COLOUR

- IN ADDITION TO ELECTRIC CHARGE

  QUARKS CARRY COLOUR CHARGE

  SOURCE OF STRONG INTERACTION

- γ COUPLE TO ELECTRIC CHARGE
- g COUPLE TO COLOUR CHARGE

- ONE ELECTRIC CHARGE +e
- THREE COLOUR CHARGES RED GREEN BLUE

- SINGLE FREE QUARKS NEVER OBSERVED

  ONLY COLOUR NEUTRAL BOUND STATES

  PROTON u d u + PERMUTATIONS

  PION π^+ u $\bar{d}$ → ANTIRED

Table 1.6. *Quark composition of some meson and baryon states (masses in MeV/c^2 in parentheses), together with values of strangeness, S*

Meson	Composition	S	Baryon	Composition	S
$\pi^+(140)$	$u\bar{d}$	0	$p(931)$	uud	0
$K^0(498)$	$d\bar{s}$	+1	$\Lambda(1116)$	uds	−1
$K^-(494)$	$\bar{u}s$	−1	$\Xi^0(1315)$	uss	−2
$\rho^-(770)$	$\bar{u}d$	0	$\Sigma^+(1189)$	uus	−1
$\omega^0(783)$	$u\bar{u}$	0	$\Omega^-(1672)$	sss	−3

ANTIPARTICLES

- SAME MASS, SPIN, LIFETIME
  OPPOSITE CHARGE, COLOUR, FLAVOUR

- PROFOUNDLY CONNECTED WITH LORENTZ INVARIANCE
  (SEE FORCE OF SYMMETRY!)

$$E^2 = p^2 + m^2 \qquad c = 1$$

$$E = \pm\sqrt{p^2 + m^2}$$

- MEANINGLESS FOR CLASSICAL PARTICLE

- QM $\rightarrow$ FREE PARTICLES $\rightarrow$ PLANE WAVE

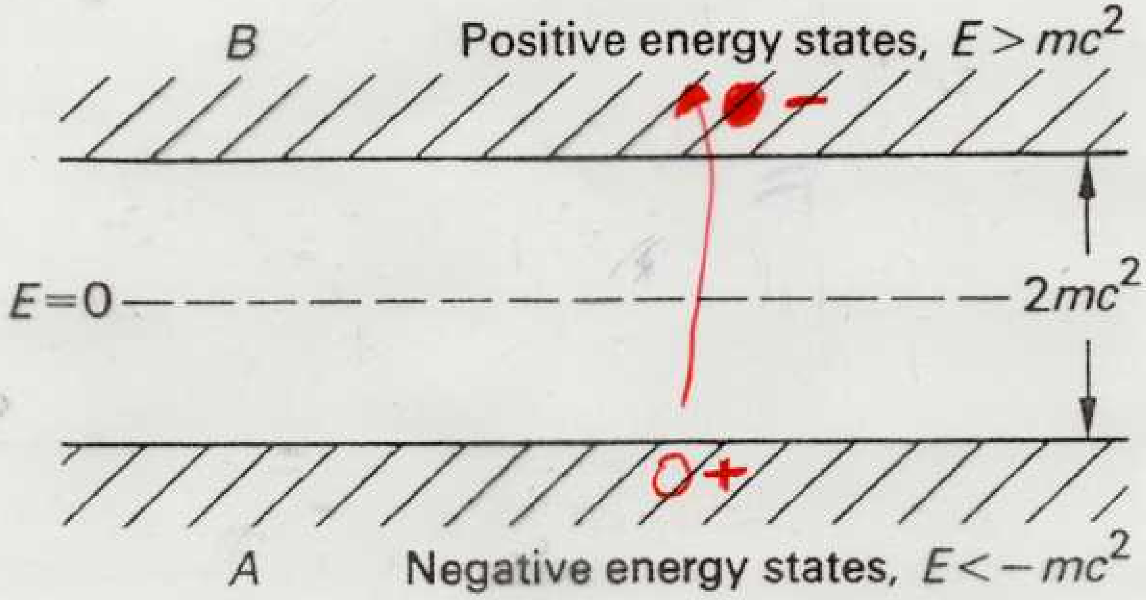
$$\psi = A e^{-i(Et - px)} \qquad \hbar = 1$$

$t \rightarrow +ve$; PHASE INCREASE FOR x +ve

$t \rightarrow -ve$; " . x -ve

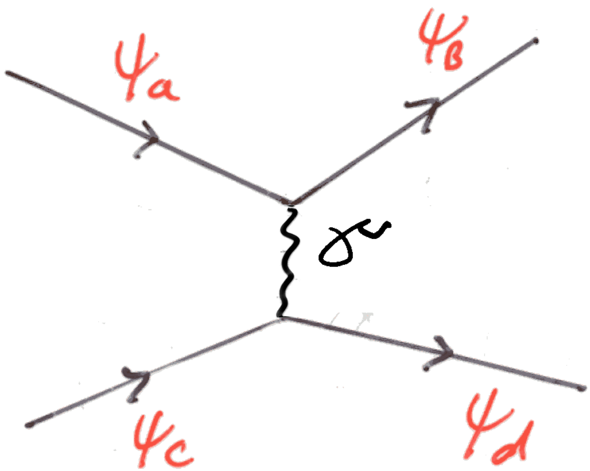
$$Et \rightarrow (-E)(-t) \qquad px \rightarrow (-p)(-x)$$

$\xrightarrow{\qquad\qquad}$	$\xleftarrow{\qquad\qquad}$
$E > 0$	$E < 0$
$t_1 < t_2$	$t_1 < t_2$



- −VE ELECTRONS FLOWING BACK IN TIME ($E<0$) ≡ +VE FLOWING FORWARD WITH $E>0$
- EQUAL & OPPOSITE CHARGES
- DIRAC "HOLE IN SEA"
  −VE SEA FULL FROM PAULI
- NON REL QM $\psi \rightarrow$ SINGLE PARTICLE
- FIELD THEORY $\rightarrow$ ψ CREATES OR DESTROYS PARTICLES

FIELD THEORY



$$\text{AMP} \sim \bar{\psi}_B \, \psi_A \, \frac{1}{q^2} \, \bar{\psi}_d \, \psi_c$$

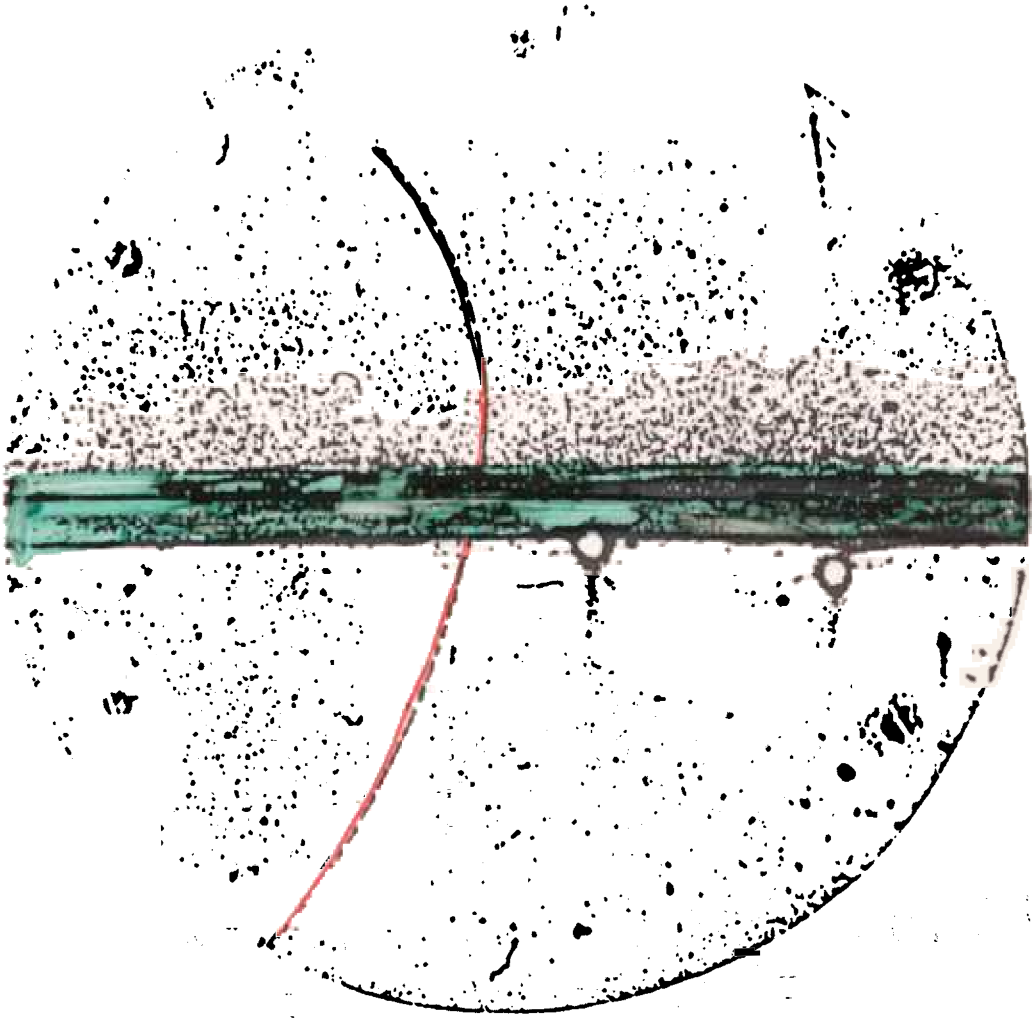
CREATE B

ANNIHILATE A

PROPAGATE γ^μ

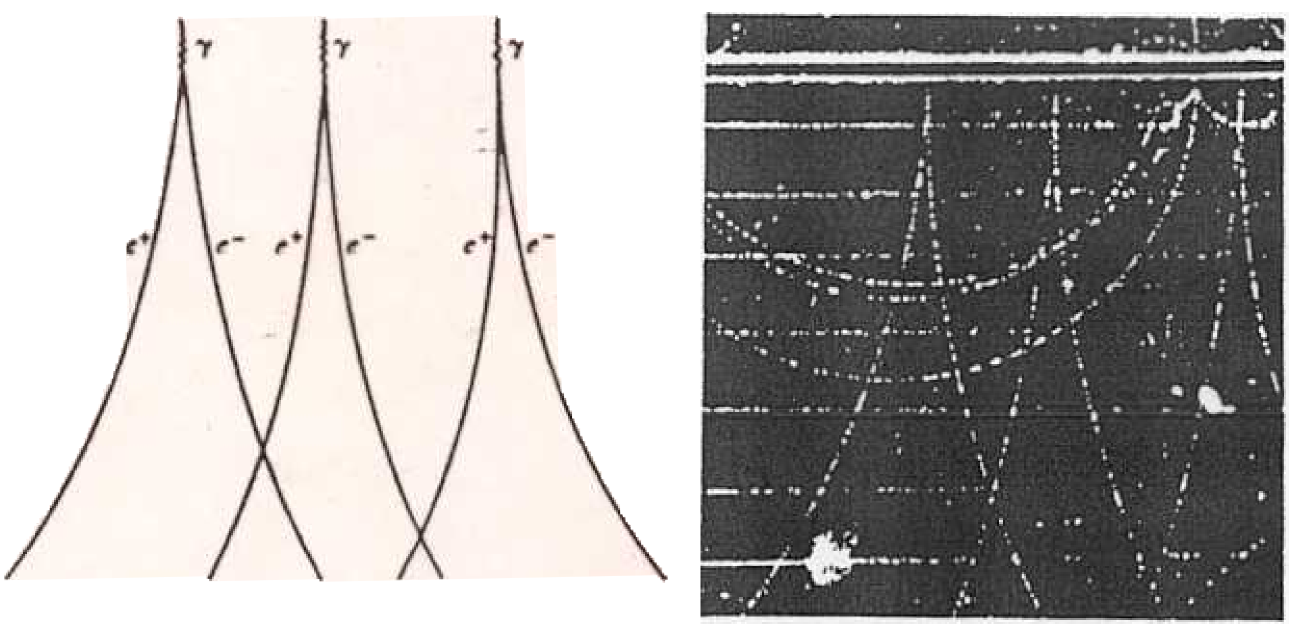
- ABOVE IS VERY SCHEMATIC!

⊗ $\vec{B}$



. A 63 million volt positron ($H\rho = 2.1 \times 10^5$ gauss-cm) passing through a 6 mm lead plate and emerging as a 23 million volt positron ($H\rho = 7.5 \times 10^4$ gauss-cm). The length of this latter path is at least ten times greater than the possible length of a proton path of this curvature.

10 X POSSIBLE PROTON PATH LENGTH



Tracks of electron-positron pairs produced by 300-MeV synchrotron x rays. (*Courtesy Lawrence Radiation Laborato[ry,] University of California, Berkeley.*)

Fig. 3.

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RELATIVISTIC WAVE EQUATION

SCHRÖDINGER – NON RELATIVISTIC

START FROM NON-RELATIVISTIC ENERGY MOMENTUM

$$E = \frac{p^2}{2m} - V$$

QUANTIZE:

$$p \to -i\hbar \nabla$$

$$E\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - V\psi$$

$$E \to i\hbar\frac{\partial}{\partial t}$$

$$i\hbar\frac{\partial\psi}{\partial t} + \frac{\hbar^2}{2m}\nabla^2\psi = -V\psi$$

$= 0$ FOR FREE PARTICLE

CAN GO THRU SAME PROCESS FOR RELATIVISTIC WAVE EQUATION.

KLEIN – GORDON – RELATIVISTIC

START FROM RELATIVISTIC ENERGY MOMENTUM

$$E^2 = p^2 + m^2 \quad \text{(FREE)}$$

$$E \to i\hbar \frac{\partial}{\partial t} \qquad p \to -i\hbar\nabla$$

$$-\hbar^2 \frac{\partial^2 \psi}{\partial t^2} = -\hbar^2 \nabla^2 \psi + m^2$$

$$\frac{\partial^2 \psi}{\partial t^2} = \nabla^2 \psi - m^2 \psi$$

$$\frac{\partial^2 \psi}{\partial t^2} = (\nabla^2 - m^2)\psi$$

IN 4-VECTOR NOTATION

$$p^\mu p_\mu = m^2$$

$$p^\mu \to i\partial^\mu$$

OK FOR "SPINLESS" PARTICLES

$$(\partial^\mu \partial_\mu + m^2)\psi = 0$$

$$= -V\phi$$

IF POTENTIAL

$$\frac{\partial^2 \psi}{\partial t^2} = (\nabla^2 - m^2)\psi \quad ①$$

- $E = \pm\sqrt{p^2 + m^2}$ SEEN AS BOTHERSOME

DIRAC LOOKED FOR EQUATION

1ST ORDER IN SPACE-TIME

FOR A MASSLESS PARTICLE, ①

$$\frac{\partial^2 \psi}{\partial t^2} = \nabla^2 \psi$$

IF YOU "TAKE $\sqrt{\ }$"

SOME PARAMETER

$$\frac{\partial \psi}{\partial t} = \pm \bar{\sigma}\, \nabla \psi$$

$$\frac{\partial \psi}{\partial t} = \pm\left[\sigma_1 \frac{\partial \psi}{\partial x} + \sigma_2 \frac{\partial \psi}{\partial y} + \sigma_3 \frac{\partial \psi}{\partial z}\right] \quad ②$$

① IS JUST ENERGY-MOMENTUM

SO MUST STILL BE SATISFIED

↳ SQUARE ②

$$\frac{\partial^2\psi}{\partial t^2} = \underbrace{\sigma_1^2\frac{\partial^2\psi}{\partial x^2} + \sigma_2^2\frac{\partial^2\psi}{\partial y^2} + \sigma_3^2\frac{\partial^2\psi}{\partial z^2}}_{\rightarrow \nabla^2 \text{ IF } \sigma_1^2+\sigma_2^2+\sigma_3^2=1}$$

$$\nabla^2\psi$$

0

$$+ \sigma_1\sigma_2\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial y} + \sigma_2\sigma_1\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial y}$$

$$+ \sigma_1\sigma_3\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial z} + \sigma_3\sigma_1\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial z}$$

$$+ \sigma_2\sigma_3\frac{\partial\psi}{\partial y}\frac{\partial\psi}{\partial z} + \sigma_3\sigma_2\frac{\partial\psi}{\partial y}\frac{\partial\psi}{\partial z}$$

$\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial y}$, $\left[\frac{\partial\psi}{\partial x}\frac{\partial\psi}{\partial x}\right]$

ALL THESE MUST BE 0

$$\left.\begin{aligned} \sigma_1\sigma_2 + \sigma_2\sigma_1 &= 0 \\ \sigma_1\sigma_3 + \sigma_3\sigma_1 &= 0 \\ \sigma_2\sigma_3 + \sigma_3\sigma_2 &= 0 \end{aligned}\right\}$$

CANNOT

NUMBERS

MATRICES