ELECTROWEAK INTERACTIONS

RECAP WHAT WE KNOW ABOUT QED (EM)

x, y, z, t

LOCAL GAUGE INVARIANCE

$$\psi(x) \longrightarrow e^{i\alpha(\bar{x})}\,\psi(x)$$

INVARIANT UNDER $U(1)$ 1-D ROTATIONS

→ CONSERVED CHARGE — GLOBAL GAUGE INVARIANCE

→ $\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi$ ← DIRAC

↓

$$\bar{\psi}(i\gamma^\mu \partial_\mu - m)\psi + e\bar{\psi}\gamma^\mu\psi$$

A_μ

PHOTON

EM FIELD.

MAYBE WEAK INTERACTION IS INVARIANCE UNDER ROTATIONS IN 2-d ! ⇒ $SU(2)$

- FOR WEAK INTERACTION

- LEFT HANDED DOUBLETS

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix} \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

WEAK ISOSPIN UP AND DOWN ?

$$\begin{pmatrix} W^+ \\ Z^0 \\ W^- \end{pmatrix}$$

ISOTRIPLET OF WEAK BOSONS ?

GUESS:

SU(2)

VECTOR IN WEAK ISOSPACE

$$\psi \longrightarrow e^{ig\,\bar{\tau}\cdot\bar{\Lambda}}\,\psi$$

ANALAGOUS TO ANGULAR MOMENTUM

$$\psi \rightarrow e^{ig\,\tau_1\Lambda_1}\,\psi$$

$$\psi \rightarrow e^{ig\,\tau_2\Lambda_2}\,\psi$$

PROJECTION

$$\psi \rightarrow e^{ig\,\tau_3\Lambda_3}\,\psi$$

QED GAUGE INVARIANCE

$$D_\mu \equiv \partial_\mu - i e A_\mu$$

γ PHOTON

$$A_\mu \to A_\mu + \frac{1}{e}\partial_\mu \alpha$$

WEAK GAUGE INVARIANCE

$$D_\mu \to \partial_\mu - i g \bar{\sigma}\cdot\bar{W}_\mu$$

$$\partial_\mu - i g \tau_i W_\mu$$

W^+
W^0
W^-

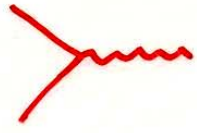
WEAK CHARGE

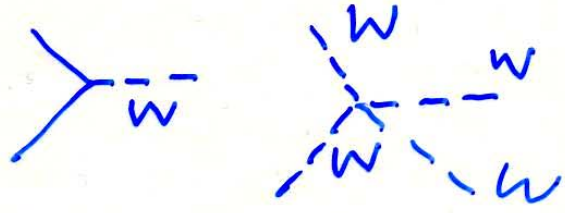
$$W_\mu \to W_\mu + \partial_\mu \bar{\Lambda} - g\, \underline{\bar{\Lambda}\times\bar{W}_\mu}$$

cf $-ie A_\mu$ $\quad -ig\, \underline{\tau_i W_\mu}$

U(1) GENERATORS COMMUTE

SU(2) GENERATORS DO NOT COMMUTE



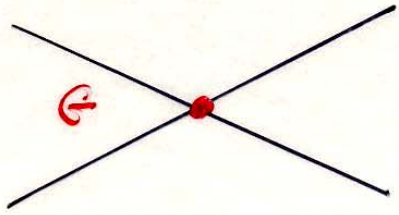


ABELIAN

NON ABELIAN

DIVERGENCES IN WEAK INTERACTIONS

- IN THE 4-FERMION V-A WEAK THEORY



$\nu_e e^- \to \nu_e e^-$

FERMI WEAK COUPLING

PERKINS Ch.5

$$\int \frac{d\sigma}{dq^2} = \int \frac{G^2}{\pi}$$

CM ENERGY SQUARED

$$\sigma_{TOT} = \frac{G^2}{\pi} q^2_{MAX} = \frac{2G^2 m E_\nu}{\pi} = \frac{G^2 s}{\pi}$$

$\therefore \quad \sigma \propto G \times$ DENSITY OF FINAL STATES

$\propto G \times s$ RISES INDEFINITELY ?

$\propto G \times p^{*2} \times 4$

UNITARITY $\sigma_{MAX} = \frac{\pi}{2}\left(\frac{\hbar}{p^*}\right)^2 \quad 1$

$$\sigma_{TOT} = \frac{4 G^2 p^{*2}}{\pi}$$

$$\sigma_{MAX} = \frac{\pi}{2 p^{*2}}$$

WHEN DOES σ_{TOT} EXCEED σ_{MAX}?

$$\frac{4 G^2 p^{*2}}{\pi} > \frac{\pi}{2 p^{*2}}$$

$$p^* > \left(\frac{\pi}{G} \cdot \frac{1}{\sqrt{8}} \right)^{\frac{1}{2}} \approx 300 \frac{GeV}{c}$$

- AT 300 GeV/c HAVE UNITARITY VIOLATION
  - PROBABILITY NOT CONSERVED
  - SCATTERED FLUX > INCIDENT FLUX

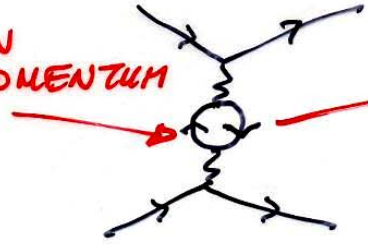
$\Rightarrow$ CANNOT BE TRUE.

RENORMALIZATION & QED

- RENORMALIZATION IS A BIT OUT OF OUR SCOPE

- IN QED EVERY LOOP $\to \infty$

LOOP CAN BE ANY MOMENTUM



$$\sim \int_0^\infty \frac{d^4q}{q^2} \to \text{FINITE}$$

$q^2 \to \infty$

- ∞ CAN BE CONTROLLED IN QED

  ↳ ABSORB INTO PHYSICAL QUANTITIES m_e, e

- RENORMALIZATION IS A PROPERTY OF GAUGE INVARIANT THEORIES.

  ↓

  INTIMATELY CONNECTED TO CANCELLATIONS INHERENT IN GAUGE INVARIANCE

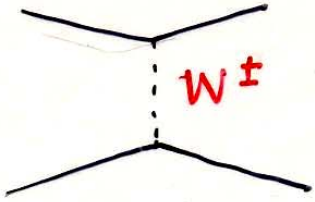
POINT 4-FERMI WEAK

- BAD HIGH ENERGY BEHAVIOUR

  WORSE AS HIGHER ORDER IN G

  $$[G] = [E]^{-2}$$

- INTRODUCE $W^{\pm}$



$$\frac{1}{\left(1+\frac{q^2}{M_W^2}\right)}$$

→ NO LONGER POINT INTERACTION

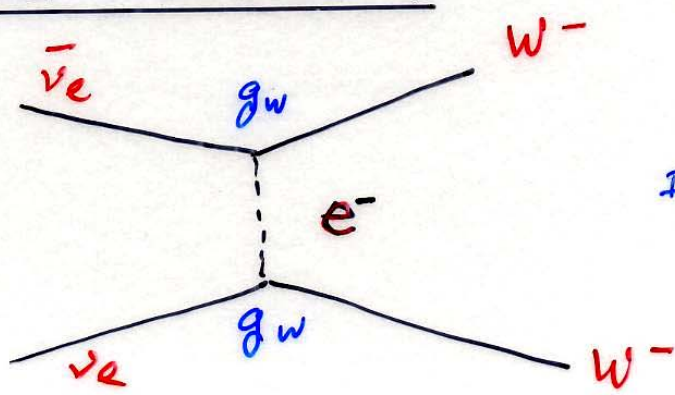
$$\sigma_{TOT} \to \frac{G^2 M_W^2}{\pi} \quad \text{FOR } s \text{ LARGE}$$

- STILL HAVE DIVERGENCES IN HIGHER ORDER – LOGARITHMIC

- NOT A GAUGE THEORY, NOT RENORMALIZABLE

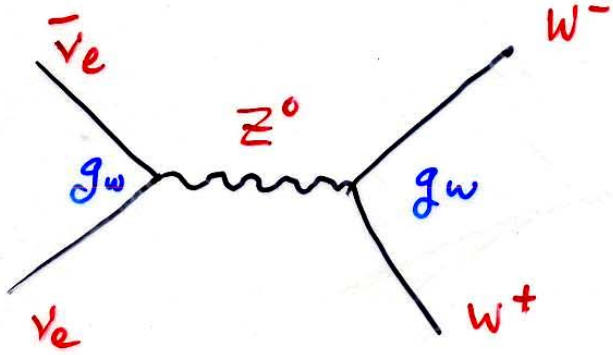
  ↳ NEED SOME CANCELLATION MECHANISM

NEUTRAL CURRENTS



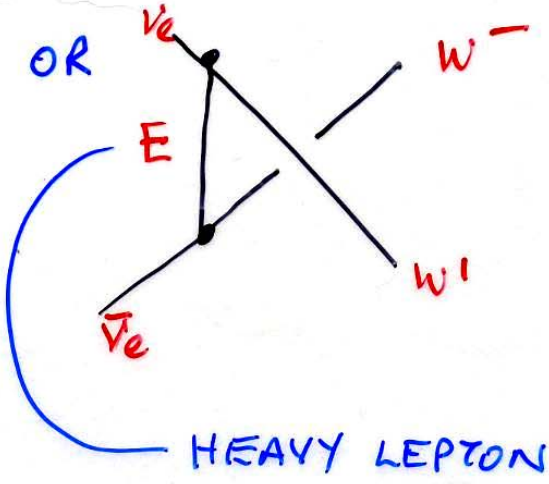
DIVERGES

CANCEL WITH



NEUTRAL CURRENT IN GIM MECHANISM

OR

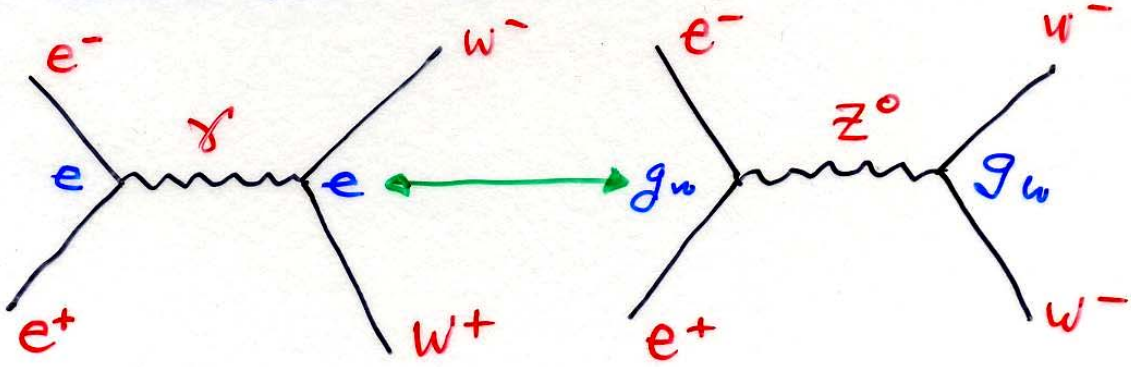


HEAVY LEPTON WITH ELECTRON FLAVOUR

↑

NO SIGN OF THIS

CANCELLATION OF



OBSERVE THAT THERE WILL BE SOME MIXING $\gamma \leftrightarrow Z^0 \Rightarrow g_w$

- FOR CANCELLATION $e \sim g_w$

RECALL

$$\lim_{q^2 \to 0} \frac{g_w^2}{(q^2 + M_W^2)} \equiv \frac{G}{\sqrt{2}}$$

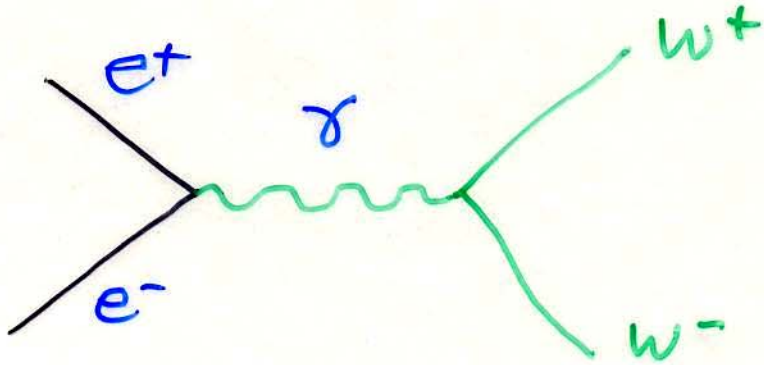
PUT $g_w = e$

$$M_W^2 = \frac{g_w^2 \sqrt{2}}{G} = \frac{e^2 \sqrt{2}}{G}$$

$$\Rightarrow M_{W,Z^0} \approx 100 \text{ GeV}$$

BUT STILL HAVE DIVERGENCES DUE TO LEPTON MASSES $\neq 0$

WHAT ABOUT HEAVY LEPTON ?

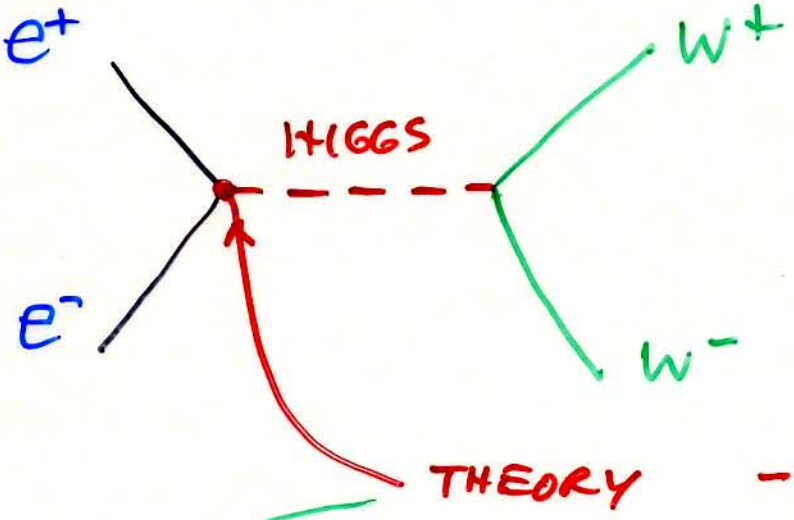


DIVERGES ~ $\frac{s}{M}$

($m_e \neq 0$)

IN STANDARD ELECTRO WEAK MODE

THIS DIVERGIEANCE EXACTLY CANCELLED BY :



HIGGS PREDI

FOR MASSIVE

W, Z

$-i m_e \sqrt{\sqrt{2} G_F}$

THIS CANCELLATION ONLY WOR

IN STANDARD MODEL

HEAVY LEPTON WOULD HAVE TO

"JUST HAPPEN" TO HAVE CORRE

1998 Electoweak Summary
CERN EP/99-15 8 Feb 1999 (11)

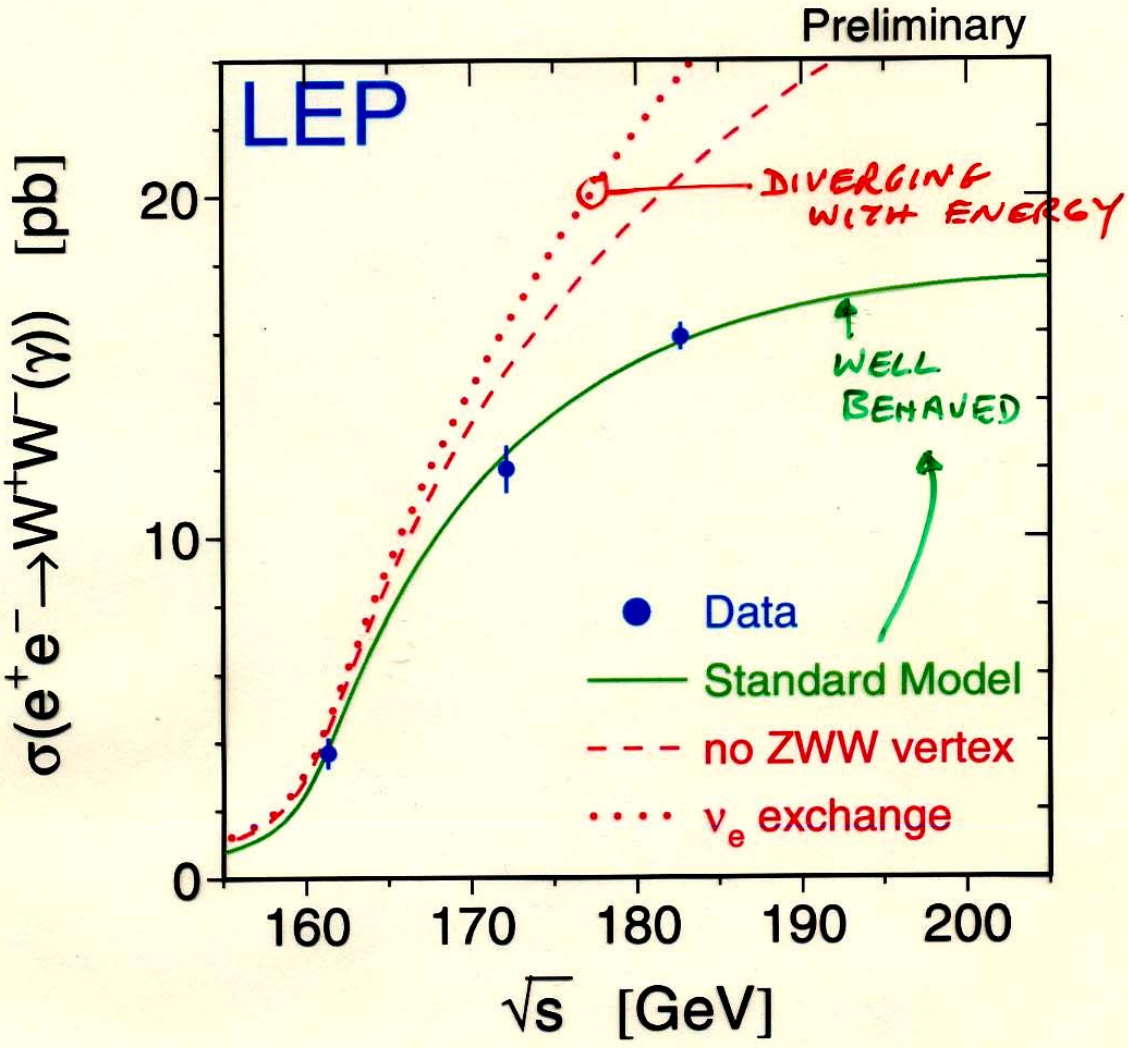


Figure 5: The W-pair cross-section as a function of the centre-of-mass energy. The data points are the LEP averages. Also shown is the Standard Model prediction (solid line), and for comparison the cross-section if the ZWW coupling did not exist (dashed line), or if only the t-channel ν_e exchange diagram existed (dotted line).

The W decay branching ratios have been determined from individual channels cross-section values measured by the four experiments (at all centre-of-mass energies [91–94]), with and without lepton universality assumption and are shown in Table 24. Correlated errors between the individual channels have been taken into account. $B(W \to \tau\nu)$ is 25% anti-correlated with the other two leptonic branching ratios, $B(W \to \mu\nu)$ and $B(W \to e\nu)$ are 1.0% correlated. Under the assumption of lepton universality,