University of Toronto PHY357S

Problem Set 4 Answers

due Monday, 8 April 2002

(Late penalty is 10% per day, and no problem set is accepted after April 10.)

Each problem is of equal weight, but **not all problems may be marked**.

Note: The last lecture for this term (April 10) will be held in Room 408 instead of Room 137.

These problems are based on Martin and Shaw and on the lectures.

1) a) Martin & Shaw Problem 10.4

> The solution to this is basically as given in the "Hints" in Martin & Shaw. The last point is very important: "The desired result follows on summing over all possible final state momenta and spins." *i.e.* The equality of the total decay rate follows because there is a one-to-one mapping by CP of every decay configuration of a K^0 onto an allowed decay configuration of a $\bar{K}^0$. In every case the rate for the mapped configurations are equal since the magnitudes of the momenta and angles are the same.

b) Martin & Shaw Problem 10.5

> As given in the "Hints" in Martin & Shaw.

2) Martin & Shaw Problem 11.2

> As given in the "Hints" in Martin & Shaw.

3) The binding energy of an atomic nucleus with Z protons and N neutrons (atomic number A=N+Z) is approximately given for heavier nuclei by a semi-empirical formula:

> Note typo, A is not the atomic number, it is the mass number or nucleon number.

$$E_b = C_{Volume}A - C_{Surface}A^{2/3} - C_{Coulomb}\frac{Z(Z-1)}{A^{1/3}} - C_{symmetry}\frac{(N-Z)^2}{A}$$

The origin of each term and the value of the constant coefficients are given below:

- The binding energy due to the short range strong nuclear force is almost entirely due to how many immediate neighbours a nucleon has, so if each nucleus has the same number of neighbours then this contribution to the binding energy is simply proportional to the number of nucleons in the nucleus.

$$C_{Volume} = 15.7\ MeV$$

- Nucleons on the surface have fewer neighbours, so the binding energy must be reduced by a term proportional to the surface area of a nucleus. If we assume a nucleus is spherical and of constant density, then the surface area is proportional to $4\pi A^{2/3}$.

$$C_{Surface} = 17.8\ MeV$$

- Protons repel each other and the corresponding potential energy is inversely proportional to the average distance between protons which is proportional to $A^{1/3}$.

$$C_{Coulomb} = 0.71\ MeV$$

- Nucleons are fermions, so the Pauli Exclusion Principle requires that you can't have two protons (or two neutrons) in the same quantum state. This means it costs less energy to make a nucleus which is symmetric in N and Z.

$C_{symmetry} = 23.6$ MeV

This formula roughly describes the data, but more terms must be added if more accuracy is required.

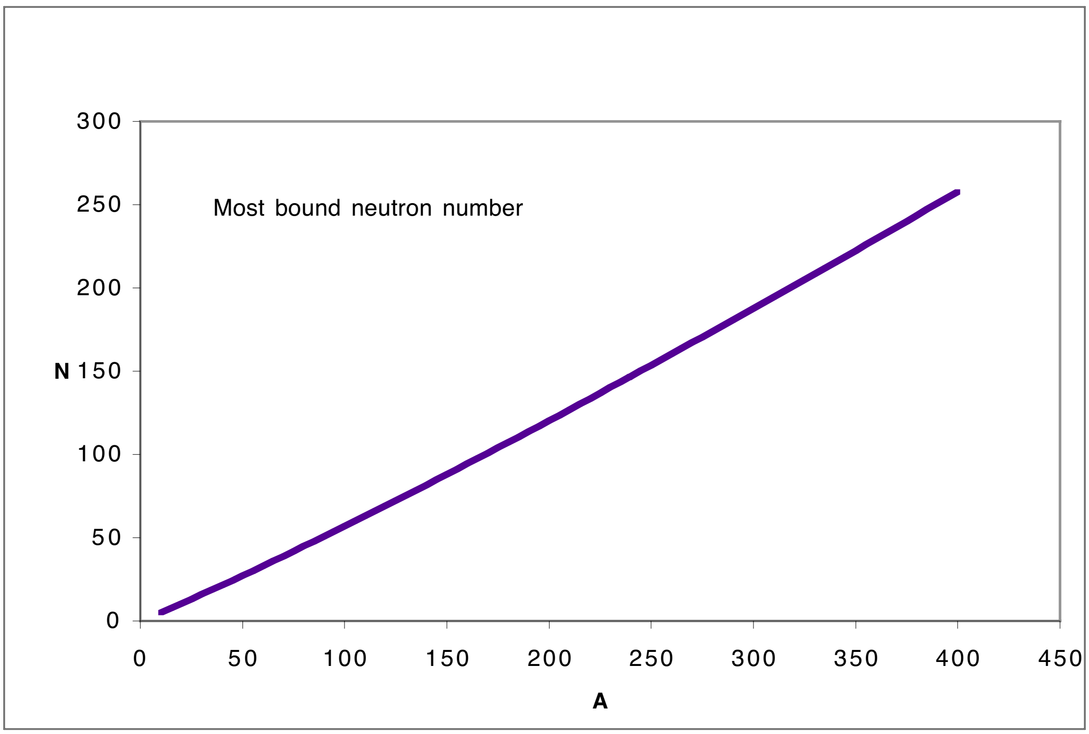
(a) Calculate and plot the neutron number which the formula predicts will give the most strongly bound nucleus as a function of atomic number *A*, for values from A=10 up to A=400.

So for fixed *A*, we want the value of *N* (or equivalently *Z*) which gives the most binding energy. We can find that the maximum binding energy by differentiating w.r.t. *Z* and solving for the zeros, *i.e.*

$$\frac{dE_b}{dZ} = -C_{Coulomb}\frac{2Z-1}{A^{1/3}} - C_{symmetry}\frac{-4(A-2Z)}{A} = 0$$

$$\therefore Z = \frac{1}{2}\frac{\left(C_{Coulomb}A + 4C_{symmetry}A^{4/3}\right)}{\left(C_{Coulomb}A + 4C_{symmetry}A^{1/3}\right)}$$

$$\therefore N = A - Z = A - \frac{1}{2}\frac{\left(C_{Coulomb}A + 4C_{symmetry}A^{4/3}\right)}{\left(C_{Coulomb}A + 4C_{symmetry}A^{1/3}\right)}$$



(b) Assume that when a nucleus undergoes fission, it splits into two strongly bound nuclei and some free neutrons. (I define a "strongly bound" nucleus to be one on the line you plotted for part (a).) Assume the most likely fission products are those which result in the maximum energy release.. Use this **approximate** model to estimate the most likely number of neutrons produced and the most likely energy release for such fission of ${}^{239}_{94}Pu$.

Electric charge has to be conserved, and neutrons carry no charge, so we have to find the two strongly bound nuclei with a total charge $Z_1+Z_2=Z=94$ which have the the largest energy release, *i.e.*

$$
\begin{aligned}
E_{released} &= M_A - M_{A_1} - M_{A_2} - (A - A_1 - A_2)m_{neutron} \\
&= \left[Zm_{proton} + (A-Z)m_{neutron} - E_{b_A}\right] - \left[Z_1 m_{proton} + (A_1 - Z_1)m_{neutron} - E_{b_1}\right] \\
&\quad - \left[Z_2 m_{proton} + (A_2 - Z_2)m_{neutron} - E_{b_2}\right] - (A - A_1 - A_2)m_{neutron} \\
&= (Z - Z_1 - Z_2)m_{proton} + (A - A_1 - A_2 - Z + Z_1 + Z_2)m_{neutron} \\
&\quad - (A - A_1 - A_2)m_{neutron} - E_{b_A} + E_{b_1} + E_{b_2} \\
&= E_{b_1} + E_{b_2} - E_{b_A}
\end{aligned}
$$

(The last line is actually pretty obvious, but I like to check.) The binding energy of the intial nucleus is fixed, so we want to find the two fission product nuclei which give the largest total binding energy. Since A=239, Z=94 is known, for each strongly bound fission product nucleus A_1, Z_1, the fission partner (in our very simple model) is fixed (A_2=A-A_1, Z_2=Z-Z_1) and the binding energy can be calculated and the maximum found. This can be done by carefully looking at the data used to make the plot in part(a), and the result is that the two fission products should be identical, *i.e.* Z_1=Z_2=Z/2=47. The fact that the fission products should be identical also follows from simple considerations, *i.e.* The binding energy of strongly bound nuclei is described by some function A_{strong}=f(Z), so the total binding energy is just B(Z)=f(Z_1)+f(Z_2)=f(Z_1)+f(Z-Z_1). The maximum of this sum must occur when B′=0, *i.e.* when f′(Z_1)-f′(Z-Z_1)=0. There may be more than one solutoin, but the obvious one is when Z_1=Z-Z_1, *i.e.* Z_1=Z/2. From the data for part (a) I get that A_1=A_2=110, so the number of free neutrons is A-A_1-A_2=19.