Note

The last lecture for this term (on April 10) will be held in Room 408 instead of Room 137.

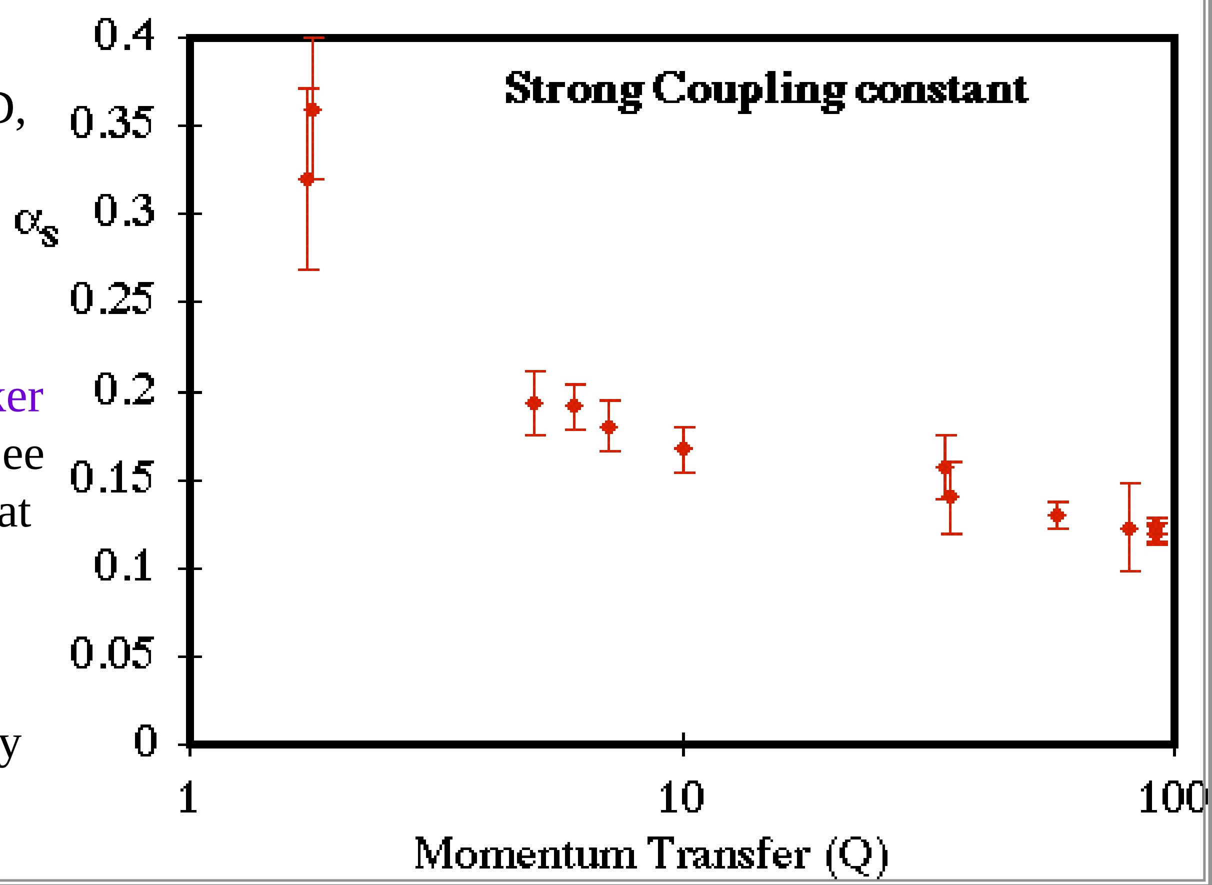
QCD Potential

$$V_{QCD}(r) = -\left(\frac{1}{2}\right)\frac{8}{3}\frac{\alpha_S}{r} + kr$$

(8 gluons, 3 flavours, historical factor of 1/2)

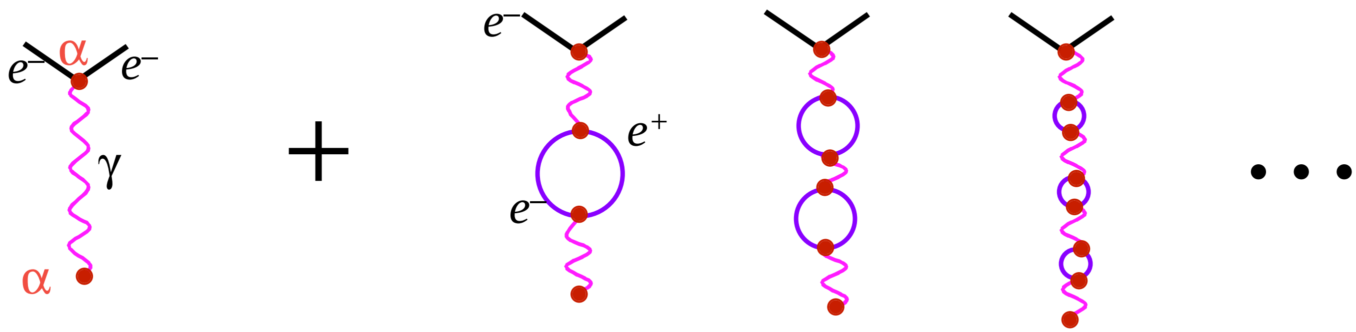
This is a phenomenological potential and the string part is not, as yet, derivable from the fundamental theory of QCD, although numerical calculations and plausibility arguments suggest that QCD is in agreement with the observations.

At low energies the coupling constant of QCD, α_S, is of order unity, so perturbation theory is not valid and it is almost impossible to calculate anything. QCD gets weaker with increasing energy (See M&S 7.12), however, so at high energies QCD is similar to QED, and experimental tests are possible. QCD is currently tested at about the 10% level.(*c.f.* QED!)



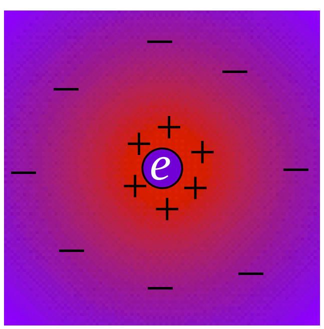
Vacuum Polarization and Running Couplings

At small enough distances (corresponding to large virtual photon momentum), the basic electron coupling to another charge is modified by virtual photons that are massive enough to form e^+e^- pairs (also all other charged fermion-antifermion pairs, depending on the virtual photon mass, *Q*.)



These virtual e^+e^- pairs naturally arrange themselves to form a polarized vacuum which partially shields the electron charge.

The closer you get the bigger the electron charge.

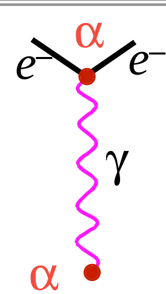


$$\boxed{\alpha\left(Q^2\right) = \alpha\left(Q_0^2\right)\frac{1}{1-\dfrac{\alpha\left(Q_0^2\right)}{3\pi}\ln\left(\dfrac{Q^2}{Q_0^2}\right)}}$$

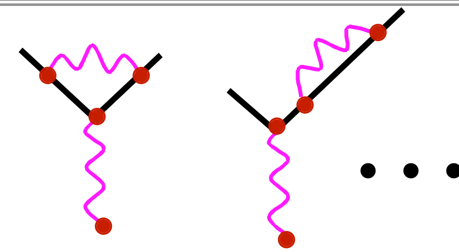
$\alpha(Q^2=0) = 1/137.03599976(50)$,
$\alpha(Q=M_Z) = 1/128.9$

Everything is in the loops!

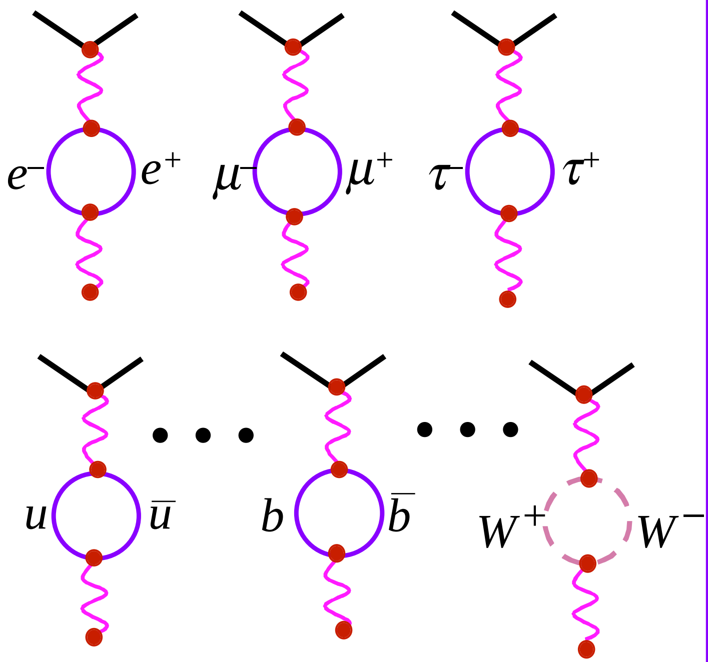
e.g. electron coupling to another charge has Next to Leading Order ($\sim\alpha^3$) contributions from interference between lowest order ($\sim\alpha^2$)



and loop ($\sim\alpha^4$) processes



all charged particles



(mass suppressed)

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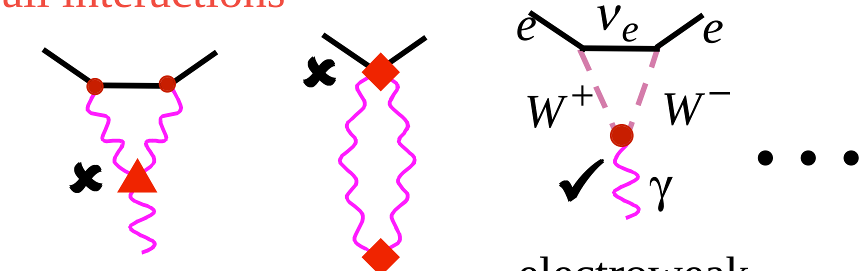
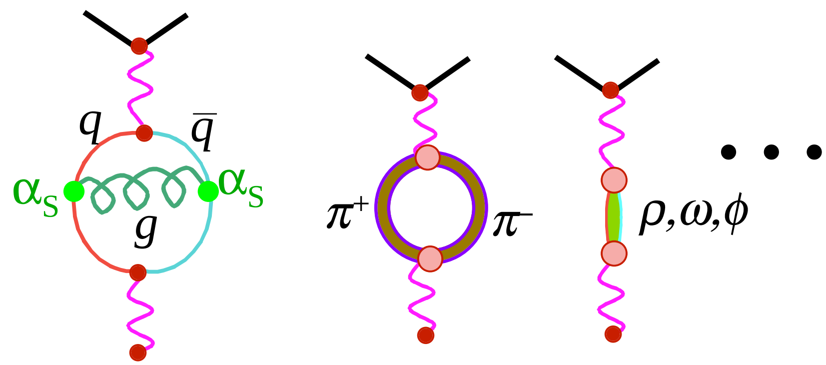
all interactions



electroweak (mass suppressed)



QCD

(from $e^+ e^- \rightarrow$ hadron data)

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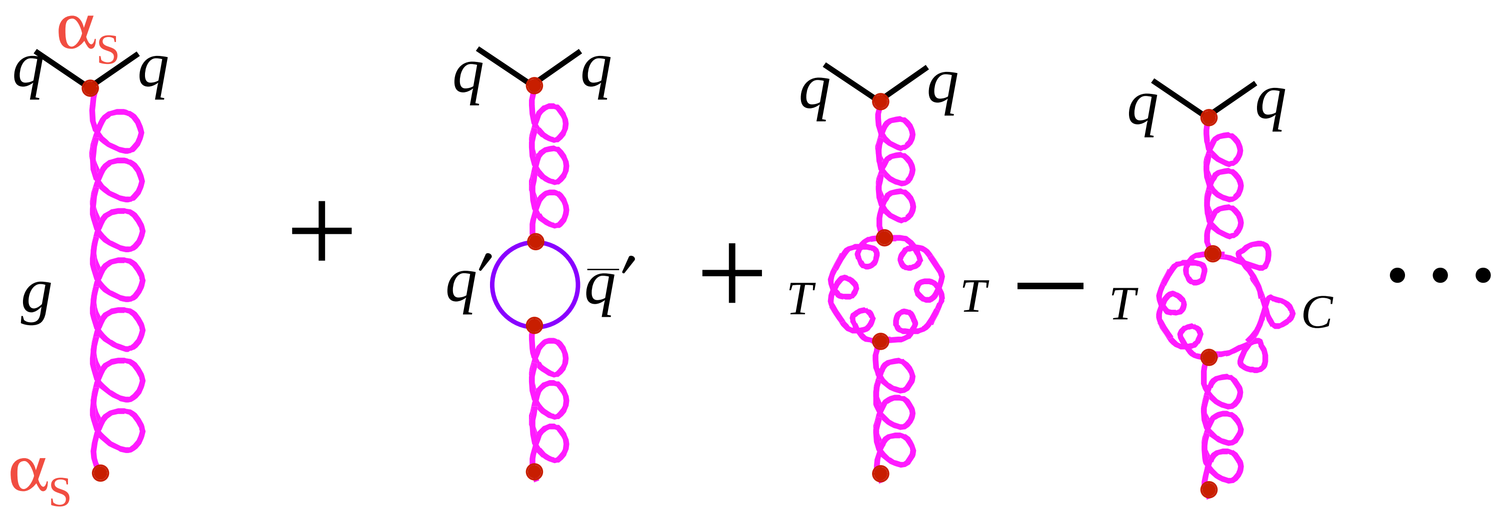
α_S Running Coupling

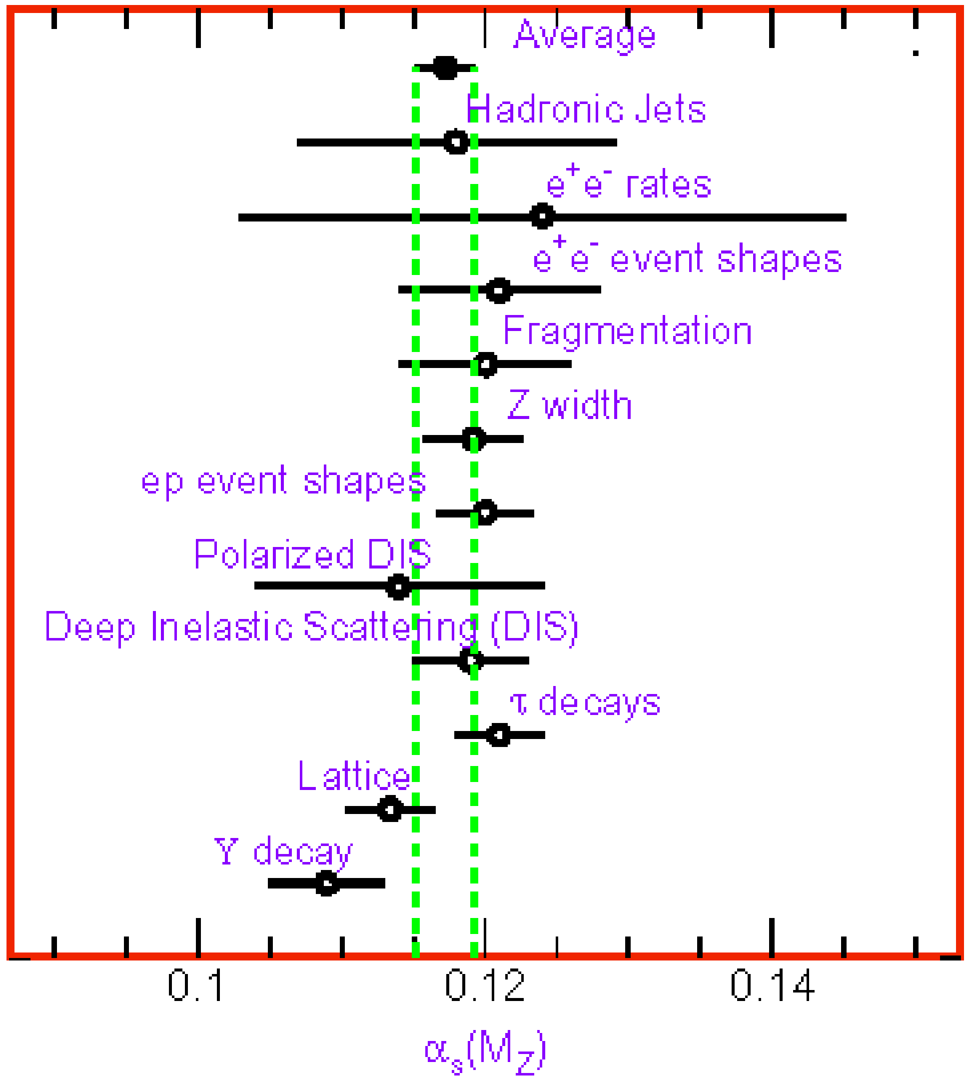
Gluon loops contribute to the running of α_S. Loops with quarks or transversely polarized gluons (*T*) shield the quark colour charge, but loops with a longitudinally (*C*="Coulomb") polarized gluon produce an anti-shielding effect.



$$\alpha(Q^2) = \alpha_S(Q_0^2)\,\frac{1}{1+\dfrac{\alpha(Q_0^2)}{4\pi}\left(-\frac{1}{2}\,\frac{8}{3}\,n_{flavours}-5+16\right)\ln\left(\dfrac{Q^2}{Q_0^2}\right)}$$

$\alpha(Q<1\text{GeV}) \approx 1 \quad\rightarrow\quad \alpha(Q=M_Z) = 0.121(3)$

QCD fits data!



http://pdg.lbl.gov/2001/qcdrpp.pdf

Neutral Kaons (again)

Strangeness eigenstates: $K^0 = d\bar{s}$ $\overline{K^0} = \bar{d}s$

but the weak interaction does not conserve strangeness:

e.g. K^0 — $\pi^+ \pi^-$ — $\overline{K^0}$

CP eigenstates:

$$\boxed{K_1 = \frac{K^0 + \overline{K^0}}{\sqrt{2}} \quad , \quad K_2 = \frac{K^0 - \overline{K^0}}{\sqrt{2}}}$$

(CP=+1, -1)

but the weak interaction also does not conserve CP:

Mass eigenstates:

$$\boxed{K_S = \frac{K_1 + \varepsilon K_2}{\sqrt{1+|\varepsilon|^2}} \quad , \quad K_L = \frac{K_2 + \varepsilon K_1}{\sqrt{1+|\varepsilon|^2}}}$$

The K_L ("K-long") is mostly K_2 and has a much longer lifetime (τ=51.7±0.4ns) than the K_S ("K-short", τ=0.08935±0.000008ns) because there is so little phase space for $K \to \pi\pi\pi$ decays relative to $K \to \pi\pi$ decays.
($K_2 \to \pi\pi\pi$ is allowed, but $K_2 \to \pi\pi$ is forbidden by CP invariance.)

BR($K_L \to \pi^+ \pi^-$)=0.2056(33)% $\longrightarrow$ $|\varepsilon| = 2.7\times10^{-3}$
BR($K_L \to \pi^0 \pi^0$)=0.0927(19)%

Kaon Oscillations

Amplitudes for K_1 and K_2 states

$$a_1(t) = a_1(t=0)e^{-\left(\frac{1}{2}\Gamma_1 + im_1\right)t}$$

$$a_2(t) = a_2(t=0)e^{-\left(\frac{1}{2}\Gamma_2 + im_2\right)t}$$

For an initial pure K^0 beam propagating in vacuum, the K^0 intensity is

$$I\left(K^0 = \frac{K_1^0 + K_2^0}{\sqrt{2}}\right) = \frac{a_1(t) + a_2(t)}{\sqrt{2}}\left(\frac{a_1^*(t) + a_2^*(t)}{\sqrt{2}}\right)$$

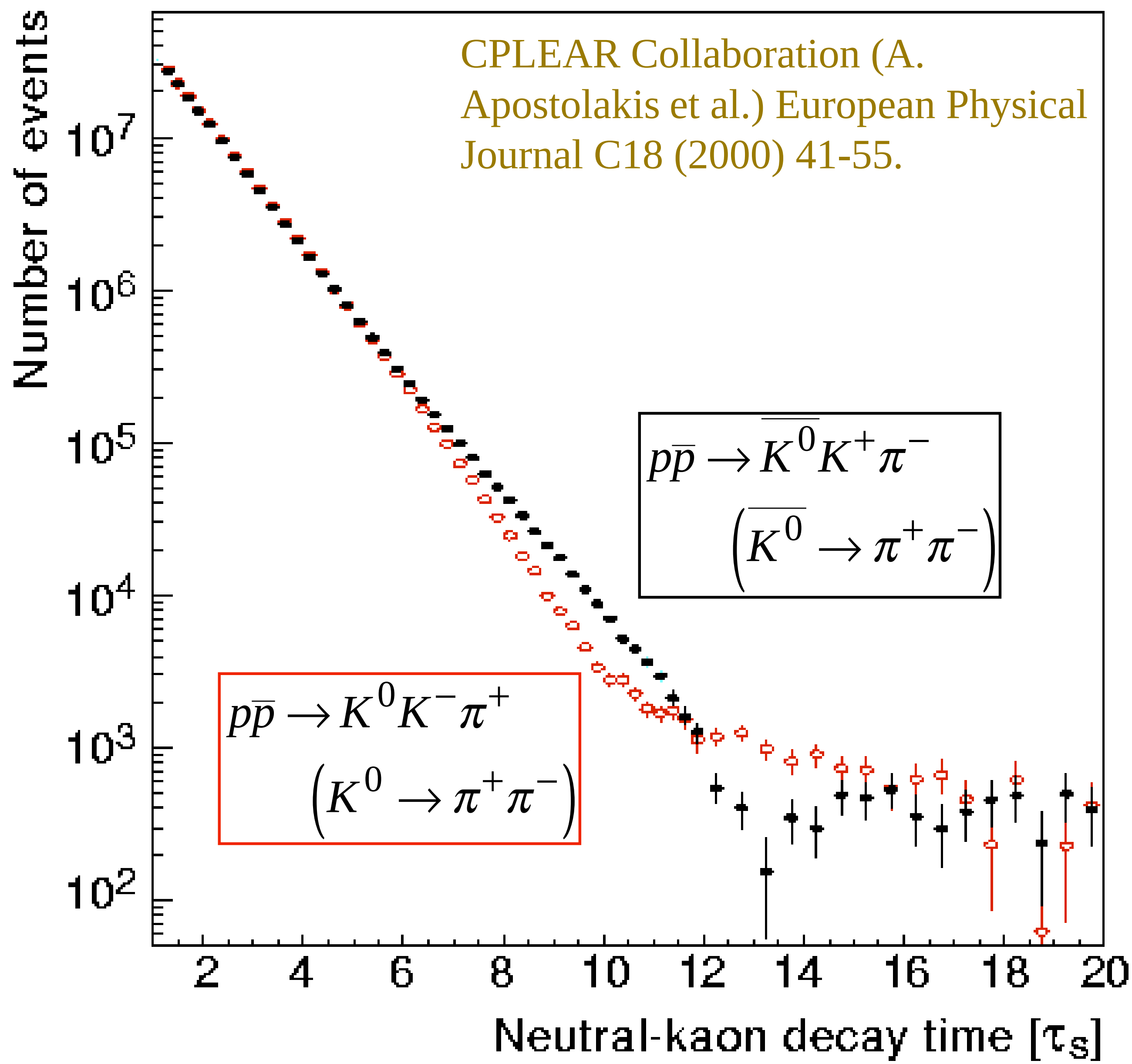
$$= \frac{\frac{1}{2}e^{-\left(\frac{1}{2}\Gamma_1 + im_1\right)t} + \frac{1}{2}e^{-\left(\frac{1}{2}\Gamma_2 + im_2\right)t}}{\sqrt{2}}\left(\frac{\frac{1}{2}e^{-\left(\frac{1}{2}\Gamma_1 - im_1\right)t} + \frac{1}{2}e^{-\left(\frac{1}{2}\Gamma_2 - im_2\right)t}}{\sqrt{2}}\right)$$

$$= \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\frac{1}{2}(\Gamma_1 + \Gamma_2)t}\left(e^{i(m_1 - m_2)t} + e^{i(m_2 - m_1)t}\right) + e^{-\Gamma_2 t}\right]$$

$$= \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} + 2e^{-\frac{1}{2}(\Gamma_1 + \Gamma_2)t}\cos\Delta m t\right] \qquad \{\Delta m \equiv (m_2 - m_1)\}$$

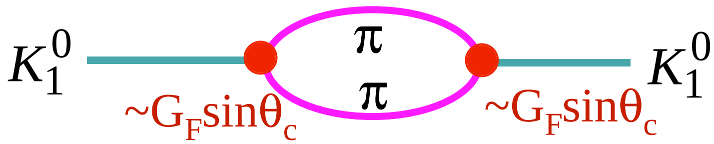
Similarly $$I\left(\overline{K^0}\right) = \frac{1}{4}\left[e^{-\Gamma_1 t} + e^{-\Gamma_2 t} - 2e^{-\frac{1}{2}(\Gamma_1 + \Gamma_2)t}\cos\Delta m t\right]$$

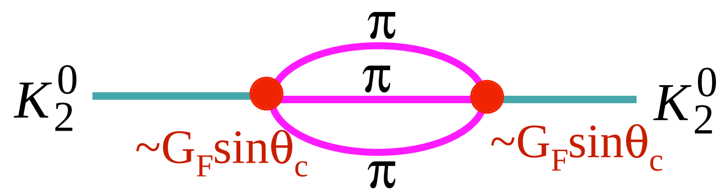
K $\rightarrow$ ππ decays



Neutral Kaon mass difference: Δm

Contributions from weak interactions to neutral kaon mass





(or can be drawn in more modern language as quark box diagrams)

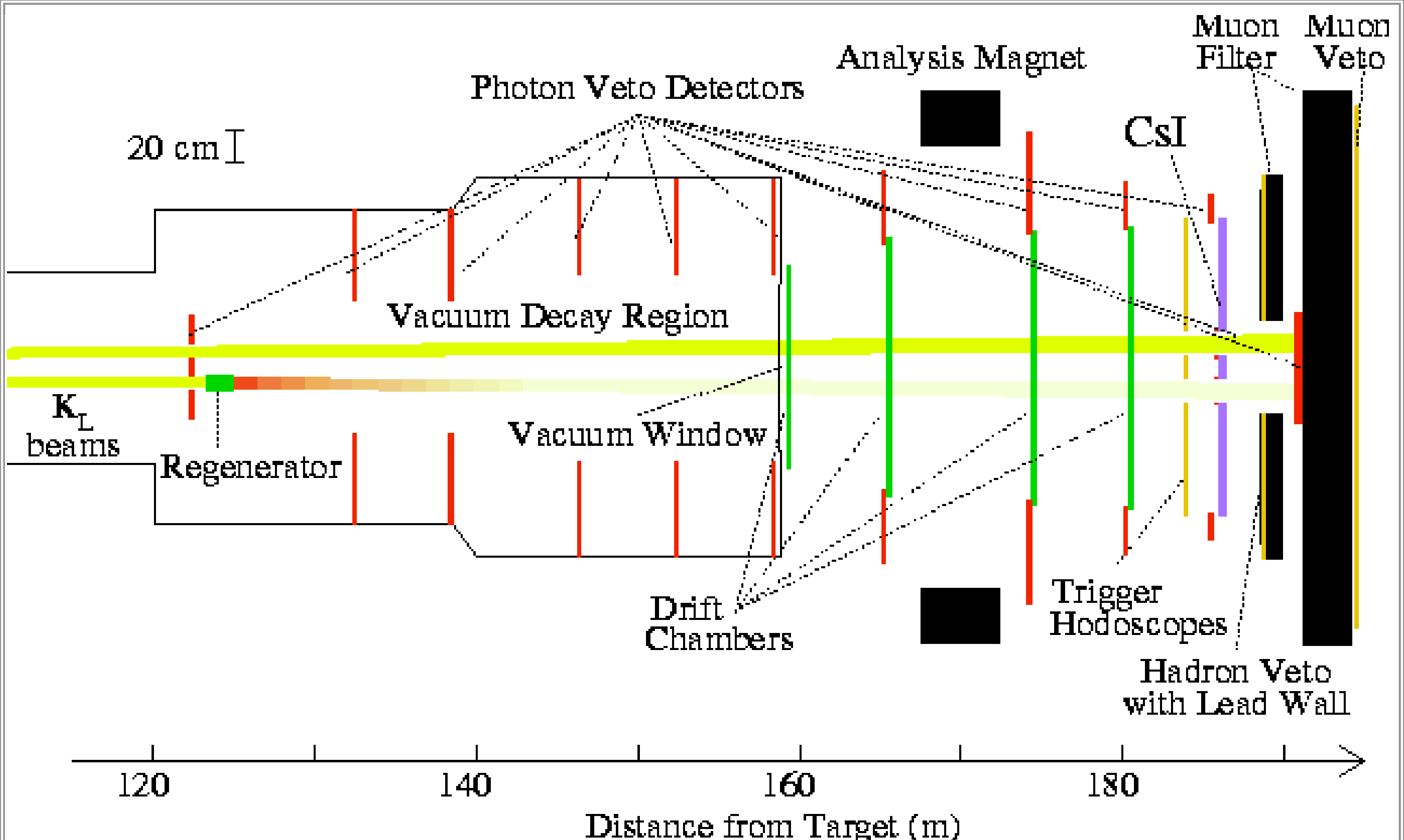
Expect $\Delta m \sim G_F^2 \sin^2 \theta_c m_K^5 \sim 10^{-4} eV$

Experimentally

$$\begin{aligned}\Delta m &= m_{K_L} - m_{K_L} \\ &= 3.489(8)\mu\text{eV} \\ &= 0.7 \times 10^{-14} m_K\end{aligned}$$

Fantastic interferometer

KTeV Experiment



(http://kpasa.fnal.gov:8080/public/ktev.html)

$\pi^+\pi^-$ and $\pi^0\pi^0$ events