Prob Set #1

① Remember $E^2 = p^2 + m^2$

so it doesn't matter if 2 GeV refers to total Relativistic energy or kinetic. But.... it's the same.

- Rest mass is same in all frames
- Proper lifetime is lifetime in Rest frame.

$\tau_{REST\ FRAME} = \tau = t_2 - t_1$

in an ~~accelerated~~ frame moving at velocity β in z direction

$$t' = \gamma'\left(t - \frac{\beta}{c} z\right)$$

where $\gamma' = \frac{1}{\sqrt{1-\beta^2}}$ $\qquad \beta = \frac{v}{c}$.

$\tau_{LAB} = t_2' - t_1' = \gamma' \tau$.

Easiest way to calc γ is $\gamma = \frac{E}{m}$ $c = 1$

$(E = \gamma m)$

$$\tau^{\pi}_{LAB} = \frac{200}{0.14} \times 2.6 \times 10^{-8} s = 3.7 \times 10^{-5} s$$

$$\tau^{K}_{LAB} = \frac{200}{0.494} \times 1.2 \times 10^{-8} s = 4.86 \times 10^{-6} s.$$

(2)

Flux = 2 cal cm^{-2} min^{-1}

$= 2 \text{ cal } cm^{-2} min^{-1} \times \frac{4.19 \text{ J}}{\text{cal}} \times \frac{1 \text{ min}}{60 \text{ s.}} \times \frac{10000 \text{ cm}^2}{\text{m}^2}$

$= 1397 \text{ J. } m^{-2}. s^{-1}$

• Total flux from sun passes thru the surface of sphere Radius R.

Area = $4\pi R^2$

R ← Earth – Sun Distance

∴ Ratio $\frac{\text{Flux thru Total Solid Angle}}{\text{" " " 1m at Earth}}$

$= 4\pi R^2$

∴ Total flux: $4\pi R^2$

$= 4\pi (93 \times 10^6 \text{ miles})^2 \left(\frac{1609 \text{ m}}{\text{mile}}\right)^2$

$= 2.81 \times 10^{23} \text{ m}^2.$

$$\frac{\text{Total Energy lost}}{\text{sec}} = \frac{E}{\Delta t}$$

$$= (1397\ J m^{-2} s^{-1})(2.8 \times 10^{23}\ m^2)$$

$$= 3.93 \times 10^{26}\ J.\ s^{-1}$$

$$\text{Total Mass lost/sec} : \frac{m}{\Delta t} = \frac{E}{c^2 \Delta t} = \frac{3.93 \times 10^{26}\ J s^{-1}}{(3.0 \times 10^8\ m s^{-1})^2}$$

$$= 4.36 \times 10^{9}\ kg\ s^{-1}$$

which doesn't matter compared to 10^{33} g.

b) Helium synthesis

$$p n \rightarrow d \gamma \quad (d = \text{deuteron})$$

$$d + p \rightarrow {}^3He$$

$$d\ {}^3He \rightarrow p\ {}^4He$$

Basically Helium production is $4H \rightarrow {}^4He$.

$$\Delta m = 4(1.6735 \times 10^{-24}\ g) - 6.6456 \times 10^{-24}\ g$$

$$= 0.0484 \times 10^{-24}\ g.$$

4.36×10^{12} g/s lost in this way

$\therefore$ no of processes / sec = $\frac{4.36 \times 10^{12} \text{ g s}^{-1}}{0.0484 \times 10^{-24} \text{ g}}$

$= 9.01 \times 10^{37}$ processes.

Amount of Helium / sec $= 9.01 \times 10^{37} \times 6.645 \times 10^{-24}$ g

$= 5.99 \times 10^{14}$ g / sec

$= 5.99 \times 10^{8}$ tonnes / sec.

3) a) Minimum beam energy is when final state particles produced at Rest in CM.

1) Total 4-momentum is Conserved

2) (Total 4-momentum)^2 is frame indep.

INITIALLY IN LAB

$$p_{TOTAL} = p_{A_L} + p_{B_L}$$

$$(p)^2 = (p_{A_L} + p_{B_L})^2$$

$$= p_{A_L}^2 + p_{B_L}^2 + 2\, p_{A_L} p_{B_L}$$

$$= m_A^2 + m_B^2 + 2\left(E^A_{LAB}, \vec{p}^{\,A}_L\right)\cdot\left(E^B_{LAB}\ \vec{p}^{\,B}_L\right)$$

$$\left(E^B_{LAB} = m_B,\ \vec{p}^{\,B}_L = 0\right)$$

$$\therefore (p)^2 = m_A^2 + m_B^2 + 2 E^A_{LAB} \cdot m_B$$

↳ This is just M^2

$$\frac{M^2 - m_A^2 - m_B^2}{2 m_B} = E^A_{LAB}.$$

3)b) Second part is correct!

$$\vec{p}_{e^+} = -\vec{p}_{e^-}$$

$$(E_{CM})^2 = M_{\pi}^2 = (p)^2$$

p ← TOTAL 4-MOMENTUM

$$p^2 = (p_{e^+} + p_{e^-})^2$$

p_1 p_2

$$= p_1^2 + p_2^2 + 2\, p_{e^+} p_{e^-}$$

$$= m_e^2 + m_e^2 + 2(E_{e^+}\ \vec{p})(E_{e^-}\ -\vec{p})$$

$$= 2m_e^2 + 2E_{e^+}E_{e^-} + 2|\vec{p}|^2.$$

m_e is ≈ 0 $\quad E_{e^+} = E_{e^-} = p.$

$$\therefore\ p^2 = 4p^2 = M_{\pi}^2.$$

p ← beam momentum.

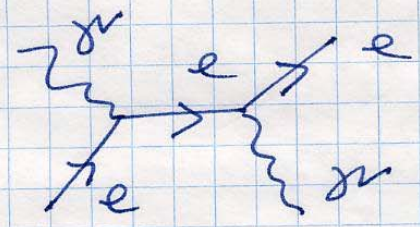
3)a) Use result of 3a) for LAB system.

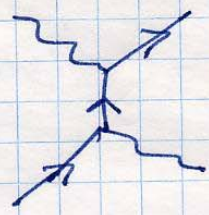
3)c) just 3a again

Prob Set #1

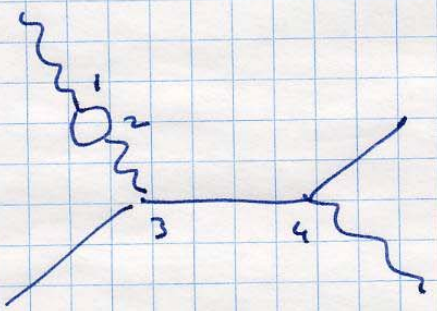
(8)

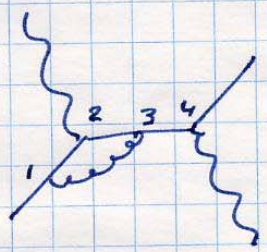
4) a) Add corrections to two diagrams



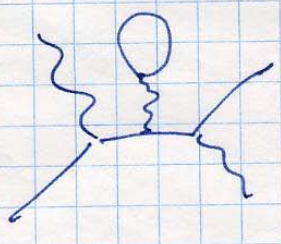


typical 4_p - vertex diagrams





etc - - .



does not contribute

↳ I explained in class. Ask me if you forget.

4 b) $e^+ e^- \rightarrow e^+ e^-$

electron + positron have equal + opposite 3 momenta.
and they have total $E_{CM.} = E$

After collision γ must be at rest
i.e 3 momenta conserved.
$\therefore$ total energy $= E$

$$E^2 = p^2 + m^2$$

$p = 0 \therefore$ velocity $= 0$!

γ^* virtual has $m = E \neq 0$.

VIRTUAL PARTICLE