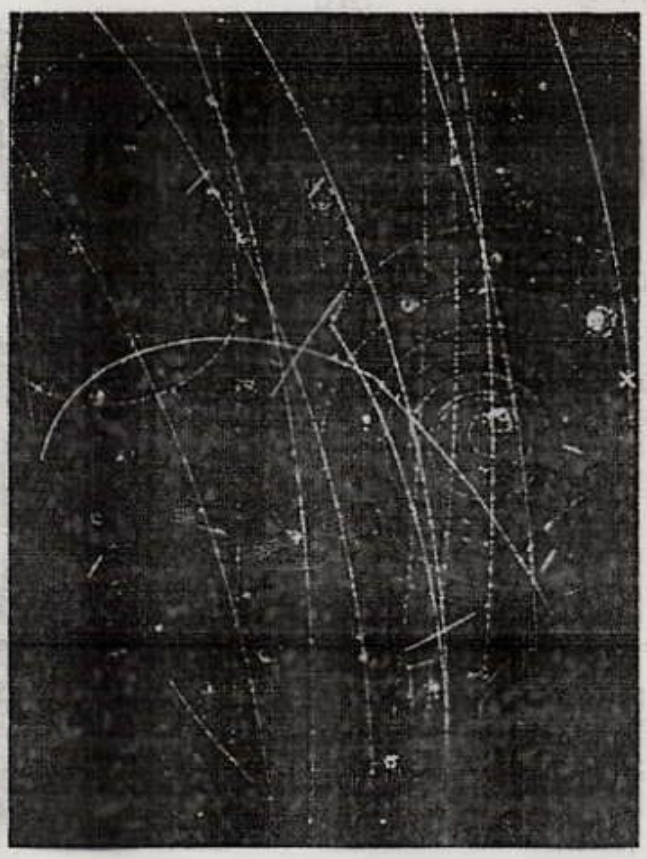
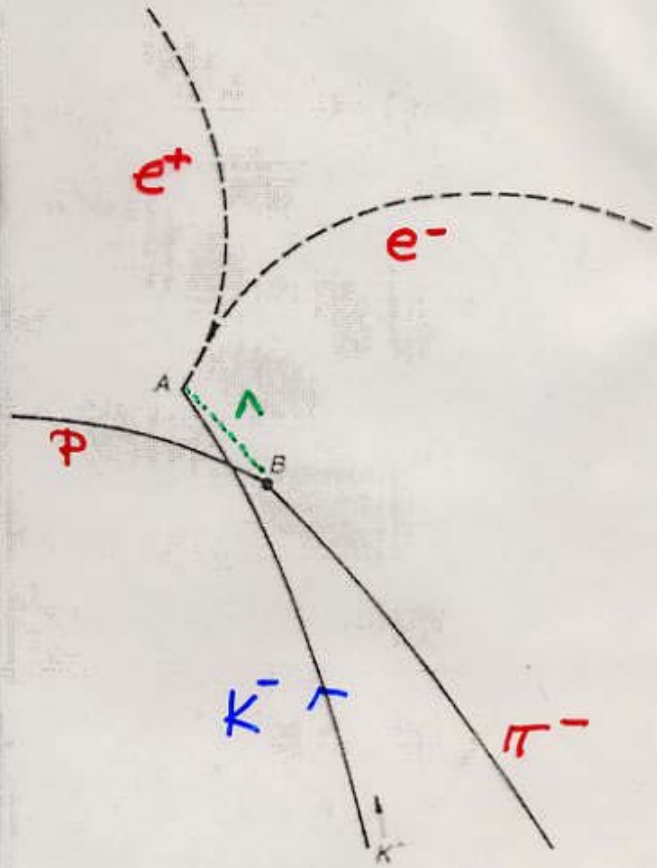
ANOTHER EXAMPLE OF STRANGENESS CONSERVATION





$$\overset{S}{K^-} \overset{-1}{} + \overset{0}{p} \rightarrow \overset{-1}{\Lambda} + \overset{0}{\pi^0}$$

$\pi^0 \rightarrow \gamma^* e^+ e^-$ $\quad S = 0$

10^{-16} s

EM

$\Lambda \rightarrow p\pi^-$

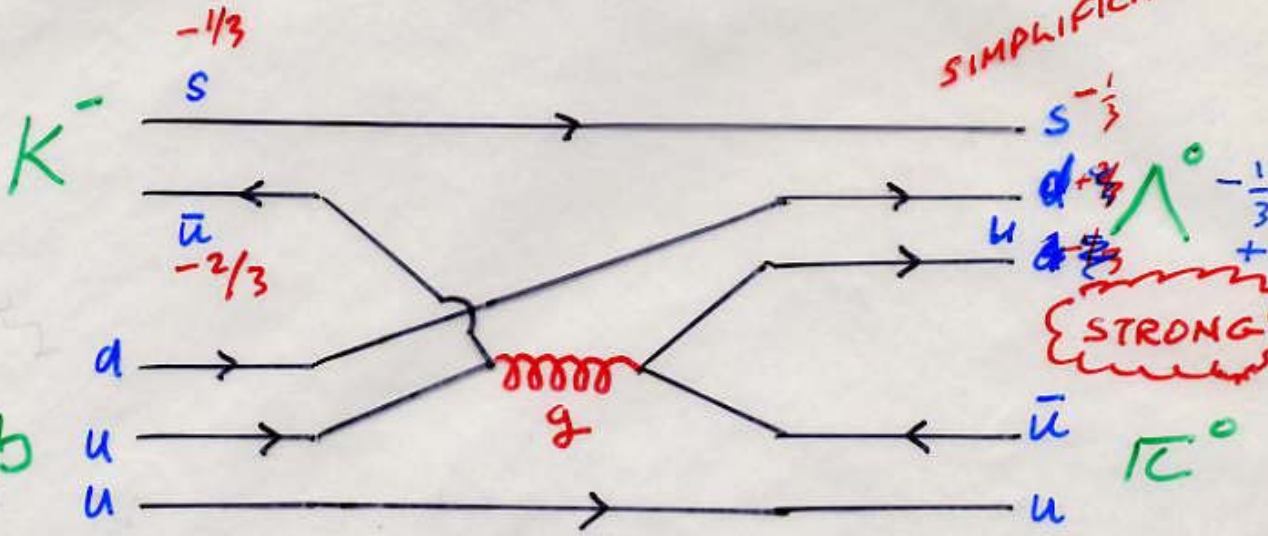
$S = 0$

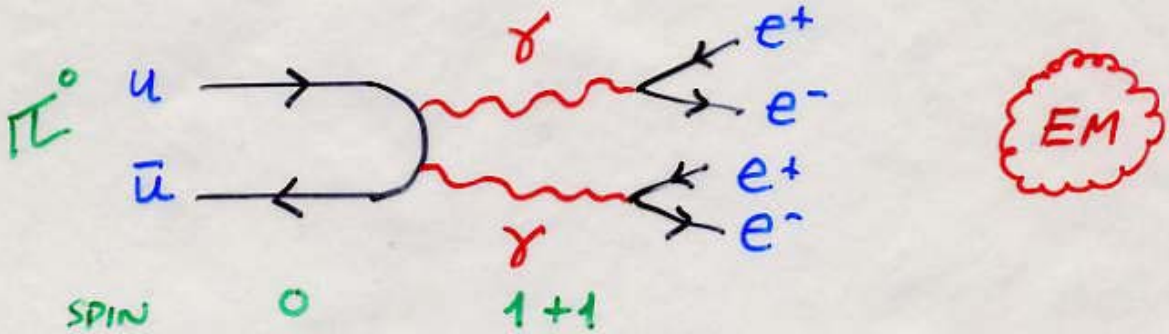
10^{-10} s

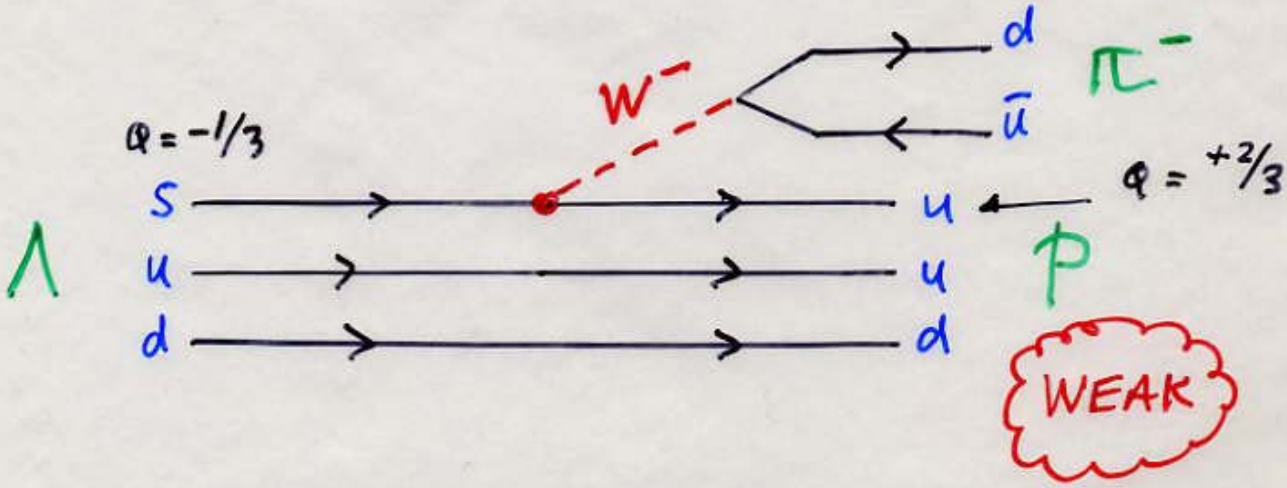
WEAK

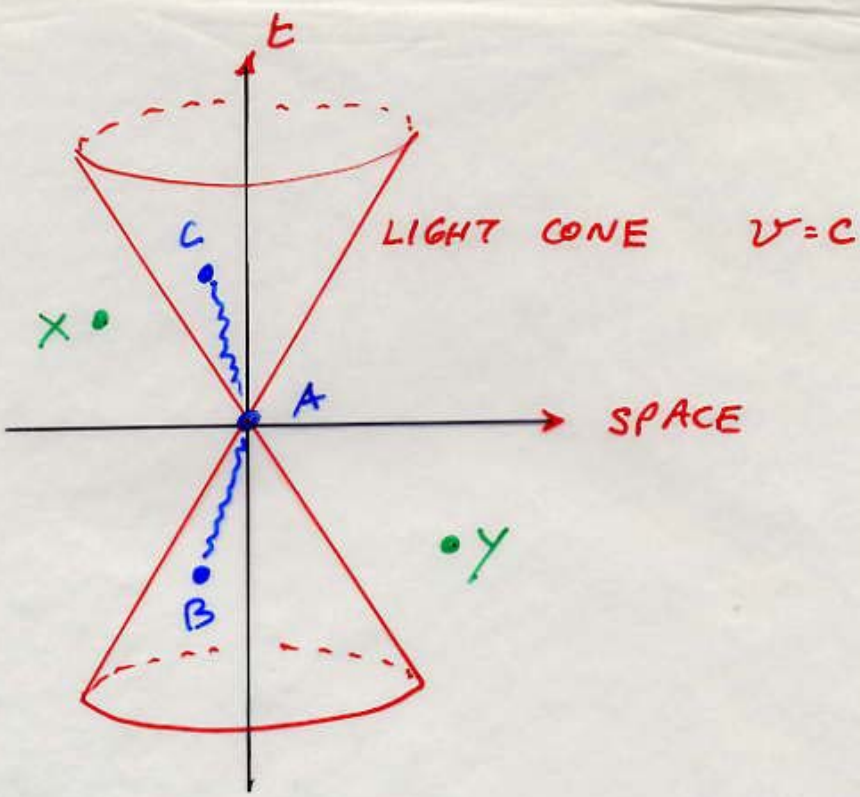
$K^- p \to \Lambda \pi^0$

IN QUARK LANGUAGE







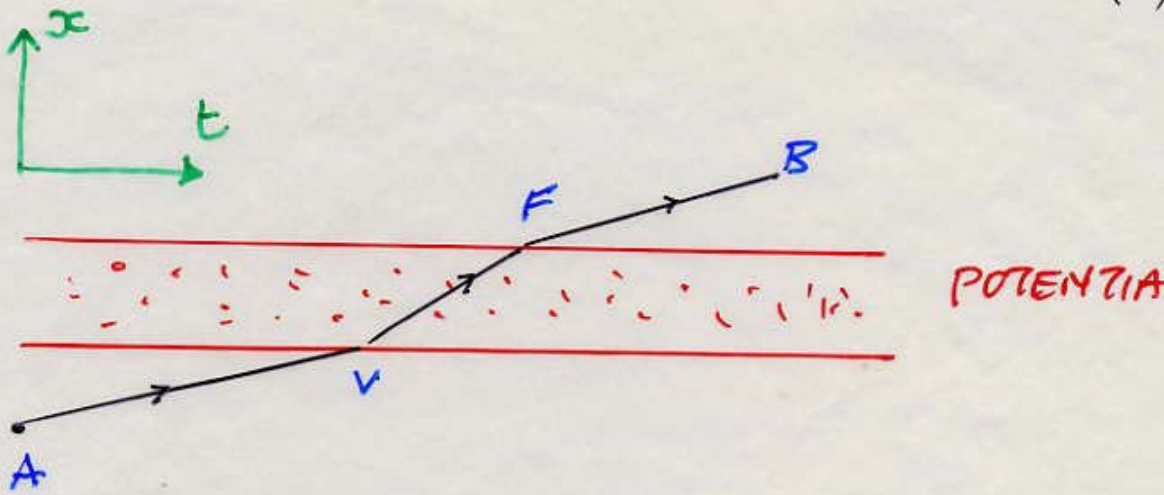


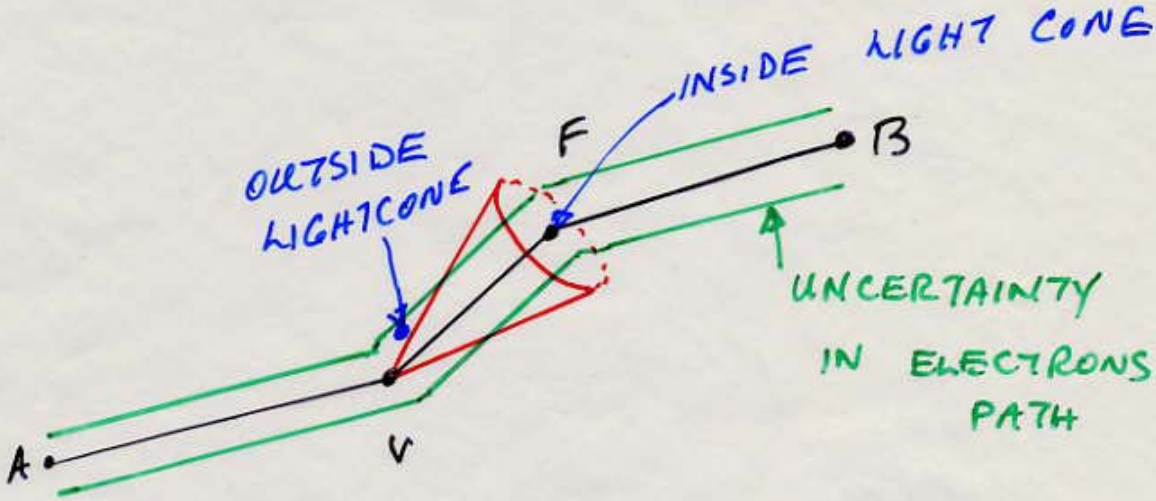
- ONLY EVENTS INSIDE LIGHTCONE CAN BE CAUSALLY CONNECTED

$$B \rightarrow A \rightarrow C$$

- X, A, Y CAN HAVE NO CAUSAL CONNECTION

  ↓ REQUIRES $v > c$

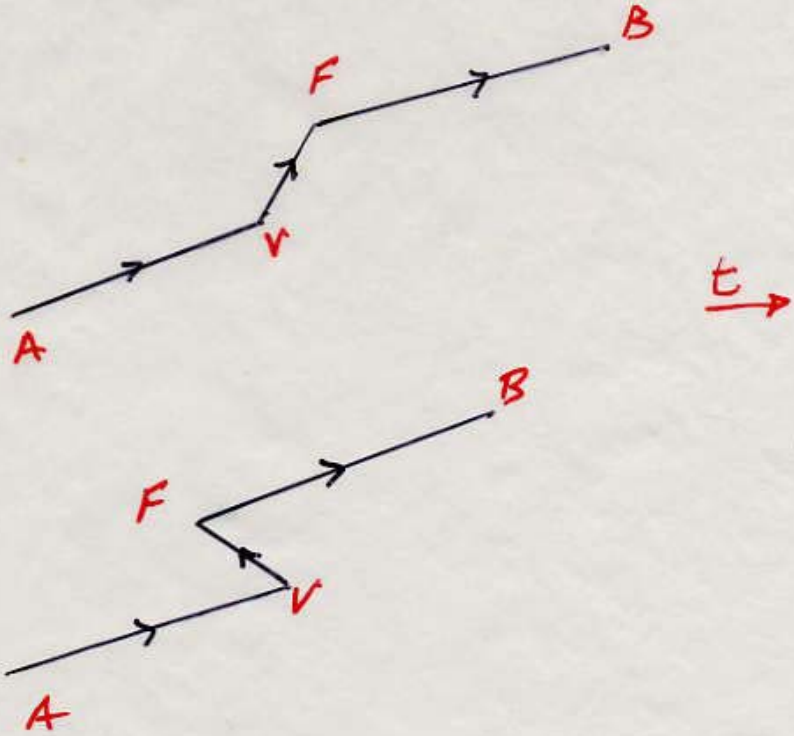




- CLASSICALLY ALL OBSERVERS SEE $V \rightarrow F$
- QM $\rightarrow$ CAN HAVE ELECTRON OUTSIDE LIGHT CONE

$\therefore$ THERE MUST BE OBSERVERS WHO SEE $F \rightarrow V$

DEPENDING ON OBSERVER



? PARTICLE WITH -VE ENERGY RUNNIG BACK IN TIME ?

UMM....

? 2 PARTICLES APPEAR AT F

- ONE GOES TO B
- ONE MEETS 'A' PARTICLE AT V

→ THEY MUTUALLY ANNIHILATE

→ PROPERTIES OF ANTIPARTICLES OBVIOUS

- SPECIAL RELATIVITY

+

QUANTUM MECHANICS

↓

ANTIPARTICLES

+

TOTAL NUMBER OF PARTICLES CAN CHANGE IN AN INTERACTION

ANTIPARTICLES

- SAME MASS, SPIN, LIFETIME
  OPPOSITE CHARGE, COLOUR, FLAVOUR

- PROFOUNDLY CONNECTED WITH LORENTZ INVARIANCE
  (SEE FORCE OF SYMMETRY!)

$$E^2 = p^2 + m^2 \qquad c = 1$$

$$E = \pm\sqrt{p^2 + m^2}$$

- MEANINGLESS FOR CLASSICAL PARTICLE

- QM → FREE PARTICLES → PLANE WAVE

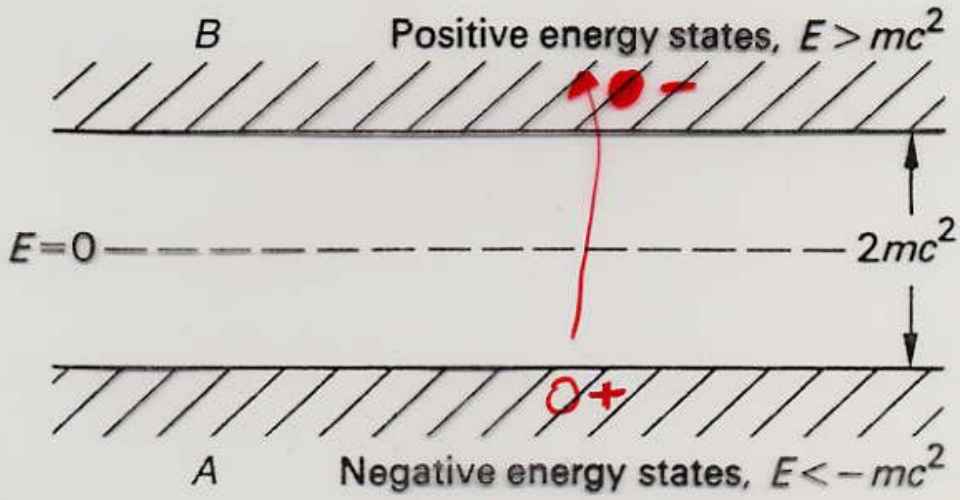
$$\psi = A e^{-i(Et - px)} \qquad \hbar = 1$$

$t \to +ve$; PHASE INCREASE FOR x +ve
$t \to -ve$; " . x -ve

$$Et \to (-E)(-t) \qquad px \to (-p)(-x)$$

$E > 0$ $\qquad$ $E < 0$

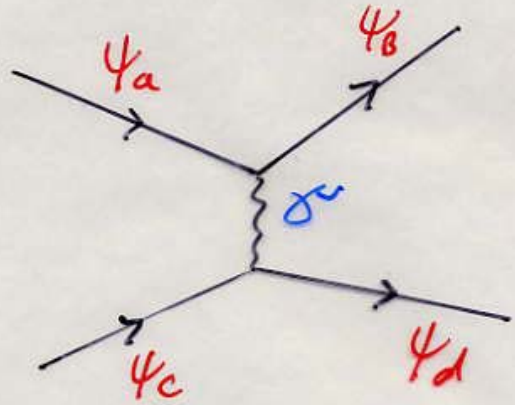
$t_1 < t_2$ $\qquad$ $t_1 < t_2$



$E<0$

- −VE ELECTRONS FLOWING BACK IN TIME ≡ +VE FLOWING FORWARD WITH $E>0$
- EQUAL & OPPOSITE CHARGES
- DIRAC "HOLE IN SEA"
  −VE SEA FULL FROM PAULI
- NON REL QM $\psi \rightarrow$ SINGLE PARTICLE
- FIELD THEORY $\rightarrow$ ψ CREATES OR DESTROYS PARTICLES

FIELD THEORY



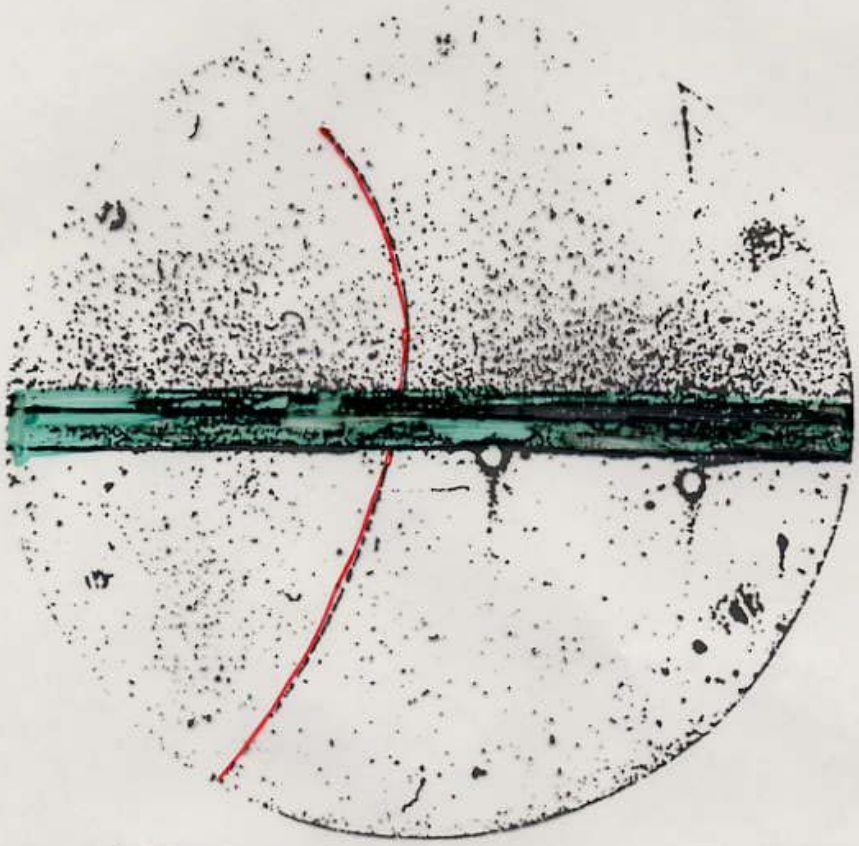
$$\text{AMP} \sim \bar{\psi}_B \psi_A \frac{1}{q^2} \bar{\psi}_d \psi_c$$

CREATE B

ANNIHILATE A

PROPAGATE γ^μ

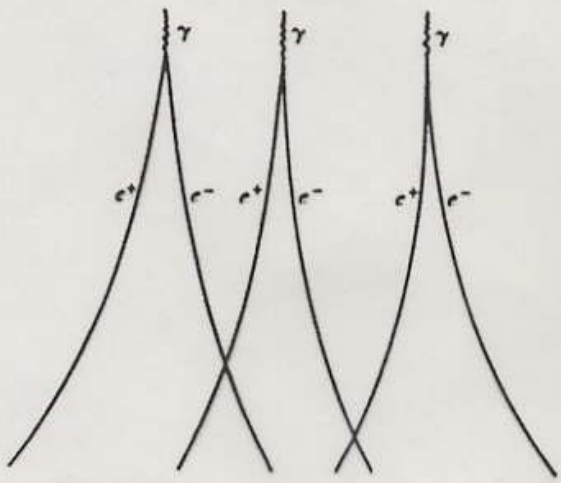
- ABOVE IS VERY SCHEMATIC!



. A 63 million volt positron ($H\rho = 2.1 \times 10^4$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

10X POSSIBLE PROTON PATH LENGTH

Tracks of electron-positron pairs produced by 300-MeV synchrotron x rays. (*Courtes Lawrence Radiation Laborat University of California, Berkeley.*)



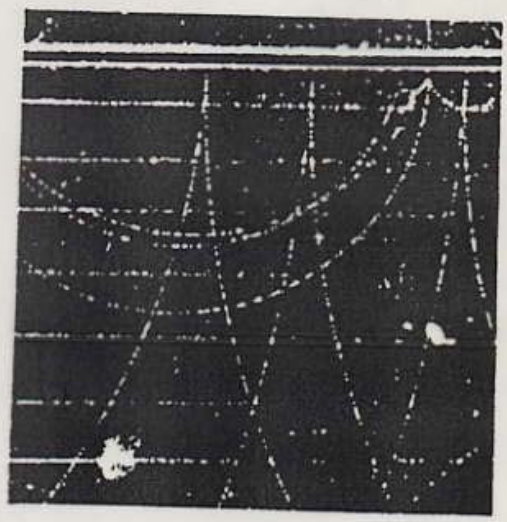


Fig. 3.

Reprinted by permission from *Phys. Rev.* **43** (1933) 491, © 1933 The American Pl Society.

RELATIVISTIC WAVE EQUATION

SCHRÖDINGER – NON RELATIVISTIC

START FROM NON-RELATIVISTIC ENERGY MOMENTUM

$$E = \frac{p^2}{2m} - V$$

QUANTIZE:

$$p \rightarrow -i\hbar \nabla$$

$$E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - V\psi$$

$$E \rightarrow i\hbar\frac{\partial}{\partial t}$$

$$i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\psi = -V\psi$$

$= 0$ FOR FREE PARTICLE

CAN GO THRU SAME PROCESS FOR RELATIVISTIC WAVE EQUATION.

KLEIN - GORDON - RELATIVISTIC

START FROM ENERGY, MOMENTUM OF A FREE PARTICLE

$$E^2 = p^2 + m^2$$

$$E \to i\hbar \frac{\partial}{\partial t} \qquad p \to -i\hbar \nabla$$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -\hbar^2 \nabla^2 \psi + m^2$$

$$\frac{\partial^2 \psi}{\partial t^2} = \nabla^2 \psi - m^2 \psi \qquad \hbar \to 1$$

$$\frac{\partial^2 \psi}{\partial t^2} = (\nabla^2 - m^2)\psi$$

IN 4-VECTOR NOTATION

$$p^\mu p_\mu = m^2$$

$$p^\mu \to i\partial^\mu$$

$$\partial^\mu = \left(\frac{\partial}{\partial t}, -\bar{\nabla}\right)$$

$$\partial_\mu = \left(\frac{\partial}{\partial t}, \bar{\nabla}\right)$$

$$(\partial^\mu \partial_\mu + m^2)\psi = 0$$

$= -V\psi$ IF POTENTIAL

OK FOR SPINLESS PARTICLE

WHY?

COVARIANT

$$\frac{\partial^2 \psi}{\partial t^2} = (\nabla^2 - m^2)\psi \quad (1)$$

BEFORE ANTIPARTICLES

- $E = \pm\sqrt{p^2+m^2}$ SEEN AS BOTHERSOME

DIRAC LOOKED FOR EQUATION
1ST ORDER IN SPACE-TIME

FOR A MASSLESS PARTICLE, (1)

$$\frac{\partial^2 \psi}{\partial t^2} = \nabla^2 \psi$$

IF YOU "TAKE $\sqrt{\ }$"

SOME PARAMETER

$$\frac{\partial \psi}{\partial t} = \pm \bar{\sigma} \nabla \psi$$

$$\frac{\partial \psi}{\partial t} = \pm\left[\sigma_1 \frac{\partial \psi}{\partial x} + \sigma_2 \frac{\partial \psi}{\partial y} + \sigma_3 \frac{\partial \psi}{\partial z}\right] \quad (2)$$

(1) IS JUST ENERGY-MOMENTUM
SO MUST STILL BE SATISFIED

↳ SQUARE (2)

$$\frac{\partial^2 \psi}{\partial t^2} = \underbrace{\sigma_1^2 \frac{\partial^2 \psi}{\partial x^2} + \sigma_2^2 \frac{\partial^2 \psi}{\partial y^2} + \sigma_3^2 \frac{\partial^2 \psi}{\partial z^2}}_{\rightarrow \nabla^2 \text{ IF } \sigma_1^2 + \sigma_2^2 + \sigma_3^2 = 1}$$

$$\nabla^2 \psi$$

$$\left[\frac{\partial \psi}{\partial x}, \frac{\partial \psi}{\partial y}\right] = 0$$

$$+ \sigma_1 \sigma_2 \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y} + \sigma_2 \sigma_1 \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial y}$$

$$+ \sigma_1 \sigma_3 \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial z} + \sigma_3 \sigma_1 \frac{\partial \psi}{\partial x} \frac{\partial \psi}{\partial z}$$

$$+ \sigma_2 \sigma_3 \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial z} + \sigma_3 \sigma_2 \frac{\partial \psi}{\partial y} \frac{\partial \psi}{\partial z}$$

ALL THESE MUST BE 0

$$\left.\begin{aligned} \sigma_1 \sigma_2 + \sigma_2 \sigma_1 &= 0 \\ \sigma_1 \sigma_3 + \sigma_3 \sigma_1 &= 0 \\ \sigma_2 \sigma_3 + \sigma_3 \sigma_2 &= 0 \end{aligned}\right\} \text{CANNOT BE NUMBERS}$$

$\downarrow$

MATRICES

TRUE FOR EITHER
+VE OR −VE SOLUTIONS

- σ CANNOT BE NUMBER

$$\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = 1$$

$$\sigma_1\sigma_2 + \sigma_2\sigma_1 = 0$$

DEFINES THEIR PROPERTIES

$$[\sigma_i, \sigma_j] = i\varepsilon_{jkl}\sigma_l$$

→ ALGEBRA → LOOKS LIKE ANGULAR MOMENTUM

σ → PAULI SPIN MATRICES

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

- NOW WE HAVE A NEW OPERATON INVOLVED WHICH

"ACTS ON SPIN LABEL"

good!

NOW WRITE

$$\frac{\partial \psi}{\partial t} = \pm \bar{\sigma} \cdot \frac{\partial \psi}{\partial \bar{r}}$$

AS:

$$\overset{i\frac{\partial}{\partial t}}{E} \chi = - \bar{\sigma} \cdot \overset{-i\frac{\partial}{\partial \bar{r}}}{\bar{p}} \chi$$

$$E \phi = + \bar{\sigma} \cdot \bar{p} \phi$$

$E \rightarrow$ ENERGY OPERATOR

$\bar{p} \rightarrow$ MOMENTUM OPERATOR

$\bar{\sigma} \rightarrow$ INSTRINSIC ANGULAR MOMENTUM (SPIN) OPERATOR

$$\psi(\bar{p}, t) \rightarrow \begin{bmatrix} \psi_1(\bar{p}, t) \\ \psi_2(\bar{p}, t) \end{bmatrix}$$

EXTRA LABLE 1, 2

$\rightarrow$ TWO COMPONENT SPINOR

$$\chi \rightarrow \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}$$ +VE SOLUTION

$$\phi \rightarrow \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$ -VE SOLUTION

$$E\chi = -\bar{\sigma}\cdot\bar{p}\,\chi$$

$$E\phi = +\bar{\sigma}\cdot\bar{p}\,\phi$$

- WEYL EQUATIONS
- SPINLESS FERMIONS
- 4 SOLUTIONS
  - PARTICLE → 2 SPIN STATES
  - ANTIPARTICLE — 2 SPIN STATES
- MASSIVE FERMIONS

DIRAC EQUATION

$$E\psi = (\underline{\alpha}\,\bar{p} + \underline{\beta}\,m)\psi$$

$$E\psi = (\alpha \bar{p} + \beta m)\psi$$

$$\alpha = \begin{pmatrix} 0 & \bar{\sigma} \\ \bar{\sigma} & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\beta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$\psi \rightarrow$ SPINOR, 4 COMPONENTS

COVARIANT FORM:

$$\left(i\gamma_\mu \frac{\partial}{\partial x_\mu} - m\right)\psi = 0$$

$$\gamma_k = \begin{pmatrix} 0 & \sigma_k \\ -\sigma_k & \end{pmatrix}$$

$$\left(i\gamma^\mu \partial_\mu - m\right)\psi = 0$$

$k = 1, 2, 3$

$$\gamma_4 = \beta$$

$$cf\left(\partial^\mu \partial_\mu + m^2\right)\phi = 0$$

SPINLESS

OPERATORS ACT ON INDICES

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi = 0$$

$E, \bar{p}$

SPIN

$$\gamma^{\mu} \equiv (\beta, \beta\bar{\alpha})$$

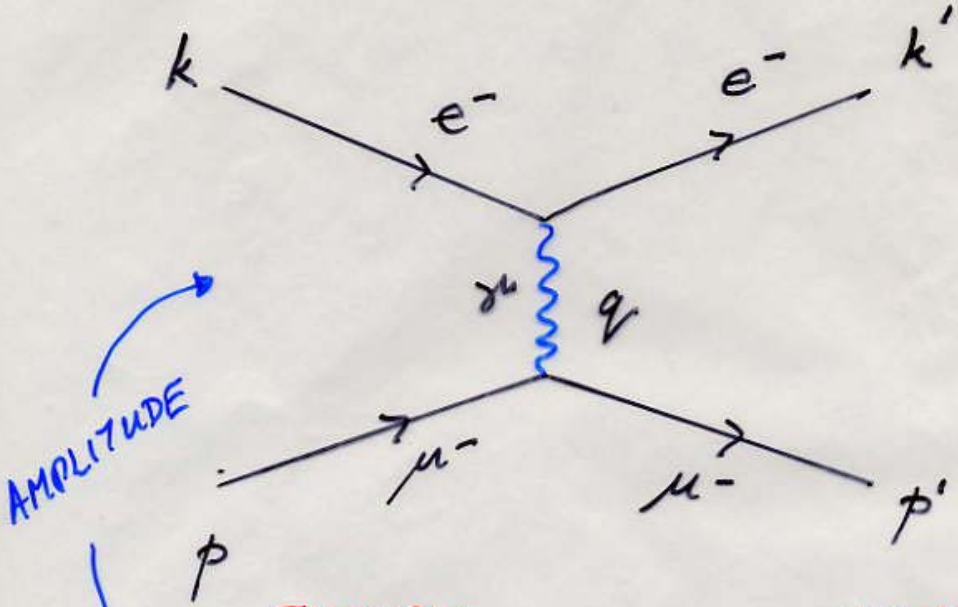
DIRAC γ^{μ} MATRICES

γ^{μ} – 4 INDICES – LIKE 4 VECTOR

EM → VECTOR INTERACTION

$$\left(\partial^{\mu}\partial_{\mu} + m^2\right)\phi = 0$$

SCALAR → SPINLESS PARTICLES – SCALAR INTERACTION



AMPLITUDE

ELECTRIC CHARGE

VECTOR INTERACTION

$$-i\mathcal{M} = -i e^2 \bar{u}(k') i \gamma^\mu u(k)$$

ANNIHILATE INCOMING e^-
CREATE OUTGOING

$$\times \quad g^{\mu\nu}/q^2$$

PHOTON 4-MOMENTUM

$$\times \quad \bar{u}(p') i \gamma_\nu u(p)$$

ANNIHILATE INCOMING μ
CREATE OUTGOING

$$\begin{pmatrix} \text{PROB OF} \\ \text{INTERACTION} \end{pmatrix} \propto (-iM)^2$$

HELICITY

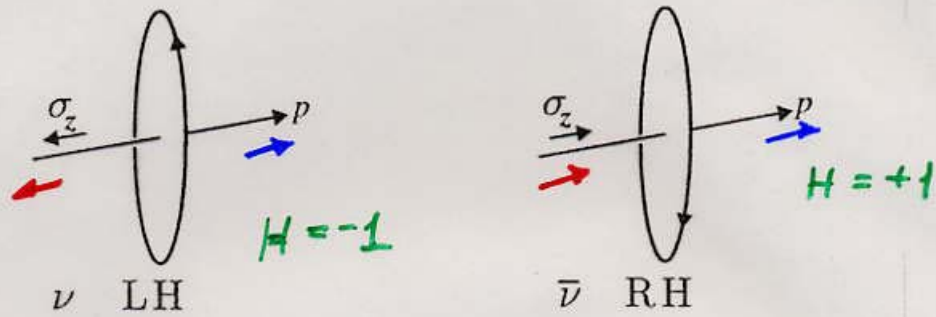


Fig. 1.5. A neutrino has LH polarisation, while an antineutrino is RH.

- MASSLESS, +VE E FERMION

$$E = |\bar{p}|$$

$$E\chi = -\bar{\sigma}\cdot\bar{p}\,\chi \rightarrow \frac{\bar{\sigma}\cdot\bar{p}}{|\bar{p}|}\chi = -\chi$$

$$H = \frac{\bar{\sigma}\cdot\bar{p}}{|\bar{p}|} = -1 \rightarrow \text{HELICITY}$$

↳ OPERATOR MEASURES SPIN COMPONENT IN DIRECTION OF MOTION

$$j_z = \pm\tfrac{1}{2}\hbar$$

$$E\chi = -\bar{\sigma}\cdot\bar{p}\,\chi \quad \cdot \; (1)$$

$\chi \rightarrow$ L.H. +ve E PARTICLE

χ ALSO $\quad -E \quad -\bar{p}$

i.e. $\quad -E\chi = -\bar{\sigma}\cdot(-\bar{p})\,\chi$

$$E\chi = \bar{\sigma}\cdot\bar{p}\,\chi$$

$$\therefore \quad H = +1$$

(1) REPRESENTS EITHER

LH PARTICLE

RH ANTIPARTICLE

$E\phi = +\bar{\sigma}\cdot\bar{p}\,\phi$ REPRESENTS

RH PARTICLE

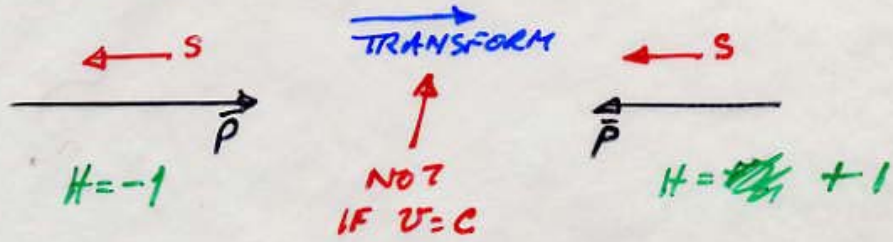
LH ANTIPARTICLE

LORENTZ INVARIANT

- HELICITY WELL DEFINED FOR MASSLESS PARTICLE

- MASSLESS PARTICLE $\rightarrow$ $v = c$

  $\therefore$ CAN NEVER TRANSFORM INTO FRAME WHERE HELICITY CHANGES SIGN



- WEYL EQUATIONS DESCRIBE MASSLESS PARTICLES OF DEFINITE HELICITY

- DIRAC EQUATIONS DESCRIBE MASSIVE PARTICLES MIXED HELICITY

DIRAC $\quad$ RELATIVISTIC LIMIT $\rightarrow$ WEYL

$e = e_R + e_L \qquad m \approx 0 \qquad \nu_L, \bar{\nu}_R$

MASS HIERARCHY — WHY?

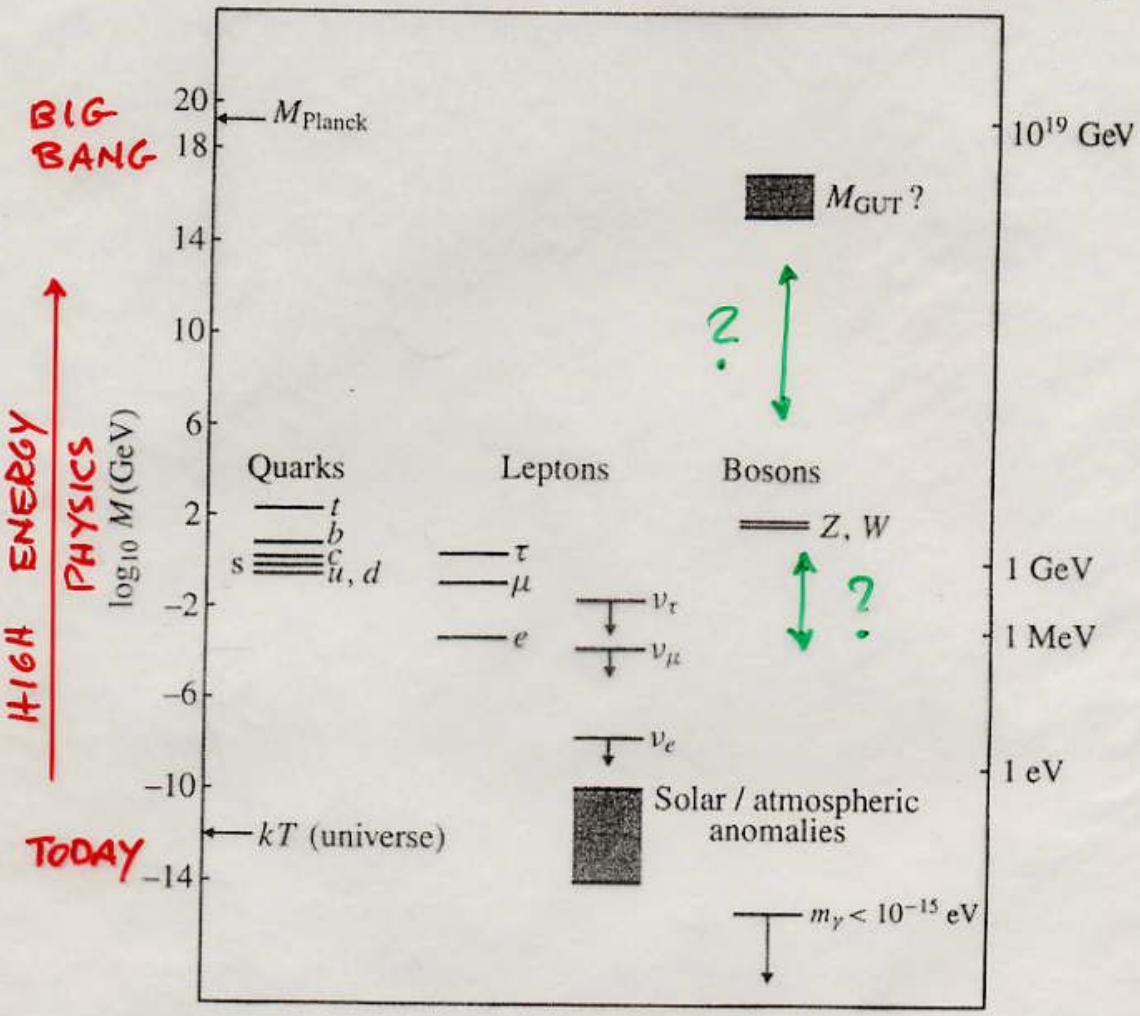


Fig. 1.7. The mass spectrum of leptons and quarks. The values shown for neutrinos are upper limits from direct measurements, and the solar and atmospheric neutrino anomalies (see Chapter 9) suggest even smaller masses. Other important mass scales are also shown: the Fermi or electroweak scale at 100 GeV, typified by the $W^{\pm}$ and Z^0 boson masses; the Planck mass scale, of order 10^{19} GeV, at which gravitational interactions are expected to become strong (see Chapter 2): and the value, $kT \simeq 1$ meV, of the cosmic microwave radiation ($T = 2.7$ K) in the universe today.

$$M_P = \left(\frac{\hbar c}{G_N}\right)^{1/2} = 1.22 \times 10^{19}\ \text{GeV}$$