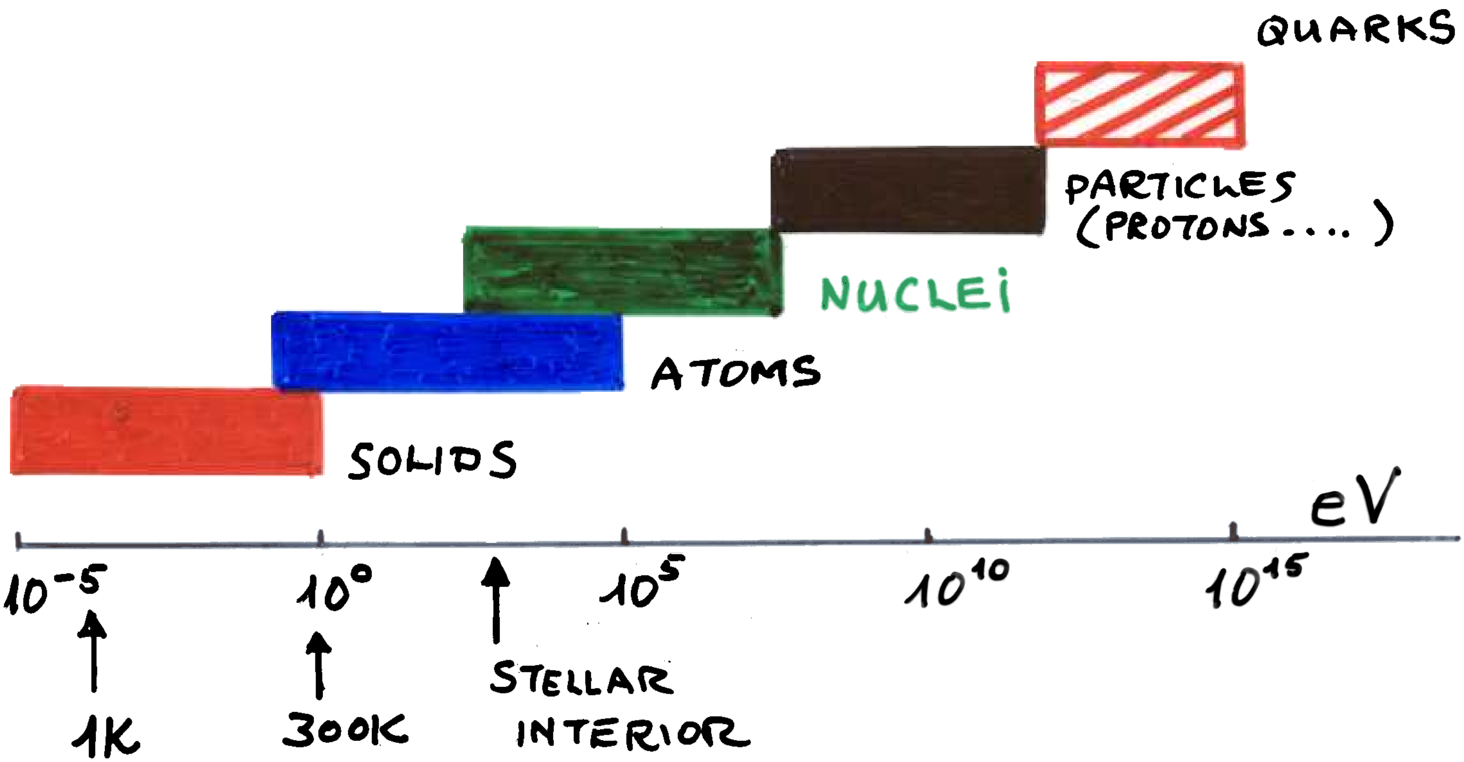
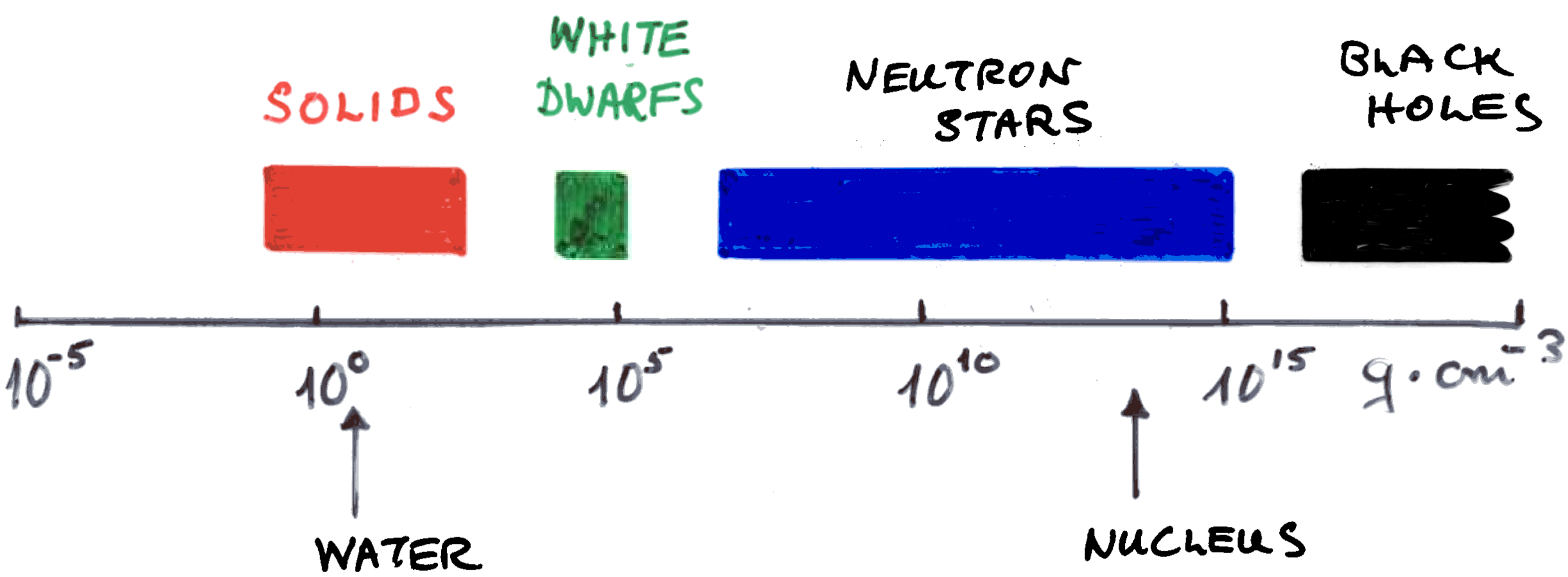
# ENERGY DENSITIES

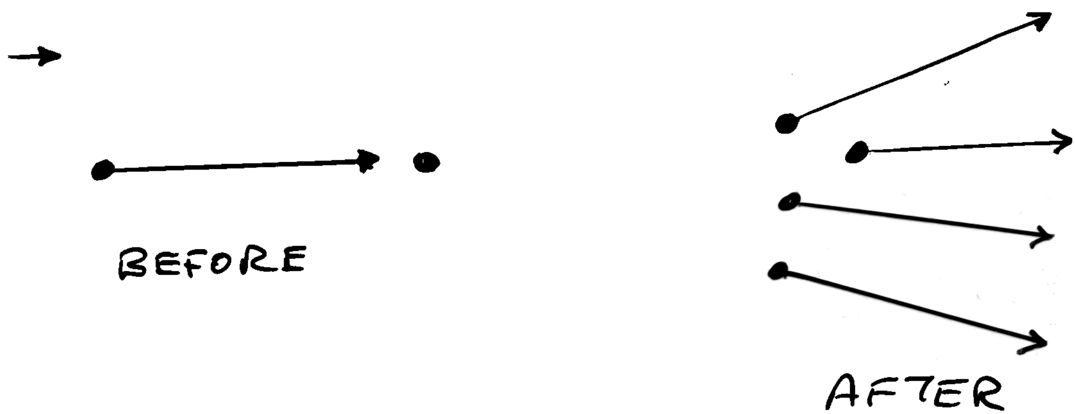



# DENSITIES



# WHY DO WE NEED HIGH ENERGIES?

1) WANT TO SEARCH FOR MASSIVE UNSTABLE PARTICLES



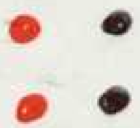

USE KINETIC ENERGY OF BEAM TO PRODUCE NEW PARTICLES

TO PRODUCE MASS $m$, NEED ENERGY AVAILABLE IN CENTRE OF MOMENTUM FRAME:

$$E = mc^2$$

2) WANT TO PROBE STRUCTURE OF MATTER AT SMALLER & SMALLER DISTANCE SCALES

SAY WANT TO RESOLVE STRUCTURE INSIDE A HELIUM NUCLEUS



$^4He_2$

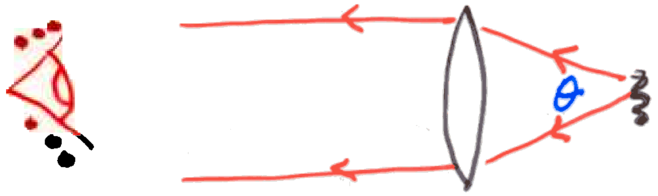

- FROM QM WE KNOW THAT BEAM HAS DE BROGLIE WAVE LENGTH


$$\psi \approx e^{-i\vec{p}\cdot\vec{r}/\hbar}$$

$$\lambda = \frac{h}{p}$$

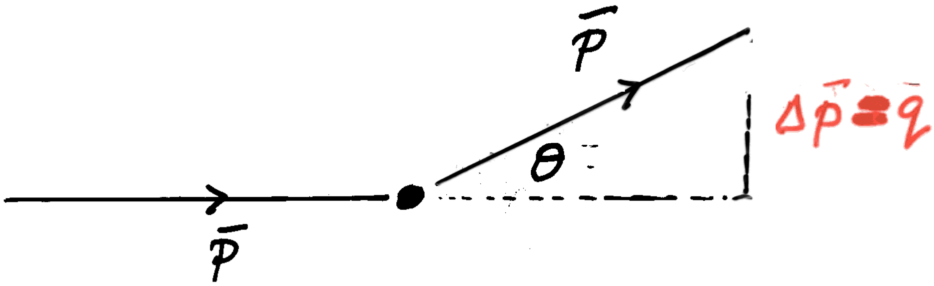
SO, $\lambda \to p$ DETERMINE SPATIAL RESOLUTION OF BEAM

RESOLUTION ? cf OPTICS



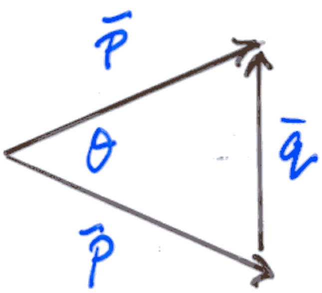

RESOLUTION $\Delta r \sim \frac{\lambda}{\sin\theta}$

- PARTICLE SCATTERING




$$\Delta r \approx \frac{\lambda}{\sin\theta} = \frac{h}{p\sin\theta}$$

ELASTIC




$$|\bar{q}| = 2|\bar{p}|\sin\frac{\theta}{2} \sim p\sin\theta$$

$$\Delta r \sim \frac{h}{q}$$

HIGH RESOLUTION

~ HIGH MOMENTUM TRANSFER

- PROTON DIAM $\sim 1\,fm = 10^{-15}\,m$

? WHAT BEAM MOMENTUM NEED TO RESOLVE PROTONS INSIDE NUCLEUS

NEED $1\,fm = \frac{\hbar}{q} = \frac{\hbar c}{qc}$

$$\hbar c = 1.23\ GeV \cdot fm$$

$$qc = \frac{1.23\ GeV\ \underline{fm}}{1\ fm}$$

$q \sim 1\ GeV/c$ MOMENTUM TRANSFER

- LIMIT ON QUARK DIAM $\sim 10^{-18}\,m$

THIS LIMIT NEEDED A BEAM WITH AN EFFECTIVE MOMENTUM

$$\sim 1000\ GeV/c = 1\ \frac{TeV}{c}$$

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Table 1.1. *Units in high energy physics*

(a)

| Quantity | High energy unit | Value in SI units |
|---|---|---|
| length | 1 fm | $10^{-15}$ m |
| energy | 1 GeV = $10^9$ eV | $1.602 \times 10^{-10}$ J |
| mass, $E/c^2$ | 1 GeV/$c^2$ | $1.78 \times 10^{-27}$ kg |
| $\hbar = h/(2\pi)$ | $6.588 \times 10^{-25}$ GeV s | $1.055 \times 10^{-34}$ J s |
| $c$ | $2.998 \times 10^{23}$ fm s$^{-1}$ | $2.998 \times 10^{8}$ m s$^{-1}$ |
| $\hbar c$ | 0.1975 GeV fm | $3.162 \times 10^{-26}$ J m |

~ PROTON MASS

(b)

| | |
|---|---|
| **natural units, $\hbar = c = 1$** ← HIGH ENERGY UNITS | |
| CHOOSE → **mass, $Mc^2/c^2$** | 1 GeV |
| **length, $\hbar c/(Mc^2)$** | 1 GeV$^{-1}$ = 0.1975 fm |
| **time, $\hbar c/(Mc^3)$** | 1 GeV$^{-1}$ = $6.59 \times 10^{-25}$ s |
| **Heaviside–Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$** | |
| **fine structure constant** | $\alpha = e^2/(4\pi) = 1/137.06$ |
| **Relations between energy units** | |
| 1 MeV = $10^6$ eV   **1 GeV = $10^3$ MeV** | 1 TeV = $10^3$ GeV |

$\hbar c = 1 = 0.1975$ GeV. fm

$\hbar = 1 = 6.59 \times 10^{-25}$ GeV. s

# SPECIAL RELATIVITY

- AT HIGH ENERGY / MOMENTUM $v \simeq c$

RESULTS OF MEASUREMENTS WILL DEPEND ON LORENTZ FRAME

SYMMETRY

→ ANY SENSIBLE THEORY MUST BE LORENTZ INVARIANT

BASED ON QUANTITIES THAT DO NOT DEPEND ON FRAME

eg MASS
LIFETIME

FORMULATED IN TERMS OF 4-VECTORS

→ A FREE PARTICLE MASS

$$E, \bar{p}, m$$

$$E^2 = \bar{p}^2 + m^2$$

$$m^2 = E^2 - \bar{p}^2$$ LORENTZ SCALAR

$$m^2 = (E, \bar{p}) \cdot (E, \bar{p})$$

HOWEVER.....

THE TEXT USES

MINKOWSKI NOTATION

I PREFER METRIC ON LAST PAGE

DIGRESSION

MINKOWSKI 4-VECTOR — 3 REAL SPACE COMP. 1 IMAGINARY ENERGY COMP.

$p_x \quad p_y \quad p_z \quad E$ ENERGY MOMENTUM

$\downarrow$

$p_\mu \qquad \mu = 1, 2, 3, 4$

$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = iE$

$$p^2 = \sum_\mu p_\mu^2 = p_1^2 + p_2^2 + p_3^2 + p_4^2$$

$$= \bar{p}^2 - E^2 = -m^2$$

SCALAR PRODUCT LORENTZ INVARIANT $\rightarrow$ MASS IS A USEFUL CONCEPT

THE MINKOWSKI NOTATION DEFINES
THE METRIC $\Rightarrow$ SQUARE OF 4-VECTOR

$$p = (\bar{p}, iE)$$

$$p^2 = (\bar{p})^2 - E^2$$

MORE USUALLY ONE USES METRIC

$$AB = g_{\mu\nu} A^\mu B^\nu = g^{\mu\nu} A_\mu B_\nu = A^\mu B_\mu$$

$$A^\mu = (A^0, \bar{A}) \qquad g_{\mu\nu} = \begin{pmatrix} +1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

$$A_\mu = (A^0, -\bar{A})$$

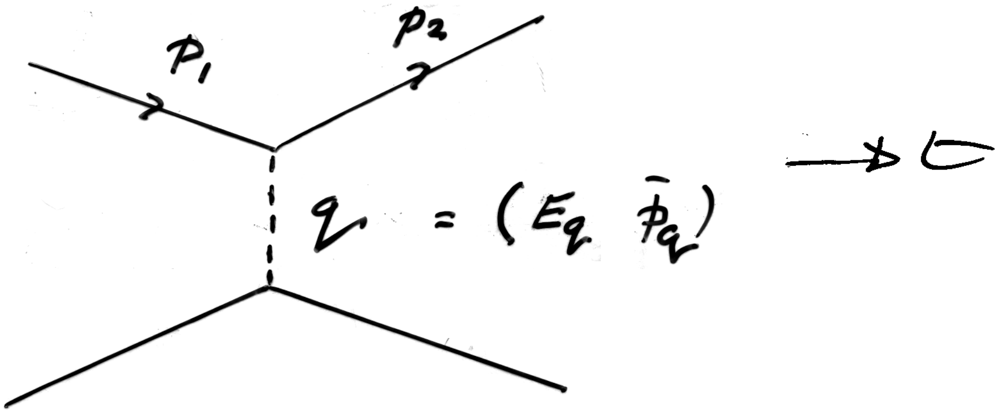
$\rightarrow$ FOR LORENTZ INVARIANCE IN AN EXPRESSION UPPER & LOWER INDICES MUST BALANCE

ENERGY - MOMENTUM $\underset{p_0}{(E}, \underset{\bar{p}}{\bar{p})}$

$$p \cdot p = p^\mu p_\mu = E^2 - p^2 = m^2$$

$p^2 = \text{MASS}^2$ NOT $p^2 = -\text{MASS}^2$

FOR A SCATTERING PROCESS

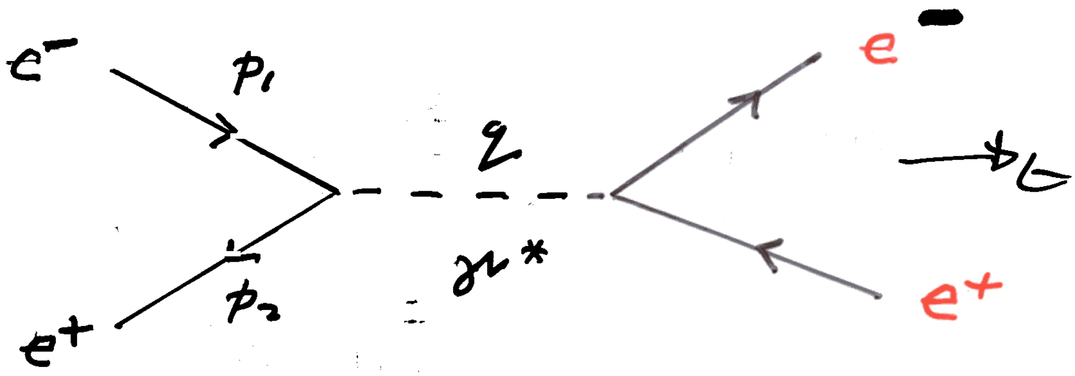



$$q = (E_q, \bar{p}_q)$$

$$q^2 = (p_1 - p_2)^2 = -ve$$

$$= E_q^2 - \bar{p}_q^2 \rightarrow \text{SPACELIKE}$$

FOR AN ANNIHILATION PROCESS




$$q^2 = (p_1 + p_2)^2 = +ve$$

$$= E_q^2 - p_q^2 = m_{\gamma^*}^2$$

TIMELIKE

SIGNS REVERSED FOR MINKOWSKI

# LORENTZ TRANSFORMATION

$$p_\mu' = \sum \alpha_{\mu\nu} p_\nu$$

FOR BOOST ALONG $x$- AXIS $\beta$

MINKOWSKI

$$\begin{pmatrix} p_x' \\ p_y' \\ p_z' \\ iE' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \\ iE \end{pmatrix}$$

"MODERN" METRIC

$$\begin{pmatrix} E' \\ p_x' \\ p_y' \\ p_z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

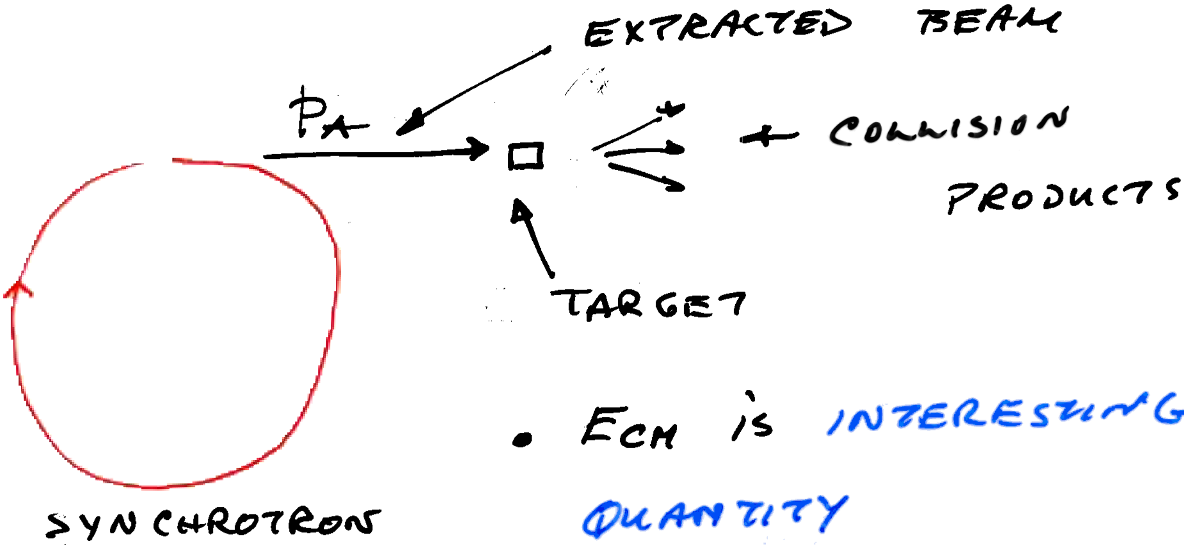
$$E' = \gamma(E - \beta p_x)$$

$$p_x' = \gamma(p_x - \beta E)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

# FIXED TARGET v. COLLIDING BEAM.

ASSUME FAMILIAR WITH CONCEPT OF SYNCHROTRON ACCELERATOR




- $E_{CM}$ is INTERESTING QUANTITY
  → MOST OF $p_A$ GOES INTO MOTION OF C.M. FRAME


- COLLIDING BEAMS
  – ACCELERATORS IN CM FRAME

ALL OF $p_A$ GOES INTO $E_{CM}$

- IN ANY ARBITRARY FRAME CONSIDER COLLISION of TWO PARTICLES A & B

$$m_A \quad (E_A, \bar{p}_A)$$

$$m_B \quad (E_B, \bar{p}_B)$$

- TOTAL $p^2$ OF SYSTEM IS

$$p^2 = (p_A + p_B)^2$$

$$= p_A^2 + p_B^2 + 2\,p_A p_B$$

$$= m_A^2 + m_B^2 + 2E_A E_B - 2\bar{p}_A \cdot \bar{p}_B$$

$$p_A^2 = E_A^2 - \bar{p}_A^2 = m_A^2$$

$$p^2 = m_A^2 + m_B^2 + 2E_A E_B - 2|\bar{p}_A||\bar{p}_B|\cos\theta$$

IN CMS FRAME $\sum \bar{p} = 0$

FROM $p^{*2} = E^{*2} - |\bar{p}^*|^2$

$$p^{*2} = E^{*2}$$

$$E^{*2} = m_A^2 + m_B^2 + 2E_A E_B - 2|\bar{p}_A||\bar{p}_B| \cos\theta$$

- FIXED TARGET — B AT REST

$$|\bar{p}_B| = 0 \; ; \quad E_B = m_B$$

$$E^{*2} = m_A^2 + m_B^2 + 2m_B E_A$$

$$E^* \propto \sqrt{E_A}$$

ACCELERATOR ENERGY

- COLLIDING BEAMS

$$|\bar{p}_A| = |\bar{p}_B| \qquad \cos\theta = -1$$

$\theta$

$$E^{*2} = m_A^2 + m_B^2 + 2E_A E_B + 2|\bar{p}_A||\bar{p}_B|$$

FOR $E >> m$ ;

$$E^{*2} = 4E_A^2$$

$$E^* = 2E_A$$

ACCELERATOR ENERGY

$\$ \propto E_A$

$E^*_{FIXED} \propto \sqrt{\$}$ ☹

$E^*_{COLLIDER} \propto \$$ ☺