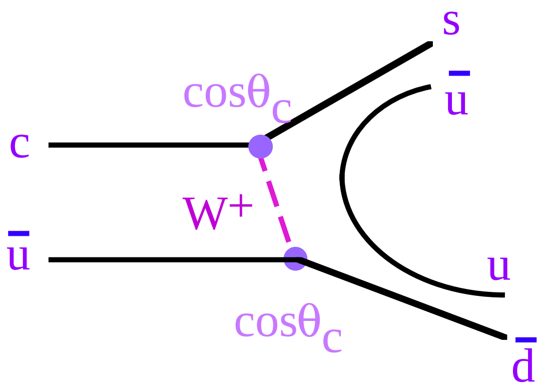
e.g.

$D_s^+ \rightarrow \mu^+ \nu_\mu$ annihilation decay



$G_F^2 \cos^2\theta_c$

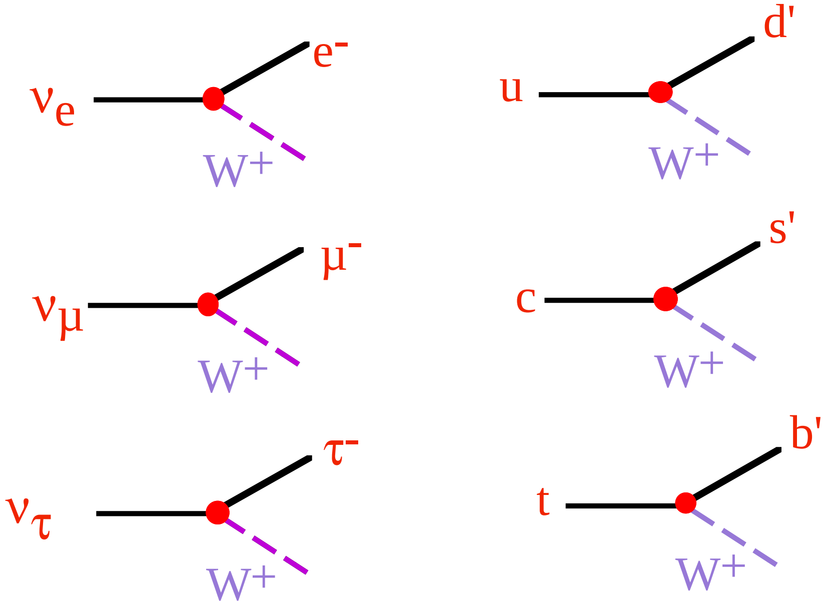
$D^0 \rightarrow K^- \pi^+$ exchange decay



$G_F^2 \cos^2\theta_c \cos^2\theta_c$

Weak couplings for 3 generations

The couplings of all known fundamental fermions:



where the Cabibbo-Kobayashi-Maskawa mixing matrix is

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

The notation for the CKM matrix varies. In this case, $c_{ij} \equiv \cos\theta_{ij}$, $s_{ij} \equiv \sin\theta_{ij}$, and there are 4 parameters: $\theta_{12} \cong \theta_c$, θ_{13}, θ_{23}, and δ. The additional quark mass eigenstates are:

b ≡ charge -1/3 quark with mass ~ 4.5 GeV

t ≡ charge +2/3 quark with mass ~ 175 GeV

Lepton Mixing

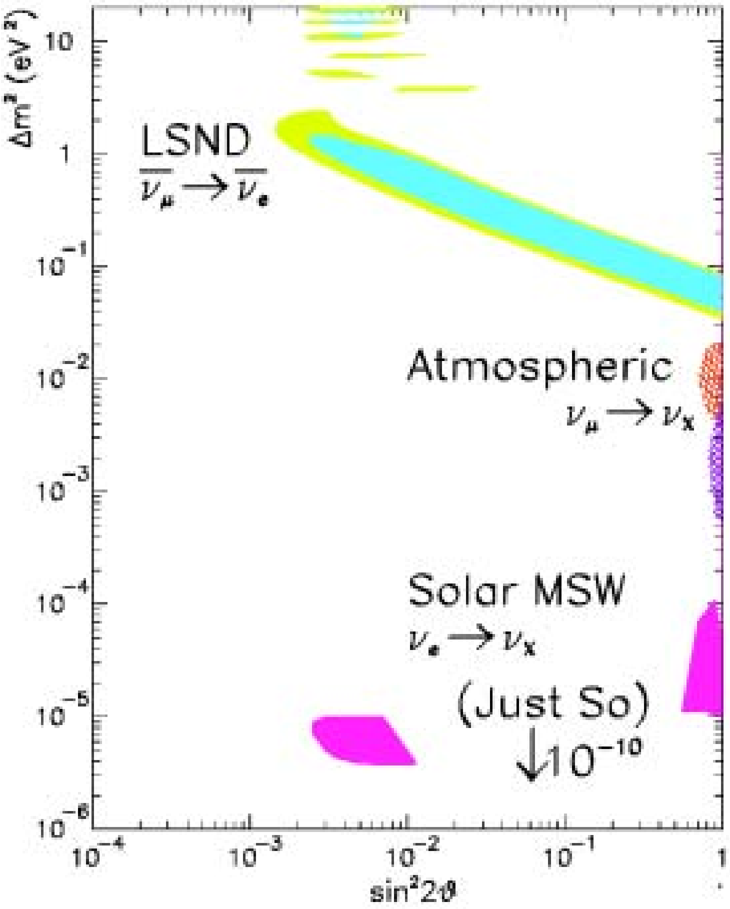
The charged lepton mass eigenstates are:

e ≡ charge -1 lepton with mass ~ 0.511 MeV

μ ≡ charge -1 lepton with mass ~ 106 MeV

τ ≡ charge -1 lepton with mass ~ 1.8 GeV

If neutrinos do not all have the same mass, then their mass and weak eigenstates may differ, and neutrino oscillations may occur. There is now good evidence that the neutrinos do not all have zero (and hence equal) masses, but the situation is not yet clear enough to identify the mass eigenstates.



Generations and Weak Isospin

The weak interaction couples the fermions in pairs, and the pairs are usually described as belonging to different generations

$$\begin{array}{cccc}
1 & 2 & 3 & T_3 \\
\begin{pmatrix} \nu_e \\ e^- \end{pmatrix} & \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix} & \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix} & \begin{matrix} +\tfrac{1}{2} \\ -\tfrac{1}{2} \end{matrix} \\
\begin{pmatrix} u \\ d' \end{pmatrix} & \begin{pmatrix} c \\ s' \end{pmatrix} & \begin{pmatrix} t \\ b' \end{pmatrix} & \begin{matrix} +\tfrac{1}{2} \\ -\tfrac{1}{2} \end{matrix}
\end{array}$$

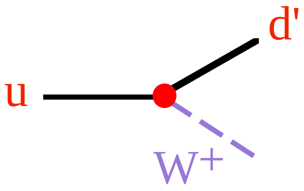
The relative assignment of the quark and lepton generations is just according to their mass.

This two-state pairing is reminisent of the the two spin states allowed for a spin 1/2 fermion, or the 2 isospin states of a nucleon (proton and neutron), so we define all known fermions to have “weak isopin” 1/2, and the “up” type fermions are assigned T_3=+1/2, and the “down” type fermions are assigned T_3=-1/2.

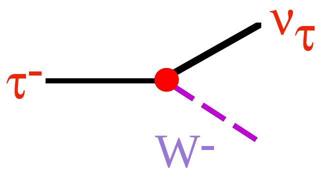
Weak SU(2) Gauge Theory

The group describing the spin states of a fermion is SU(2). So an SU(2) gauge theory is the “obvious” theory for the weak interaction. A gauge theory needs bosons to transfrom between all types of “charge” in the theory, i.e.

W^+: $T_3=+1$ $\{(T_3={}^1/_2) \rightarrow (T_3=-{}^1/_2)\}$, e.g.



W^-: $T_3=-1$ $\{(T_3=-{}^1/_2) \rightarrow (T_3={}^1/_2)\}$, e.g.



W^0: $T_3=0$ $\langle(+{}^1/_2-{}^1/_2)-(-{}^1/_2+{}^1/_2)\rangle$ $\{(T_3=\pm{}^1/_2) \rightarrow (T_3=\pm{}^1/_2)\}$, e.g.