CONTINUOUS TRANSFORMATIONS.

ANOTHER EXAMPLE WOULD BE ROTATIONS.

CAN MAKE A ROTATION ABOUT AN AXIS ARBITRARILY SMALL — ARBITRARILY CLOSE TO "DO NOTHING"

SINCE CAN BUILD UP A

FINITE CONTINUOUS TRANSFORMATION FROM $\sum$ INFINITESIMAL CONTINUOUS TRANSFORMATIO

IT IS SUFFICIENT TO STUDY THE SYMMETRY PROPERTIES OF A HAMILTONIAN UNDER

INFINITESIMAL TRANSFORMATIONS

→ USUALLY MORE TRACTABLE MATHEMATICALLY

UNITARY TRANSFORMATIONS → SYMMETRY

A MATHEMATICAL REPRESENTATION OF A UNITARY TRANSFORMATION ; $U^{\dagger}U=1$ IS

$$U = e^{i\varepsilon F}$$

REAL PARAMETER

GENERATOR OF TRANSFORMATION

IF U IS A UNITARY OPERATOR

F GENERATES A GROUP OF TRANSFORMATIONS

group MEMBER

$$g_1 \cdot (g_2 \cdot g_3) = (g_1 \cdot g_2) \cdot g_3 \in G$$

$$g \cdot I = g = I \cdot g$$

$$g \cdot g^{-1} = I = g^{-1} \cdot g$$

IF g_i AND $g_j \in G$

THEN $g_i \cdot g_j \in G$

CLOSURE AND MULTIPLICATION

GROUPS ARE ABSTRACT ENTITIES BUT CAN CORRESPOND TO PHYSICAL SYMMETRIES

SO(1) → GROUP OF ROTATIONS ABOUT X-AXIS

→ INVARIANCE LEADS TO ANGULAR MOMENTUM CONSERVATION

SU(2) → GROUP OF ALL 2×2 MATRICES WITH UNIT DETERMINANT

→ INVARIANCE OF STRONG FORCE → ISOSPIN CONSERVE

SU(3) → 3×3

INVARIANCE PROPERTY OF FORCE BETWEEN QUARKS

∴ BY ASSUMING

$\mathcal{U}$ IS UNITARY TRANSFORMATION

HAVE SHOWN THAT

GENERATOR F IS HERMITIAN

∴ OBSERVABLE

NOW LOOK AT CASE WHERE

$\mathcal{U}$ IS UNITARY TRANSFORMATION

ALSO IS A SYMMETRY OF THE HAMILTONIAN

A SYMMETRY OF THE HAMILTONIAN MEANS THAT THE TRANSFORMATION CORRESPONDING TO $\mathcal{U}$ LEAVES THE HAMILTONIAN UNCHANGED

SYMMETRY $\longrightarrow$ $[H, \mathcal{U}] = 0$

SYMMETRY $\rightarrow$ $[H, U] = 0$

NOW WE CAN USE THE TECHNIQUE OF AN INFINITESIMAL TRANSFORMATION

$$\mathcal{U} = e^{i\varepsilon F} \rightarrow \mathcal{U} = 1 + i\varepsilon F$$

for $\varepsilon F \ll 1$

$$\begin{aligned}
[H, U] &= HU - UH \\
&= H(1 + i\varepsilon F) - (1 + i\varepsilon F)H \\
&= H + i\varepsilon HF - H - i\varepsilon FH \\
&= i\varepsilon HF - i\varepsilon FH \\
&= i\varepsilon [H, F]
\end{aligned}$$

IF $[H, U] = 0$; THEN $[H, F] = 0$

HAVE SHOWN THAT

IF $\mathcal{U}$ IS A UNITARY TRANSFORM THEN THE GENERATOR OF THE GROUP OF TRANSFORMATIONS F IS AN OBSERVABLE

FURTHERMORE

IF $\mathcal{U}$ IS A SYMMETRY OF THE HAMILTONIAN $([H, \mathcal{U}] = 0)$

THEN F IS A CONSERVED QUANTITY

$$\begin{tikzcd} \text{SYMMETRY} \\ \mathcal{U} \arrow[r] & \text{CONSERVED OBSERVABLE} \\ & \langle F \rangle \arrow[l, bend left, "\text{GENERATOR}"] \end{tikzcd}$$

TRANSLATION INVARIANCE $\longrightarrow$ LINEAR MOMENTUM

ROTATIONAL INVARIANCE $\longrightarrow$ ANGULAR MOMENTUM

TRANSFORMATIONS (SYMMETRY)	CONSERVED QUANTITY (QUANTUM NO)
SPACE TRANSLATION	MOMENTUM
TIME TRANSLATION	ENERGY
SPATIAL ROTATION	ANGULAR MOMENTUM
ROTATION IN ISO-SPACE	ISOTOPIC SPIN
?	FLAVOUR (LEPTON)
	FLAVOUR (QUARK)
	BARYON NUMBER
	COLOUR CHARGE
	WEAK CHARGE
GAUGE INVARIANCE	ELECTRIC CHARGE

IN GENERAL THE HAMILTONIAN DESCRIBING PARTICLE INTERACTIONS WILL BE

$$\mathcal{H} = \mathcal{H}_{STRONG} + \mathcal{H}_{EM} + \mathcal{H}_{WEAK} + \mathcal{H}_{GRAVITY}$$

CORRESPONDING TO THE FOUR FORCES.

EACH TERM IN THE HAMILTONIAN WILL BE INVARIANT UNDER DIFFERENT TRANSFORM

$\therefore$ CONSERVES DIFFERENT QUANTUM NUMBERS

$$\left| \begin{matrix} \text{INITIAL} \\ \text{STATE} \end{matrix} \right\rangle \longrightarrow \left| \begin{matrix} \text{FINAL} \\ \text{STATE} \end{matrix} \right\rangle$$

THE QUANTUM NUMBERS OF THE INITIAL AND FINAL STATES GOVERN WHICH FORCE CAN CAUSE THE TRANSITION

$\Rightarrow$ IN SECOND LECTURE WE DISCUSSED WHICH QUANTUM NUMBERS THE DIFFERENT INTERACTIONS CONSERVE

CONSERVATION OF CHARGE

CONSERVED QUANTITIES ARE DISCOVERED BY EXPERIMENT

DISCOVERING ALL THE CONSERVED QUANTITIES $\rightarrow$ DEFINES SYMMETRIES OF HAMILTONIAN

KNOWING SYMMETRIES OF HAMILTONIAN $\rightarrow$ DEFINES ITS FORM

EXAMPLES :

WHY DOES $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$

$\mu^- \not\rightarrow e^- \gamma$

CONSERVATION OF LEPTON FLAVOUR e, μ IMPLIES A SYMMETRY OF THE HAMILTONIAN (WEAK FORCE)

$n \rightarrow p\, e^- \bar{\nu}_e$

$n \not\rightarrow \pi^+ e^- \bar{\nu}_e$

CONSERVATION OF NUCLEON (BARYON) NUMBER, IMPLIES A SYMMETRY OF HAMILTONIAN OF STRONG FORCE

$$[\mathcal{H}_{STRONG}, B] = 0$$

UNITARY TRANSFORM

ACTUALLY ACCIDENT ?

IN FACT THERE IS AN EXAMPLE SO FAMILIAR THAT WE USE IT UNCONSCIOUSLY

THE DECAY

$$e^- \to \nu\gamma$$

DOES NOT HAPPEN $(\tau > 2\times10^{21}\,y)$

QUITE APART FROM LEPTON FLAVOUR THIS IS RULED OUT BY

CONSERVATION OF ELECTRIC CHARGE

ELECTRIC CHARGE IS A CONSERVED QUANTUM NUMBER

FROM OUR FOREGOING DISCUSSION, WE KNOW THAT THE CONSERVATION OF ELECTRIC CHARGE MUST CORRESPOND TO SOME

SYMMETRY

i.e. TRANSFORMATION THAT LEAVES THE PHYSICS UNCHANGED.

ψ IS SOME STATE CORRESPONDING TO THE SCHRÖDINGER EQUATION

$$i\hbar \frac{d\psi}{dt} = H\psi$$

Q IS THE OPERATOR WHICH CORRESPONDS TO ELECTRIC CHARGE.

THE EXPECTATION VALUE $\langle Q \rangle$ IS CONSERVED IF:

$$[H, Q] = 0$$

AND ψ CAN BE AN EIGEN FUNCTION OF BOTH THE HAMILTONIAN, AND THE ELECTRIC CHARGE OPERATOR

$$Q\psi = q\psi$$

I MENTIONED BEFORE, IN PASSING, THAT PHYSICS RESULTS MUST BE INVARIANT TO CHANGES IN

PHASE OF WAVE FUNCTIONS

$$\psi \rightarrow e^{i\varepsilon Q}\psi$$

PHASE TRANSFORMATION

<u>GLOBAL</u> GAUGE TRANSFORM

INDEPENDENT OF $(\bar{x}, t)$

$$\psi' = e^{i\varepsilon Q}\psi$$

<u>NB</u> $\psi^*\psi = (\psi')^*\psi'$ ← PROBABILITIES UNCHANGED

GAUGE INVARIANCE MEANS THAT

$$i\hbar \frac{d\psi'}{dt} = H\psi'$$

JUST AS:

$$i\hbar \frac{d\psi}{dt} = H\psi$$

$$i\hbar \frac{d\psi'}{dt} = H\psi'$$

REWRITING IN TERMS FOR UNTRANSFORMED WAVE FUNCTION

$$i\hbar \frac{d}{dt} e^{i\varepsilon Q}\psi = H e^{i\varepsilon Q}\psi$$

FOR A HERMITIAN OPERATOR, $\mathcal{U}$.

$$\mathcal{U}^{+} H \mathcal{U} = H$$

$$i\hbar \frac{d}{dt}\left(e^{-i\varepsilon Q^{+}} e^{i\varepsilon Q}\right)\psi = e^{-i\varepsilon Q^{+}} H e^{i\varepsilon Q}\psi$$

SINCE WE ASSUME THAT Q IS AN OBSERVABLE, IT MUST BE HERMITIAN

$$Q = Q^{+}$$

$$i\hbar \frac{d}{dt}\psi = e^{-i\varepsilon Q} H e^{i\varepsilon Q} \cdot \psi$$

$$i\hbar \frac{d}{dt}\psi = e^{-i\varepsilon Q} H e^{i\varepsilon Q} \cdot \psi$$

IF WE COMPARE THIS TO

$$i\hbar \frac{d\psi}{dt}\psi = H\psi$$

THEN

$$e^{-i\varepsilon Q} H e^{i\varepsilon Q} = H$$

EXPAND THE EXPONENTIAL, AND JUST CONSIDER AN

INFINITESIMAL GAUGE TRANSFORM

$$\varepsilon Q \to 0$$

$$(1 - i\varepsilon Q) H (1 + i\varepsilon Q) = H$$

$$H + i\varepsilon HQ - i\varepsilon QH - \underbrace{\cancel{(i\varepsilon)^2}}_{\to 0} QHQ = H$$

$$[HQ - QH] = H - H = 0$$

$$[H, Q] = 0$$

THIS DEMONSTRATION THAT A

GLOBAL GAUGE INVARIANCE

LEADS TO

A CONSERVED CHARGE

IS EXTREMELY IMPORTANT.

IN FACT, PHYSICS MUST BE INVARIANT UNDER

ARBITRARY PHASE CHANGES AT DIFFERENT POINTS IN SPACE & TIME

$$\psi \rightarrow \psi e^{i\varepsilon(\bar{x},t)Q}$$

INVARIANCE UNDER SUCH TRANSFORMATIO LEADS TO THE INTERACTIONS WE KNOW AS:

FORCES

LOCAL GAUGE SYMMETRY

a la DAS & FERBEL

A LOCAL GAUGE SYMMETRY REQUIRES INVARIANCE UNDER THE TRANSFORMATION

$$\psi \rightarrow \psi e^{i\alpha(\bar{x},t)}$$

FOR ILLUSTRATIVE PURPOSES, LET'S JUST LOOK AT AN ARBITARY FUNCTION OF SPACE COORDINATES

CONSIDER THE SCHRÖDINGER EQUATION

$$H\psi(\bar{x}) = E\psi(\bar{x})$$

THE HAMILTONIAN IS JUST

TOTAL ENERGY = KINETIC + POTENTIAL

$$= \frac{P^2}{2m} + V(\bar{x})$$

P IS AN OPERATOR $-i\hbar\frac{\partial}{\partial x}$

OR IN 3-DIMENSIONS

$$\mathcal{H} = -\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\bar{x})$$

SINCE $\vec{\nabla}\left[e^{i\alpha(\bar{x})}\psi(\bar{x})\right] \neq e^{i\alpha(\bar{x})}\vec{\nabla}\psi(\bar{x})$

THE PHASE TRANSFORMATION EFFECTS THE RHS & LHS OF

$$\left[-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V(\bar{x})\right]\psi(\bar{x}) = E\psi(\bar{x})$$

DIFFERENTLY.

SO WE DO NOT HAVE INVARIANCE UNDER

$$\psi \rightarrow e^{i\alpha(\bar{x})}\psi$$

BUT IT MUST BE!

$$\psi\psi^* = e^{i\alpha(\bar{x})}\psi\left(e^{i\alpha(\bar{x})}\psi\right)^*$$

KNOWING THAT LOCAL GAUGE INVARIANCE MUST BE SATISFIED TELL US THAT THE

HAMILTONIAN IS

INCOMPLETE!

$$H\,\psi(\bar{x}) = E\,\psi(\bar{x})$$

$$H = \frac{-\hbar^2}{2m}\vec{\nabla}^2 + V(\bar{x})$$

THE SCHRÖDINGER EQUATIONS IS:

$$\left[\frac{-\hbar^2}{2m}\vec{\nabla}^2 + V(\bar{x})\right]\psi(\bar{x}) = E\psi(\bar{x})$$

IF $\psi(\bar{x})$ IS A SOLUTION, THEN

SO IS $e^{i\alpha(\bar{x})}\psi(\bar{x})$

CONSIDER TERM

$$\vec{\nabla}\left[e^{i\alpha(\bar{x})}\psi(\bar{x})\right]$$

$$= e^{i\alpha(\bar{x})}\left[i\,\vec{\nabla}\alpha(\bar{x})\psi(\bar{x}) + \vec{\nabla}\psi(\bar{x})\right]$$

AARGH!

$$\neq e^{i\alpha(\bar{x})}\vec{\nabla}\psi(\bar{x})$$

IF WE PUT:

$$\vec{\nabla} \to \vec{\nabla} - i\,\vec{A}(\bar{x})$$

WHERE $\vec{A}(\bar{x}) \to \vec{A}(\bar{x}) + \vec{\nabla}\alpha(\bar{x})$

UNDER THE LOCAL GAUGE TRANSFORMATION

$$\psi(\bar{x}) \to e^{i\alpha(\bar{x})}\psi(\bar{x}) \longrightarrow \text{DETAIL}$$

THEN HURRAH!

$$\left(\vec{\nabla} - i\,\vec{A}(\bar{x})\right)\psi(\bar{x}) \quad \text{UNDER SAME TRANSFORM}$$

$$\longrightarrow \left(\vec{\nabla} - i\vec{A}(\bar{x}) - i\vec{\nabla}\alpha(\bar{x})\right)\cdot e^{i\alpha(\bar{x})}\psi(\bar{x})$$

SINCE

$$\vec{\nabla}\left[e^{i\alpha(\bar{x})}\psi(\bar{x})\right] \to e^{i\alpha(\bar{x})}\left[i\vec{\nabla}\alpha(\bar{x})\psi(\bar{x}) + \vec{\nabla}\psi(\bar{x})\right]$$

SO UNDER

$$\psi \to e^{i\alpha(\bar{x})}\psi(\bar{x})$$

$$\left(\vec{\nabla} - i\,\vec{A}(\bar{x})\right)\psi(\bar{x}) \longrightarrow e^{i\alpha(\bar{x})}\left(\vec{\nabla} - i\vec{A}(\bar{x})\right)\psi(\bar{x})$$

$$\left.\begin{aligned} \psi &\to e^{i\alpha}\psi \\ \vec{\nabla} &\to \vec{\nabla} - i\vec{A}(\vec{x}) \\ \vec{A}(x) &\to \vec{A}(x) + \vec{\nabla}\alpha(\vec{x}) \end{aligned}\right\} \text{GAUGE XFORM}$$

A TERM $\nabla\psi(x)$

$$\to \left\{\vec{\nabla} - i\,\vec{A}(\vec{x})\right\}\cdot e^{i\alpha(\vec{x})}\psi(\vec{x})$$

$$\hookrightarrow \vec{A}(x) + \vec{\nabla}\alpha(\vec{x})$$

TRANSFORMED

$$\left(\vec{\nabla} - i\vec{A} - i\overset{\nabla\psi}{\nabla\alpha}\right)\cdot e^{i\alpha}\psi$$

$$= \nabla e^{i\alpha}\psi - iA e^{i\alpha}\psi - i\nabla\alpha\cdot e^{i\alpha}\psi$$

$$= \cancel{i\nabla\alpha e^{i\alpha}\psi} + e^{i\alpha}\nabla\psi - iA e^{i\alpha}\psi \cancel{- i\nabla e^{i\alpha}\psi}$$

$$= e^{i\alpha}(\vec{\nabla} - iA)\psi$$

BY REQUIRING INVARIANCE UNDER THE LOCAL GAUGE TRANSFORMATION, WE ARE FORCED TO REWRITE THE SCHRÖDINGER

$$\left\{ -\frac{\hbar^2}{2m} \left(\vec{\nabla} - i\vec{A}(\bar{x}) \right)^2 + V(\bar{x}) \right\} \psi(\bar{x}) = E\psi(\bar{x})$$

WE HAVE INTRODUCED A NEW "KINETIC ENERGY" TERM

LOCAL GAUGE INVARIANCE $\longrightarrow$ VECTOR POTENTIAL

$\downarrow$

E.M. FORCE

THE NEW FORCE (= PARTICLE IN RELATIVISTIC Q.M) IS PREDICTED BY THIS SYMMETRY

$$\psi \to \underbrace{e^{i\alpha(\bar{x})}}_{\text{GROUP } U(1)} \psi$$

$U(1) \to$ GAUGE GROUP OF ELECTROMAGNETIC INTERACTION

GAUGE THEORIES

- GLOBAL SYMMETRIES — CLASSIFICATION OF STATES ACCORDING TO A CONSERVED QUANTUM NUMBER

- LOCAL SYMMETRIES — GAUGE INVARIANCE IN E.M.
  $\Rightarrow \gamma$ AS GAUGE BOSON

GAUGE THEORIES BUILT FOR

COLOUR	—	$SU(3)$	NON ABELIAN
WEAK	—	$SU(2)$	NON ABELIAN
EM	—	$U(1)$	ABELIAN

- GENERALLY GAUGE INVARIANCE LEADS TO MASSLESS GAUGE BOSONS

U(1) LOCAL GAUGE INVARIANCE

PHYSICS INVARIANT UNDER

H & H 14.2

$$\psi(x) \longrightarrow e^{i\alpha(x)}\psi(x)$$

$\alpha(x)$ ARBITRARY FN (x,t) $\rightarrow$ LOCAL GAUGE INVARIANCE

DIRAC $\rightarrow$ $\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi - m\bar{\psi}\psi$ FREE

NOT GAUGE INVARIANT

$$m\bar{\psi}\psi \rightarrow m\, e^{-i\alpha(x)}\bar{\psi}\cdot e^{i\alpha(x)}\psi \quad \checkmark$$

BUT

$$\partial_\mu\psi \rightarrow e^{i\alpha(x)}\partial_\mu\psi + ie^{i\alpha(x)}\psi\,\underline{\partial_\mu\alpha}$$

$\times$

MUST DEFINE DERIVATIVE WHICH MAINTAINS INVARIANCE

$$D_\mu\psi \rightarrow e^{i\alpha(x)}D_\mu\psi$$

TRANSFORM $\rightarrow$

$$D_\mu = \partial_\mu - ieA_\mu$$

CONSTRUCT

$$A_\mu \rightarrow A_\mu + \frac{1}{e}\partial_\mu\alpha$$

GAUGE INVARIANT LAGRANGIAN. $\partial_\mu \rightarrow D_\mu$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu\psi - m\bar{\psi}\psi$$

$$= \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu$$

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu \psi - m\psi\bar{\psi}$$

$$= \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu \psi A_\mu$$

TRANSFORM

$$\bar{\psi}' i\gamma^\mu \partial_\mu \psi' \underbrace{- m\bar{\psi}'\psi'}_{\text{OK}} + e\bar{\psi}'\gamma^\mu \psi' \left(A_\mu + \frac{1}{e}\partial_\mu \cdot \alpha\right)$$

$$= i\bar{\psi}'\gamma^\mu \partial_\mu \psi' + i\,i\,\bar{\psi}'\psi\,\partial_\mu \cdot \alpha + e\bar{\psi}'\gamma^\mu \psi' \cdot A_\mu$$

$$+ \bar{\psi}'\gamma^\mu \partial_\mu \cdot \alpha\, \psi$$

CANCEL

$$= \bar{\psi}'(i\gamma^\mu \partial_\mu - m)\psi' + e\bar{\psi}'\gamma^\mu \psi' A_\mu$$

cf. $p^\mu \to p^\mu + eA^\mu$

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi A_\mu$$

GAUGE INVARIANCE GENERATES NEW VECTOR GAUGE FIELD A_μ

$$e\bar{\psi}\gamma^\mu\psi A_\mu \equiv -j^\mu A_\mu$$

INTERACTION OF EM CURRENT WITH FIELD

- MUST ADD KINETIC ENERGY TERM
  - GAUGE INVARIANT → INVOLVES A_μ

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$ GAUGE INVARIANT

$$\mathcal{L} = \underbrace{\psi(i\gamma^\mu\partial_\mu - m)\psi}_{\text{FREE}} + \underbrace{e\bar{\psi}\gamma^\mu A_\mu\psi}_{\text{INTERACT}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{KINETIC}}$$

- MASS TERM $\frac{1}{2}m^2 A_\mu A^\mu$ → BREAKS GAUGE INVARIANCE

∴ GAUGE BOSON MUST BE MASSLESS

⇒ VERY IMPORTANT

A SHORT DIGRESSION

- MAXWELL'S EQUATIONS ARE:

$$\bar{\nabla}\cdot\bar{E} = \rho \qquad \bar{\nabla}\times\bar{E} + \frac{\partial \bar{B}}{\partial t} = 0$$

$$\bar{\nabla}\cdot\bar{B} = 0 \qquad \bar{\nabla}\times\bar{B} - \frac{\partial E}{\partial t} = \bar{J}$$

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q} = 0 \qquad \mathcal{L} = T - V$$

- CAN WRITE THESE AS

$$\Box^2 A^\mu - \partial^\mu(\partial_\nu A^\nu) = j^\mu$$

$$j^\mu = (\rho, \bar{J}) \qquad A^\mu = (\phi, \bar{A})$$

- EM FIELD TENSOR IS $F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$

THEN MAXWELL IS

$$\partial_\mu F^{\mu\nu} = j^\mu$$

- THE LAGRANGIAN $\mathcal{L} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} - j^\mu A_\mu$

PROPAGATING EM FIELD

INTERACTION OF CURRENT WITH EM FIELD

EULER - LAGRANGE GIVES

$$\partial_\mu F^{\mu\nu} = j^\nu \longleftarrow \text{MAXWELL}$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - j^{\mu} A_{\mu} + \frac{1}{2} m A_{\mu} A^{\mu}$$

PROPAGATION OF MASSIVE FIELD

$$\left(\Box^2 + m^2\right) A^{\mu} = j^{\mu}$$

EQUATION OF MOTION OF MASSIVE γ^{μ}

- QUADRATIC TERMS IN $\mathcal{L}$ ARE PROPAGATORS

$$\frac{1}{2}(\partial_{\mu}\phi)^2$$

$$-\frac{1}{2} m^2 \phi^2$$

$$\bar{\psi}(i\gamma_{\mu}\partial^{\mu} - m)\psi$$

- OTHER TERMS ARE INTERACTION VERTICES

- FOR GAUGE INVARIANCE ON DIRAC

$$\mathcal{L} = \psi(i\gamma_{\mu}\partial^{\mu} - m)\psi + e\bar{\psi}\gamma^{\mu}A_{\mu}\psi$$

NO TERM TO DESCRIBE γ^{μ} PROPAGATION

$$\mathcal{L} = \psi(i\gamma^\mu \partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu A_\mu \psi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

→ JUST "MAXWELL" PROPAGATION OF MASSLESS EM FIELD.

- WHY NOT $\frac{1}{2}m^2 A_\mu A^\mu$
  - ↳ NOT GAUGE INVARIANT
    - $m \Rightarrow 0$!

- GAUGE INVARIANCE
  - → LOCAL PHASE VARIATION
    - → MUST NOT OBSERVE
      - → PHOTON FIELD COMPENSATES

ELECTROWEAK UNIFICATION

- QED IS A RENORMALIZABLE GAUGE THEORY

$$\psi_x \to \psi'(x) = e^{-i\alpha}\psi(x)$$

U(1) INVARIANCE

CHOOSE →

PHASE ARBITRARILY AT DIFFERENT SPACE TIME POINTS

IN

$$\mathcal{L} = \bar{\psi}(x)\left(\underbrace{i\gamma^\mu\partial_\mu}_{\text{THIS PART CHANGES UNDER TRANSFORMATION}} - m\right)\psi(x)$$

→

$$D_\mu\psi(x) = (\partial_\mu + ieA_\mu)\psi(x)$$

HOW ψ VARIES IN SPACE

HOW PHASE VARIES

↓

MASSLESS PHOTON

REPRIS ON GAUGE THEORIES

a la Dodd

QED IS GAUGE INVARIANT ($\rightarrow$ CHARGE CONSERVATION)

$$G\,\mathcal{L}(\psi_e) \rightarrow \mathcal{L}(\psi_e^*)$$

SYMMETRY GROUP

LAGRANGIAN

$\hookrightarrow$ SYMMETRY GROUP IS $U(1)$

$\Rightarrow$ PHASE TRANSFORMATION (GLOBAL)

IF APPLY LOCAL PHASE TRANSFORMATION WHICH DEPENDS ON SPACE <u>AND</u> TIME

$$G(x,t)\,\mathcal{L}(\psi_e) \rightarrow \mathcal{L}^*(\psi_e^*)$$

$$\mathcal{L}(\psi_e) \neq \mathcal{L}^*(\psi_e^*)$$

UNLESS WE INTRODUCE A FIELD CORRESPONDING TO THE PHOTON

$$G(x,t)\,\mathcal{L}(\psi_e, A) \rightarrow \mathcal{L}(\psi_e^*, A^*)$$

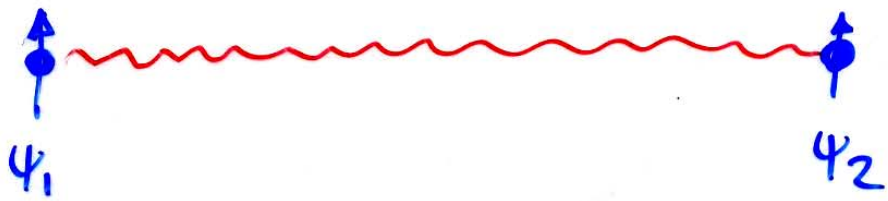
ψ DESCRIBES PROPAGATION OF ELECTRON

$\mathcal{L}(\psi_1 \psi_2)$ DESCRIBES INTERACTION OF TWO ELECTRONS

GAUGE INVARIANCE DEMANDS ψ, $\mathcal{L}$ ARE INVARIANT UNDER REDEFINITION OF PHASE INDEPENDANTLY AT ANY POINT IN SPACE TIME

$$G(\bar{x})\mathcal{L}(\psi_1 \psi_2 A) \rightarrow \mathcal{L}(\psi_1^* \psi_2^* A^*)$$

THIS REQUIRES A GAUGE BOSON WHICH COMMUNICATES LOCAL PHASE CONVENTION BETWEEN ALL POINTS IN SPACE TIME



THIS IS WHAT WE NORMALLY CALL THE ELECTROMAGNETIC FORCE

charge conservation,	$\tau(n \to p\nu_e\bar{\nu}_e) > 10^{18}$ yr
lepton conservation,	$\tau(^{76}\mathrm{Ge} \to {}^{76}\mathrm{Se} + 0\nu + e^- + e^-) > 10^{26}$ yr
baryon conservation,	$\tau(p \to e^+\pi^0) > 10^{33}$ yr

cf

- IS BARYON NUMBER CONSERVED ?

IF $[H, B] = 0$

$B \rightarrow \left\{\begin{matrix}\text{LONG RANGE} \\ \text{FORCE}\end{matrix}\right\} \left\{\begin{matrix}\text{CONSERVED} \\ \text{CHARGE}\end{matrix}\right\}$ cf EM.

BARYON NO.

"5TH FORCE"

TAKE SAME MASS ^{27}Al

^{195}Pt

GRAVITATION SAME

BARYON NO. / MASS DIFFERENT

"5TH FORCE" DIFFERENT

→ NO EFFECT

$G_{BARYON} < G_{NEWTON} \times 10^{-9}$