# Problem Set 2

## due Monday, 11 February 2002

**(Late penalty is 10% per day, and no problem set is accepted after February 11.)**

Each problem is of equal weight, but **not all problems may be marked**.

These problems are primarily based on Chapters 1-7 of Martin and Shaw and on the lectures notes up to January 23; Appendices A, B and E and the inside back cover of M&S may also be helpful. If you have any questions, ask me or Stan, preferably before the last lecture prior to the due date.

1) The "whole body dose" is the radiation dose absorbed by a person averaged over their whole body. A typical transatlantic flight will give you an additional whole body radiation dose of about 30 μSv due to cosmic rays.

a) Almost very time a $^{60}$Co ("Cobalt-60") atom decays it produces two gamma rays: one at 1173KeV and one at 1333KeV. If a typical undergraduate lab 40 KBq $^{60}$Co source is 1m away from a student for 6 three hour lab periods, approximately what whole body radiation dose will the student receive from the source?

Using the formula from Lecture 5 we have that the dose rate for the two gammas is

$$D = 2A\frac{E}{R^2} = 2(0.040Mbq)\frac{1.173MeV}{(1m)^2} + 2(0.040Mbq)\frac{1.333MeV}{(1m)^2} = 0.2\frac{\mu SV}{hr}$$

So after 18 hours the total dose would be

$$\int D = 0.2\frac{\mu SV}{hr}(18hr) = 3.6\mu SV$$

Which is only a tenth the dose of a (one way) transatlantic flight.

b) When a $^{90}$Sr ("Strontium-90") atom decays, it produces beta decay electrons with a average total energy of about 1MeV. If a beta electron enters your body it is absorbed. If student accidentally puts a 40 KBq $^{60}$Sr source in their pocket next to their skin and forgets about it for 3 days until the next lab period, what whole body radiation dose will the student receive from the source? (The student will have to fill out a whole bunch of long forms, but we won't get into that here.)

Any beta that enters the body will be absorbed because the range of a 1 MeV electron is less than a centimetre. (From M&S Section 3.2.2. we know that the minimum ionization of a unit charged particle is typically 2 MeV/gm/cm$^2$; the human body has a density about 1 gm/cm$^3$.) Half the betas will be absorbed, so the total energy absorbed by the student will be

$$\int E = \tfrac{1}{2}(1MeV)(40KBq)(3days \times 24hr/day \times 3600s/hr) = 5.2\times10^{15}eV = 0.83mJ$$

where we have assume the student is working too hard to change clothes even to sleep. We don't know how much the student weighs, but most adults are in the range of 50-150kg; I'll assumed the student has a mass of 83kg to make the things come out nice. Averaged over the whole body the radiation dose is then

$$\int D = \frac{\int E}{Mass_{student}} = \frac{0.83mJ}{83kg} = 1\times10^{-5}\frac{J}{kg} = 10\mu Gy$$

This is a very similar whole body dose as for the gammas. This dose will actually be concentrated in a very small area, and the local dose over the cubic centimetre or so absorbing the gammas will be about 100kg/1g=100,000 times larger, *i.e.* a Gy. I hope the pocket is not over a part of the human body particularly sensitive to radiation; rapidly dividing cells tend to be more sensitive, e.g. bone marrow, the mucolining of the intestine.  If you want a quick summary of the radiation sensitivity human tissue, look at http://www.google.com/search?q=cache:ubbuqDYiWFsC:www.orcbs.msu.edu/radiation/radmanual(html)/radmansensitive.html; for more gory detail, see http://www.atsdr.cdc.gov/toxprofiles/tp149-c3.pdf.

2)

a) Consider the one particle Hamiltonian in three dimensions:

$$H = -\frac{1}{2m}\nabla^2 + k_x x^{a_x} + k_y y^{a_y} + k_z z^{a_z}$$

where $k_x, a_x, k_y, a_y, k_z, a_z$ are real constants. For example, $k_x = k_y = k_z \neq 0$, $a_x = a_y = a_z = 2$ would be a three dimensional harmonic oscillator.  What are the constraints on $k_x, a_x, k_y, a_y, k_z, a_z$ if we require that three momentum, $\vec{\mathbf{p}} = \left(p_x, p_y, p_z\right)$, must be conserved?

We imediately know that $k_x = k_y = k_z = 0$ will give energy conservation, since that just gives us the free body hamiltonian.   In general to have momentum conservation, we must have

$$[H, \vec{p}] = 0$$

In this case

$$\begin{aligned}
[H, \vec{p}] &= H\vec{p} - \vec{p}H \\
&= \left(-\frac{1}{2m}\nabla^2 + k_x x^{a_x} + k_y y^{a_y} + k_z z^{a_z}\right)\left(-i\vec{\nabla}\right) - \left(-i\vec{\nabla}\right)\left(-\frac{1}{2m}\nabla^2 + k_x x^{a_x} + k_y y^{a_y} + k_z z^{a_z}\right) \\
&= i\left\{\frac{1}{2m}\vec{\nabla}^3 - k_x x^{a_x}\vec{\nabla} - k_y y^{a_y}\vec{\nabla} - k_z z^{a_z}\vec{\nabla} - \frac{1}{2m}\vec{\nabla}^3 + k_x\vec{\nabla}x^{a_x} + k_y\vec{\nabla}y^{a_y} + k_z\vec{\nabla}z^{a_z}\right\} \\
&= i\left\{\begin{matrix} -k_x x^{a_x}\vec{\nabla} - k_y y^{a_y}\vec{\nabla} - k_z z^{a_z}\vec{\nabla} + \left(k_x a_x x^{a_x - 1}, k_y a_y y^{a_y - 1}, k_z a_z z^{a_z - 1}\right) \\ + k_x x^{a_x}\vec{\nabla} + k_y y^{a_y}\vec{\nabla} + k_z z^{a_z}\vec{\nabla} \end{matrix}\right\} \\
&= i\left(k_x a_x x^{a_x - 1}, k_y a_y y^{a_y - 1}, k_z a_z z^{a_z - 1}\right)
\end{aligned}$$

So we see that this will be zero and we will have momentum conservation if

$$a_x k_x = a_y k_y = a_z k_z = 0$$

*i.e.* Only if the '*k*'s are constants and the '*a*'s are zero.  (We would have to add a term corresponding to another particle's free particle momentum and make the potential terms correspond to the difference in their position to get momentum conservation.)

b) The 1993 Nobel prize in Physics was given for observations of a binary pulsar which strongly indicate that gravitational radiation must exist.  The graviational force is presumably mediated by bosons called "gravitons", and gravitational radiation presumably consists of these gravitons.  Infer the parity (P) and charge conjugation (C) of the graviton from the behaviour of the classical Newtonian gravitational field.

The classical Newtonian gravitational field looks just like the classical electromagnetic field, except that there is not such thing as negative mass.  We can follow exactly the same steps as in M&S 4.3.4 discussing the parity of the photon, which shows that the parity of the gravition must be

$$\mathrm{P_g} = -1.$$

M&S 4.4.1 discusses the charge conjugation of the photon, and this is negative since electric charge changes sign under charge conjugation. Mass does not change sign under charge conjugation, *i.e.* the mass of an antiparticle is the same as the mass of the corresponding particle, so the graviton must have charge conjugation

$$C_g = +1.$$

3)

a) Do the masses of all the hadrons given in Appendix E.4 and E.5 indicate a consistant mass difference between the u and d quarks?

There is more than one "correct" approach for this problem. This is how I did it.

The mass differences among the Σ are discussed (as discussed in M&S 5.2.4) give

$$m_d\text{-}m_u = 3.75\text{MeV} \qquad \text{(I give one more sig. fig. than M\&S)}$$

Looking at the mass difference for hadrons differing only by the change of a u to a d quark, we have

$$M(n)\text{-}M(p) = (939.5656\pm0.0003)\text{-}(938.2723\pm0.0003)\text{MeV} = 1.2933\pm0.0004 \text{ MeV}$$
$$M(\Sigma^0)\text{-}M(\Sigma^+) = (1192.55\pm0.08)\text{-}(1189.37\pm0.07)\text{MeV} = 3.18\pm0.11 \text{ MeV}$$
$$M(\Sigma^-)\text{-}M(\Sigma^0) = (1197.44\pm0.03)\text{-}(1192.55\pm0.08)\text{MeV} = 4.89\pm0.09 \text{ MeV}$$
$$M(\Xi^-)\text{-}M(\Xi^0) = (1321.3\pm0.1)\text{-}(1314.9\pm0.3)\text{MeV} = 6.4\pm0.3 \text{ MeV}$$
$$M(\Xi_c{}^0)\text{-}M(\Xi_c{}^+) = (2470\pm2)\text{-}(2466\pm1)\text{MeV} = 4\pm2.2 \text{ MeV}$$
$$M(K^0)\text{-}M(K^+) = (497.67\pm0.03)\text{-}(493.68\pm0.02)\text{MeV} = 3.99\pm0.04 \text{ MeV}$$
$$M(D^+)\text{-}M(D^0) = (1869.3\pm0.5)\text{-}(1864.5\pm0.5)\text{MeV} = 4.8\pm0.7 \text{ MeV}$$
$$M(B^0)\text{-}M(B^+) = (5279\pm2)\text{-}(5279\pm2)\text{MeV} = 0\pm3 \text{ MeV}$$

I have added the uncertainties in quadrature, although in most cases this is an overestimate of the uncertainty on the difference since the errors on closely related particles are likely to be correlated.

If we assume the wavefunctions are similar (in size, especially), then we can use the same simple model as M&S are write down the mass differences as

$$M(n)\text{-}M(p) = M(\Sigma^0)\text{-}M(\Sigma^+) = M(\Xi_c{}^0)\text{-}M(\Xi_c{}^+) = M(B^0)\text{-}M(B^+) = (m_d - m_u) - \frac{1}{3}\delta$$

$$M(\Sigma^-)\text{-}M(\Sigma^0) = M(\Xi^-)\text{-}M(\Xi^0) = M(K^0)\text{-}M(K^+) = M(D^+)\text{-}M(D^0) = (m_d - m_u) + \frac{2}{3}\delta$$

So we expect in our simple model

$$M(n)\text{-}M(p) = M(\Sigma^0)\text{-}M(\Sigma^+) = M(\Xi_c{}^0)\text{-}M(\Xi_c{}^+) = M(B^0)\text{-}M(B^+) = 3.2\text{MeV}$$
$$M(\Sigma^-)\text{-}M(\Sigma^0) = M(\Xi^-)\text{-}M(\Xi^0) = M(K^0)\text{-}M(K^+) = M(D^+)\text{-}M(D^0) = 4.9\text{MeV}$$

The values we actually have, respectively, are

$$1.2933\pm0.0004 \text{ , } 3.18\pm0.11 \text{ , } 4\pm2.2 \text{ , } 0\pm3 \text{ MeV}$$
$$4.89\pm0.09 \text{ , } 6.4\pm0.3 \text{ , } 3.99\pm0.04 \text{ , } 4.8\pm0.7$$

And the first set are all less than or equal to the second set, so we have at least rough qualitative agreement with a constant mass difference for the up and down quarks. It is not surprising that the agreement is not exact, since the more massive strange quarks change the wave functions noticiably, and meson wave functions are different from baryon wavefunctions.

b) Estimate the mass differences between the $\Delta^-$, $\Delta^0$, $\Delta^+$, and $\Delta^{++}$ baryons.

M&S Table 6.4 gives us the quark contents of the Δ baryons, so we can write down the masses in our simple approximation:

$$M(\Delta^-) = M_0 + 3m_d + \delta(3e_d^2) = M_0 + 3m_d + \frac{1}{3}\delta$$

$$M(\Delta^0) = M_0 + m_u + 2m_d + \delta(e_d^2 + 2e_ue_d) = M_0 + m_u + 2m_d - \frac{1}{3}\delta$$

$$M(\Delta^+) = M_0 + 2m_u + m_d + \delta(e_u^2 + 2e_ue_d) = M_0 + 2m_u + m_d$$

$$M(\Delta^{++}) = M_0 + 3m_u + \delta(3e_u^2) = M_0 + 3m_u + \frac{4}{3}\delta$$

From M&S equation 5.26, we can solve for δ, *i.e.*

$$\delta = M(\Sigma^+) + M(\Sigma^-) - 2M(\Sigma^0) = 1189.37+1197.44-2(192.55)\text{ MeV} = 1.71\text{ MeV}$$

So the approximate mass differences should be about

$$M(\Delta^-) - M(\Delta^0) = (m_d - m_u) + \frac{2}{3}\delta = 5\text{ MeV}$$

$$M(\Delta^0) - M(\Delta^+) = (m_d - m_u) - \frac{1}{3}\delta = 3\text{ MeV}$$

$$M(\Delta^+) - M(\Delta^{++}) = (m_d - m_u) - \frac{4}{3}\delta = 1.5\text{ MeV}$$

4) Martin and Shaw Problem 7.3

In this question we want to integrate

$$M_C(\mathbf{q}) = \int d^3\mathbf{x}\left(-\frac{\alpha}{r}\right)e^{i\mathbf{q}\cdot\mathbf{x}},$$

but this diverges if we just try to do it, so we use a trick of taking the appropriate limit for a screened potential

$$M_C(\mathbf{q}) \xrightarrow[\mu\to 0]{} \int d^3\mathbf{x}\left(-\frac{\alpha e^{-\mu r}}{r}\right)e^{i\mathbf{q}\cdot\mathbf{x}},$$

As is discussed in the hints at the back of M&S, we can let **q** define the *z* axis, so

$$\begin{aligned}
M_C(\mathbf{q}) &= \underset{\mu\to 0}{Limit}\int_{\phi=0}^{2\pi} d\phi \int_{\cos\theta=-1}^{1} d(\cos\theta)\int_{r=0}^{\infty} r^2 dr\left(-\frac{\alpha e^{-\mu r}}{r}\right)e^{iqr\cos\theta} \\
&= \underset{\mu\to 0}{Limit}\int_{r=0}^{\infty} r^2 dr\left(-\frac{2\pi\alpha e^{-\mu r}}{iqr^2}\right)\left(e^{iqr} - e^{-iqr}\right), \\
&= -\underset{\mu\to 0}{Limit}\int_{r=0}^{\infty} dr\,\frac{4\pi\alpha}{q}e^{-\mu r}\sin(qr)
\end{aligned}$$

This can be integrated by parts, but that isn't required in this course, so I just got it using Maple, althought I just as easily used Mathematica or looked it up in a Table or Integrals. We then have

$$\begin{aligned}
M_C(\mathbf{q}) &= -\frac{4\pi\alpha}{q}\underset{\mu\to 0}{Limit}\left[-e^{-\mu r}\frac{q\cos(qr) + \mu\sin(qr)}{\mu^2 + q^2}\right]_{r=0}^{\infty} \\
&= -\frac{4\pi\alpha}{q}\underset{\mu\to 0}{Limit}\frac{q}{\mu^2 + q^2} \qquad = -\frac{4\pi\alpha}{q^2}
\end{aligned}$$