# Note

**The last lecture for this term (on April 10) will be held in Room 408 instead of Room 137.**

# Quantum Chromodynamics

The most general $SU(3)_{colour}$ based gauge theory allows interactions which violate P and T symmetries. There is no theoretical reason to exclude such interactions, but current experimental limits on P and T violations in strong interactions tell us that any $SU(3)_{colour}$ P and T violation interactions are at least 8 orders of magnitude weaker then the P and T conserving QCD interactions. Various theoretical explanations for the absence of P and T violations (e.g. axions) have not been confirmed experimentally.

Quantum ChromoDynamics (QCD) is an $SU(3)_{colour}$ gauge theory based on 3 colour charges, needing 8 gauge bosons to mediate the transformations between the 3 colours. The P and T violating parts of the interaction Hamiltonian are excluded.

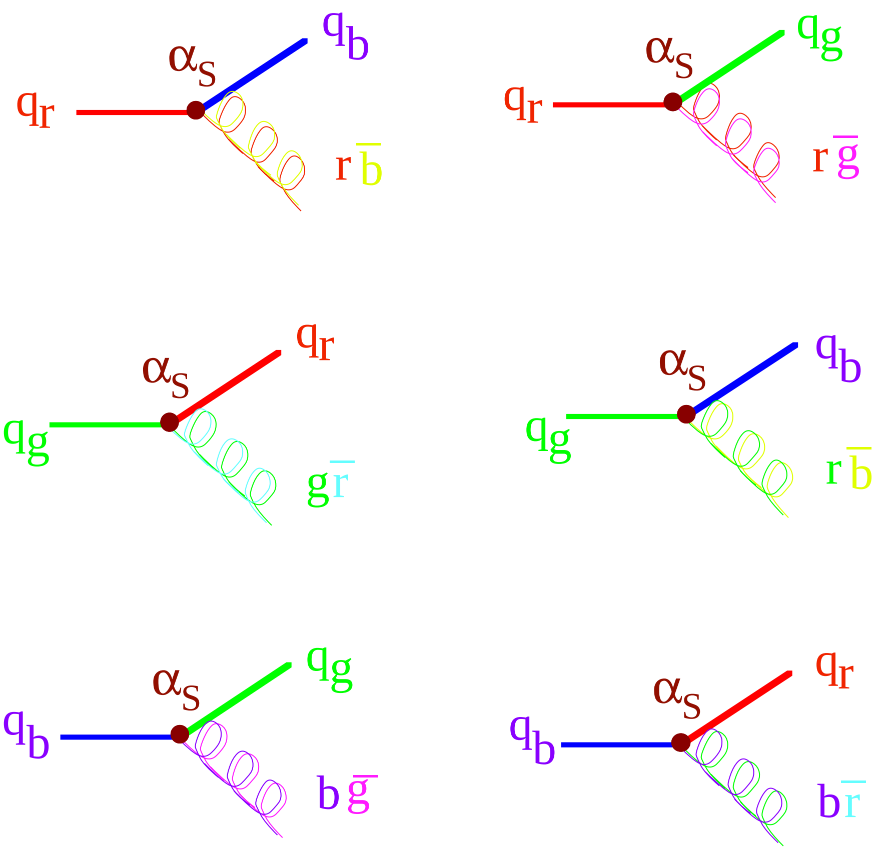
Gluons carry colour charge, unlike photons which do not carry electric charge.

$SU(3)_{colour}$ is thought to be an exact symmetry, but it is hard to set more than crude direct experimental limits on colour conservation and on the gluon mass:

- In any decay, colour conservation forced by colour confinement and energy conservation.
- $m_{gluon} < 10$ MeV

# Gluons

There are 6 explicitly coloured gluons:



There are also two antisymmetric colour-anticolour gluons from the 3 colour-anticolour possibilities.

One can choose any two colour-anticolour combinations, *e.g.*

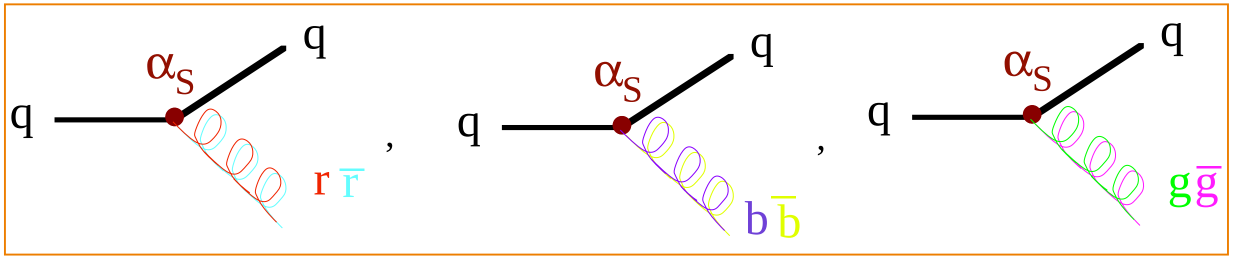
✓ $\langle r\bar{r} + b\bar{b} - 2g\bar{g}\rangle / \sqrt{6}$

and

✓ $\langle r\bar{r} - g\bar{g}\rangle / \sqrt{2}$

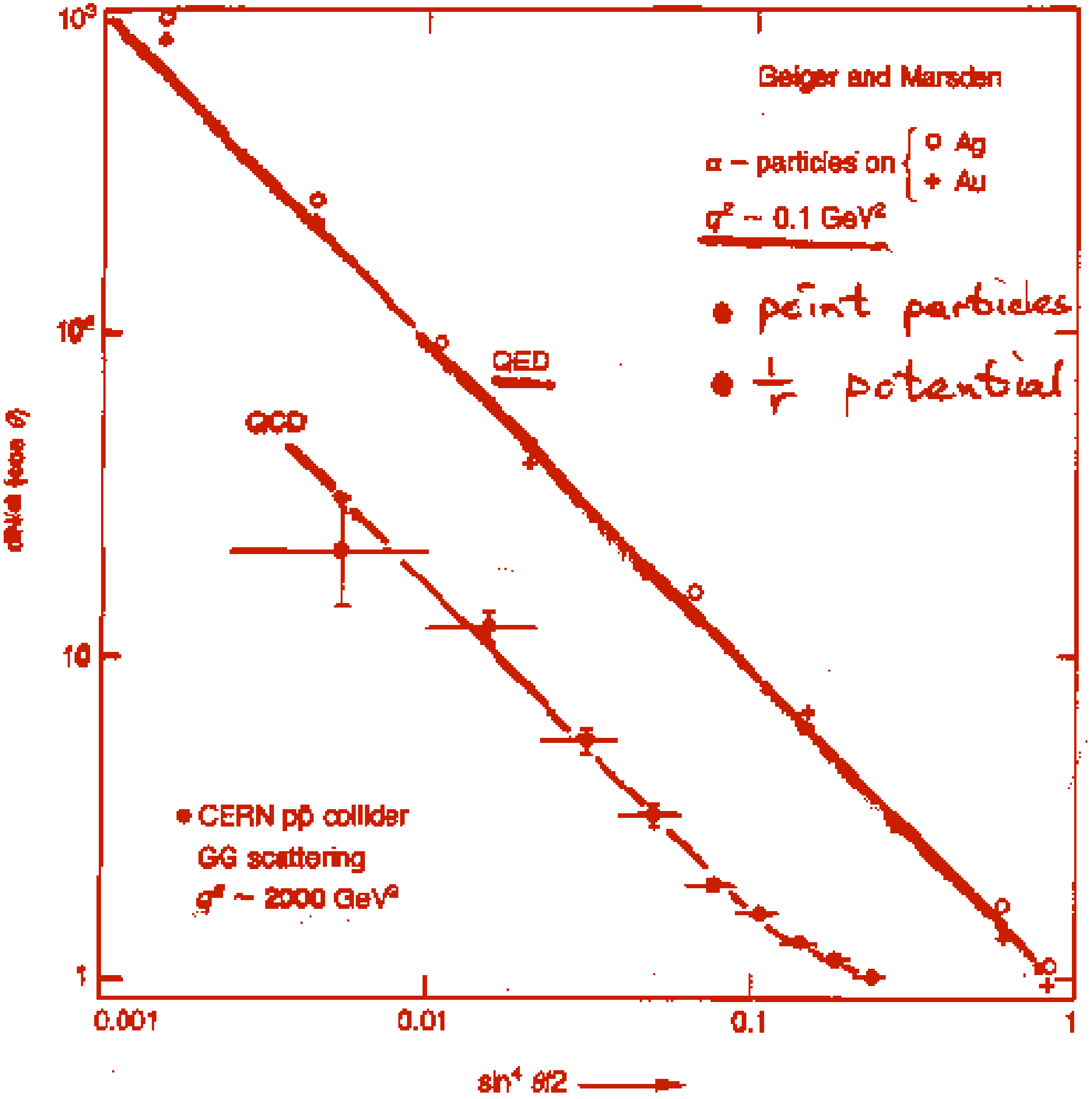
as long as they are orthogonal to each other and to the symmetric combination which is not a gluon.

✗ $\langle r\bar{r} + b\bar{b} + g\bar{g}\rangle / \sqrt{3}$

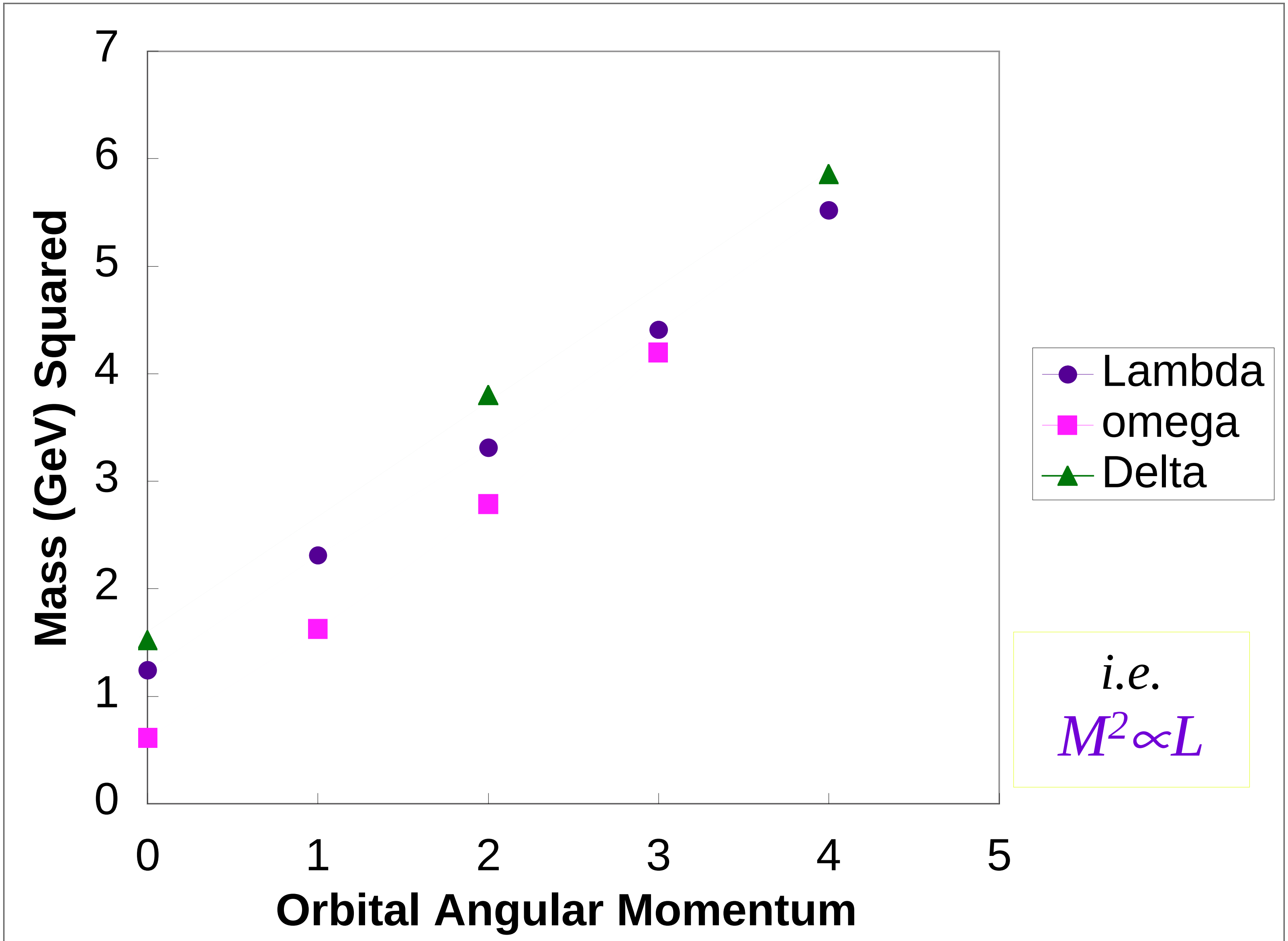


# Short Distance QCD Potential

At short distances QCD interactions obey a 1/r Coulomb potential.



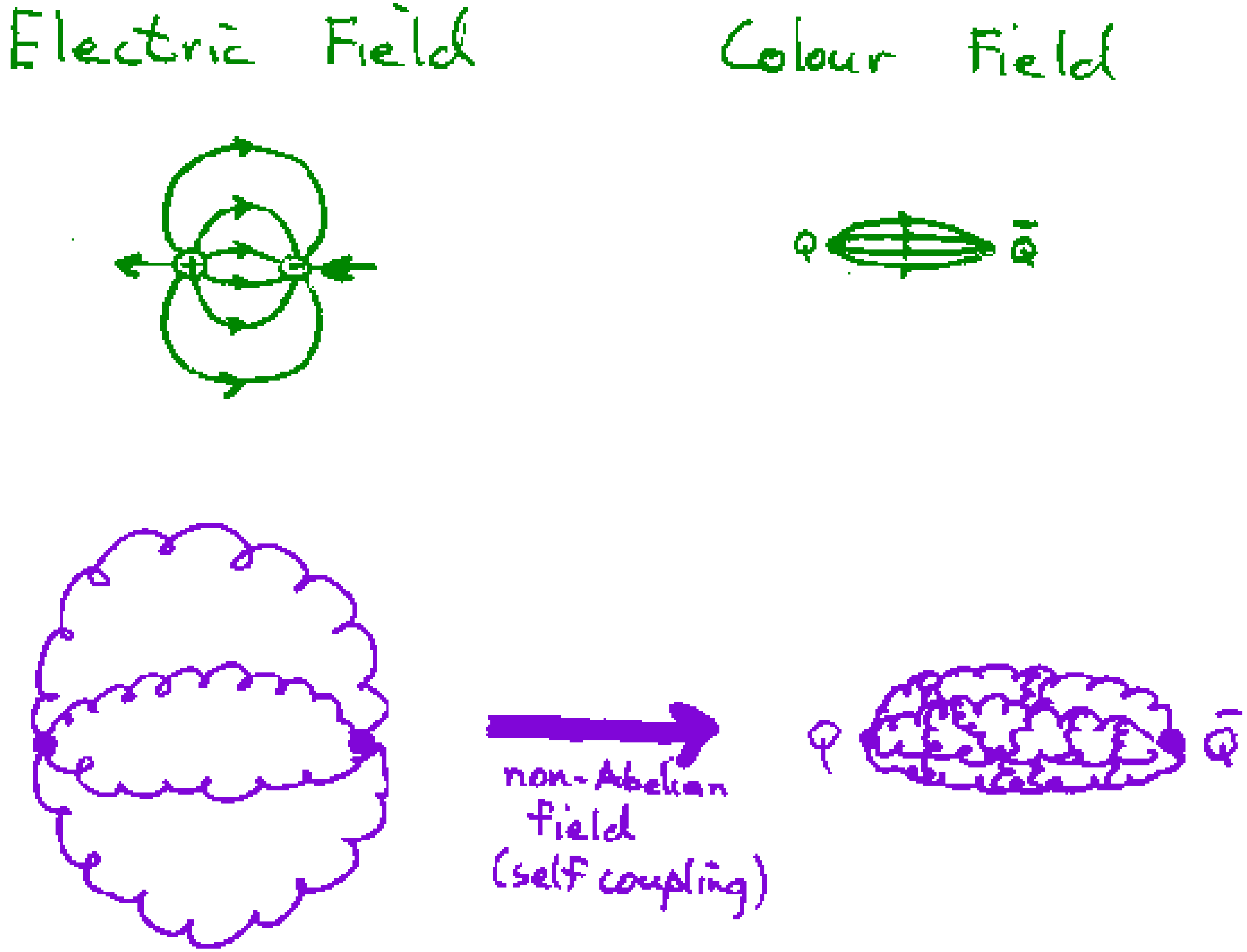
# Regge Trajectories



*i.e.*

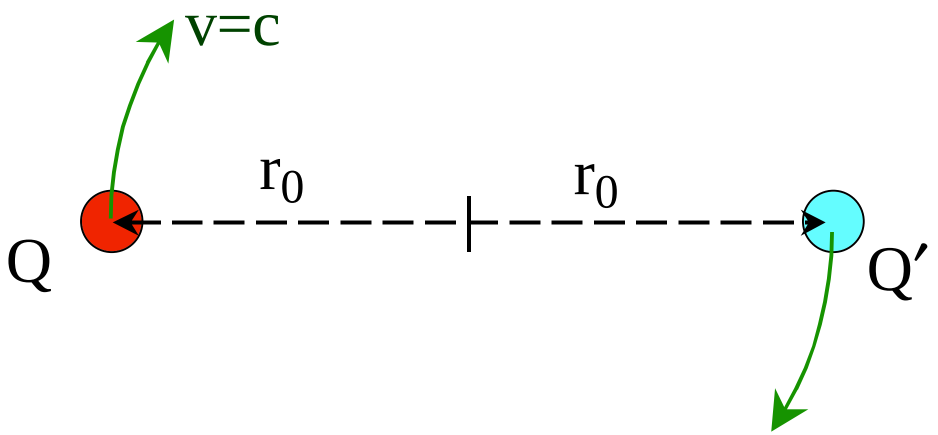
$M^2 \propto L$

# String Potential



# String Energy and Angular Momentum

For two massless quarks rotating around their common centre of mass



bound by a string with potential energy of form

$$V(r) = kr^n,$$

the mass and orbital angular momentum of the string are

$$M = \frac{E}{c^2} = \int_{-r_0}^{r_0} \gamma dm = \int_{-r_0}^{r_0} \frac{1}{\sqrt{1 - {v(r)^2}/{c^2}}} \frac{dV(r)}{c^2} = \pi k r_0^n$$

$$L = \frac{\mathbf{r} \times \mathbf{p}}{\hbar c} = \int_{-r_0}^{r_0} r\beta\gamma \frac{dm}{\hbar c} = \int_{-r_0}^{r_0} r \frac{v(r)}{c} \frac{1}{\sqrt{1 - {v(r)^2}/{c^2}}} \frac{dV(r)}{\hbar c} = \frac{\pi}{2\hbar c} k r_0^{n+1}$$

# Infrared Slavery and Quark Confinement

So if we ignore the masses of the quarks and assume that the mass and angular momentum of the quark system are determined only by the string, then

$$L = \frac{\pi}{2\hbar c} k \left[ \left( \frac{M}{\pi k} \right)^{1/n} \right]^{n+1} = \frac{\pi k}{2\hbar c} \left( \frac{M}{\pi k} \right)^{\left(1+1/n\right)} = \frac{1}{2\hbar c (\pi k)^{1/n}} M^{\left(1+1/n\right)} = \alpha_{string} M^{\left(1+1/n\right)}$$

Hence, the data indicate

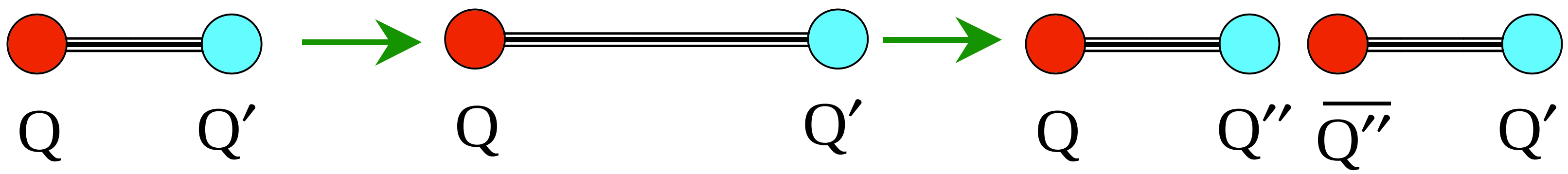
$$n=1$$

$$V(r)=kr$$

*i.e.* The interquark potential is linear for large r, and from the data the force between the quarks is

k = 0.85 GeV/fm = 1.4kN = 15 tonnes

This potential leads to quark confinement, since the string tension is so large that the string will always break and create quark-antiquark pairs if one tries to separate a pair of quarks.



i.e. quarks are confined.

# QCD Potential

$$V_{QCD}(r) = -\left(\frac{1}{2}\right)\frac{8}{3}\frac{\alpha_S}{r} + kr$$

(8 gluons, 3 flavours, historical factor of 1/2)

This is a phenomenological potential and the string part is not, as yet, derivable from the fundamental theory of QCD, although numerical calculations and plausibility arguments suggest that QCD is in agreement with the observations.

At low energies the coupling constant of QCD, $\alpha_S$, is of order unity, so perturbation theory is not valid and it is almost impossible to calculate anything. QCD gets weaker with increasing energy (See M&S 7.12), however, so at high energies QCD is similar to QED, and experimental tests are possible. QCD is currently tested at about the 10% level.( *c.f.* QED!)