Problem Set 1 Answers

1) What is the magnitude of theoretical Dirac magnetic moment of the tau lepton

a) in natural units? (*i.e.* in eV^n, where *n* is some integer)

From M&S equations 1.14 or 2.9 we have

$$\mu_\tau = \frac{q_\tau}{m_\tau} S_\tau = \frac{e}{1777 MeV/c^2}\frac{\hbar}{2} = \frac{1}{3.554 GeV} = 0.281 GeV^{-1} = 2.81\times 10^{-10} eV^{-1}$$

where we get the tau lepton parameters from Chapter 2 and Appendix E.

b) in SI units?

Here we don't use $e = c = \hbar = 1$, but insert the SI values instead

$$\mu_\tau = \frac{q_\tau}{m_\tau} S_\tau = \frac{e}{1777 MeV/c^2}\frac{\hbar}{2} = \frac{e(\hbar c)c}{3554 MeV}$$

$$= \frac{1.602\times 10^{-19} C\left(1.973\times 10^{-16} GeV\cdot m\right) 2.998\times 10^{8} m/s}{3.554 GeV}$$

$$= 2.67\times 10^{-27}\frac{C\cdot m^2}{s} = 2.67\times 10^{-27} A\cdot m^2$$

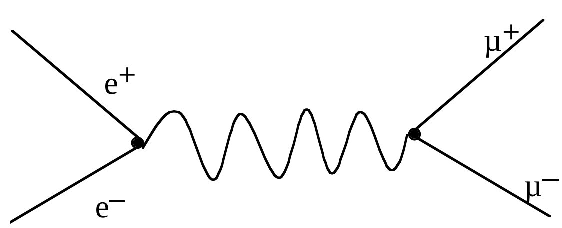
Note that eV=e×Volt so we could have used eV/e=V, but then I would have needed the SI value for Panck's constant which isn't conveniently on the back cover of M&S.

2) Consider a positron beam

a) Draw all distinct (*i.e.* not related by time ordering) Feynman diagrams for the processes

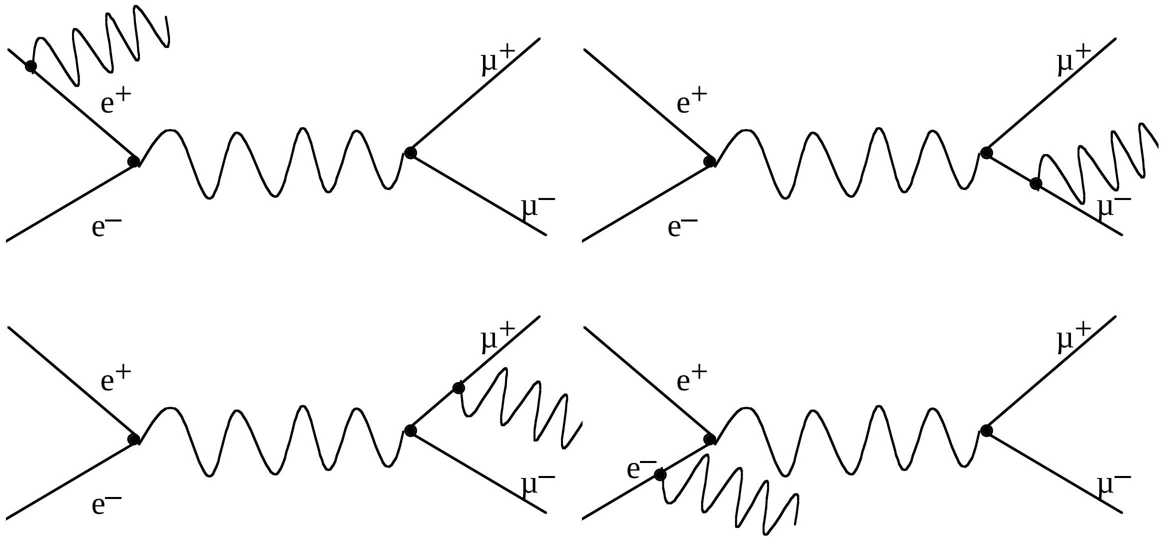
i) $e^+e^- \rightarrow \mu^+\mu^-$

From M&S section 1.3.3, and looking at the answers in Appendix F to M&S Problem 1.1, we can see there is only one diagram for this process.



ii) $e^+e^- \rightarrow \mu^+\mu^- \gamma$

For this process there are 4 possibilities:



Estimate the relative rate for process (ii) compared to process (i).

For the first process there are two photon-coupling vertices (each giving a factor α) and one time ordering; for the second process there are thress photon-coupling vertices and 4 time orderings. We thus expect process (ii) to have a relative rate of about 4α compared to process (i). (The factor of 4 is not clearly stated in M&S, and it is very approximate in any event, so it is not necessary to include it in your answer.)

b) Consider a positron passing through iron and interacting with an atomic electron to produce a $\mu^+\mu^-$ pair by the process $e^+e^- \rightarrow \mu^+\mu^-$?

i) What is the minimum (*i.e.* threshold) momentum a positron must have for this process to be possible?

The threshold centre-of-mass energy for this process is twice the muon rest mass, so using M&S Equation 3.3 (or A.12), we have

$$2m_\mu = \sqrt{m_{e^+}^2 + m_{e^\pm}^2 + 2m_{e^\pm}E_{threshold}} \approx \sqrt{2m_{e^\pm}E_{threshold}}$$

$$\therefore E_{threshold} \approx 2\frac{m_\mu^2}{m_e} = 2\frac{(0.1057GeV)^2}{0.000511GeV} = 43.7GeV$$

ii) Use M&S equation 6.3b to calculate the interaction length (*i.e.* the inverse of the probablity per unit length) for this process if the positron momentum is 100 GeV.

This equation give the cross section for the process

$$\sigma = \frac{4\pi\alpha^2}{3E_{cm}^2} = \frac{4\pi\alpha^2}{3\left(m_{e^+}^2 + m_{e^\pm}^2 + 2m_{e^\pm}E_{e^+}\right)}$$

$$\approx \frac{4\pi\alpha^2}{6m_{e^\pm}E_{e^+}} = \frac{4\pi\left(\frac{1}{137.04}\right)^2}{6(0.000511GeV)100GeV}$$

$$= 0.00218GeV^{-2} = 0.00218GeV^{-2}(\hbar c)^2$$

$$= 0.00218GeV^{-2}\left(1.973\times10^{-16}GeV\cdot m\right)^2 = 8.5\times10^{-35}m^2 = 0.85\mu b$$

using various numbers from the inside back cover of M&S.

Looking at M&S equations 3.8, we see that the interaction length is

$$l = \frac{1}{n\sigma}$$

$$\therefore l_{e^+e^-\to\mu^+\mu^-} = \frac{1}{n_{e^-}^{Fe}\sigma_{e^+e^-\to\mu^+\mu^-}}$$

where n is the density of scatterers (electrons in this case, nuclei for Equations 3.8). One could look up the density, atomic mass, and atomic number of iron, but I just scaled the nuclear collision length from M&S Table 3.3, *i.e.*

$$l_{e^+e^-\to\mu^+\mu^-} = \frac{1}{n_{e^-}^{Fe}\sigma_{e^+e^-\to\mu^+\mu^-}} = \frac{1}{Z^{Fe}n_{nuclei}^{Fe}\sigma_c\left(\frac{\sigma_{e^+e^-\to\mu^+\mu^-}}{\sigma_c}\right)}$$

$$= \frac{1}{Z^{Fe}\left(\frac{\sigma_{e^+e^-\to\mu^+\mu^-}}{\sigma_c}\right)}\frac{1}{n_{nuclei}^{Fe}\sigma_c} = \frac{\sigma_{tot}^{Fe}}{Z^{Fe}\sigma_{e^+e^-\to\mu^+\mu^-}}l_c$$

$$= \frac{1.120b}{26(0.85\mu b)}10.5cm = 5.3km$$

c) If a 100 GeV positron enters an iron calorimeter, estimate how many positrons will be produced in the shower.

Using the model of M&S Section 3.3.5(a) the number of particles at the peak of an electromganetic shower is approximately given by M&S Equation 3.23

$$N_{\max} = \frac{E_0}{E_c}$$

This model is based on a simple geometric progression (1 particle –> 2 particles –> 4 particles –> 8 particles -> …), so the total number of particles in the shower is just

$$N_{tot} = 2N_{\max} - 1 \approx 2N_{\max} = 2\frac{E_0}{E_c}$$

Looking at M&S Figure 3.11, it is obvious that the shower has approximately equal numbers of electrons, protons, and photons, so using the Critical Energy for iron from M&S Table 3.5, the number of positrons is

$$N_{e^+} = \frac{N_{tot}}{3} = \frac{2E_0}{3E_c^{Fe}} = \frac{2(100GeV)}{3(24MeV)} = 2.8 \times 10^3$$

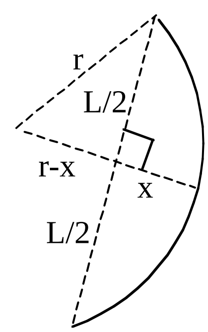
3) Figure 1.2 of Martin and Shaw shows one of the discovery pictures of the positron. The magnetic field is perpendicular to the picture and you may assume the tracks are in the plane of the picture. Your answers must include an uncertainty (*e.g.* 7±1 MeV), so you may need to look at your old *First Year Physics Laboratory Manual* or at http://www.upscale.utoronto.ca/PVB/Harrison/ErrorAnalysis/All.html

a) What is the momentum (in MeV/c) of the charged particle before it enters the lead plate? You may assume the particle has unit charge (*i.e.* $q=e$) and that it is relativisitic (*i.e.* $\beta \approx 1$).

We can use M&S equation 3.2 to determine the momentum, so we need the scale of the picture and the radius of curvature of the track.

The thickness of the lead plate is given in the text to be 6mm, and in the picture I measure it to be 4±1mm, so the picture is smaller than life size by a scale factor of 0.67±0.17.

The radius of curvature of the track can be measured either directly with a compass or by measuring the deviation from straightness and using a bit of trigonometry. Using a chord of length L along the track and the deviation from straightness x at the midpoint of the chord, the radius of curvature is



$$r = \frac{x}{2} + \frac{L^2}{8x}$$

I measured L=21±1mm and x=1.5±0.5mm. Assuming (probably incorrectly) that my L and x uncertainties are uncorrelated, then

$$dr = \frac{dx}{2} + \frac{2LdL}{8x} - \frac{L^2 dx}{8x^2} = \frac{dx}{2}\left(1 - \frac{L^2}{4x^2}\right) + \frac{LdL}{4x}$$

$$\therefore \sigma_r^2 = \frac{\sigma_x^2}{4}\left(1 - \frac{L^2}{4x^2}\right)^2 + \frac{L^2\sigma_L^2}{16x^2}$$

I have done the full error calculation, but this is unecessary since the error on L can be neglected compared to the error on x. If you used a compass, the uncertainty in r is just whatever you judged it to be. I get r=38±13mm, but we must correct by the scale factor in order to get the true radius of curvature of the track:

$$\rho = \frac{38 \pm 13mm}{0.67 \pm 17} = 57\left(1 \pm \sqrt{\left(\frac{13}{38}\right)^2 + \left(\frac{0.17}{0.67}\right)^2}\right)mm = 0.057 \pm 0.024\text{m}$$

The magnetic field is 1.5 Tesla, so using M&S equation 3.2 we have

$$p = 0.3B\rho = 0.3(1.5)(0.057 \pm 0.024) = 26 \pm 11 MeV$$

b) How much energy would be lost by the charged particle in the lead plate if it were

i) a proton?

A 26 MeV/c proton is non-relativistic and its kinetic energy is just

$$T = \frac{p^2}{2m} = \frac{(26 \pm 11 MeV)^2}{2(938.27 MeV)} = 0.36 \pm^{0.37}_{0.24} \ MeV$$

From M&S Figure 3.3 we can see that a 26 MeV/c proton is well up the $1/\beta^2$ ionization rise (see the Bethe-Bloch formula M&S Equation 3.11a) and its ionization energy losses will be many MeV/cm, so if this was a proton it would loses all its energy and stop in the lead plate.

ii) a positron?

A 26 MeV/c positron is relativistic, with $\beta \approx 1$ and $\gamma = E/m \sim \gamma$. A pion with $\gamma = 50$ would have a momentum of about 7 GeV, so from M&S Figure 3.3 we can see that the ionization energy losses would be about 17MeV/cm in lead. For particles of the same charge in the same material, their ionization energy losses depend only on γ, so the positron will also lose about 17MeV/cm, or 10MeV for the 6mm thick plat in this case.

The positron is above the critical energy for lead (7 MeV from M&S Table 3.5), so we also expect it to radiate

$$\Delta E = -\frac{dE}{dx}\Delta x = \frac{E}{L_R}\Delta x = E\,\frac{6mm}{0.56cm} \approx E\,.$$

So on average we expect the positron to radiate all its energy, but this is a random process and it is not suprising to find some positrons passing through the lead with significant energy.

4) For each part of this question, write down a process (*i.e.* an interaction or a decay) in which the initial state has strangeness S=–1 and which is:

a) allowed by the strong interaction

From M&S Section 2.2.4 we know that the K^- and the Λ have strangeness S=-1. From M&S Chapter 2 and the solutions to the problems at the end of Chapter 2 we know that strangenss must be conserved by the strong interaction, so some examples:of allowed strong processes would be:

$$K^- p \longrightarrow \pi^0 \Lambda$$

$$K^- p \longrightarrow K^- p$$

b) forbidden by the strong interaction but is allowed by the weak interaction

The weak interaction can violate strangeness conservation, so examples of allowed processes are

$$\Lambda \longrightarrow \pi^- p$$

$$K^- p \longrightarrow \pi^0 n$$

c) forbidden by the strong, electromagnetic, and weak interactions, but is not forbidden by energy-momentum conservation.

Here we want something that violates something like electric charge or baryon number or lepton number conservation. For example,

$$\Lambda \longrightarrow \pi^- e^-$$

$$K^- \longrightarrow \pi^0 e^-$$

$$K^- p \longrightarrow \pi^0 \pi^+$$