INADEQUACY OF SIMPLE LIQUID DROP

$$B(Z,A) = a_1 A - a_2 A^{2/3} - a_3 Z^2/A^{1/3}$$

THIS DEFINTION OF BINDING ENERGY IS OPPOSITE OF FERBEL + DAS SAME AS F&H TEXT

⇒ THIS IS MAGNITUDE OF B.E.

⇒ REFERRED TO FREE NUCLEONS B.E. IS −ve.

- NO EXPLANATION OF FACT THAT LIGHT NUCLEI WITH $N=Z$ ARE MOST STABLE

C
O
N

- NO EXPLANATION OF FACT THAT
  - EVEN − EVEN MOST STABLE
  - ODD − ODD RARE

- FOR FIXED A, $Z=0$ IS MOST STABLE!

SINCE WE KNOW

$p \leftrightarrow n$ VIA β DECAY

THIS MODEL WOULD PREDICT ALL $Z \neq 0$ NUCLIDES UNDERGO β-DECAY TO MAKE $Z=0$ ($B \rightarrow$ MINIMUM) NO ATOMS!

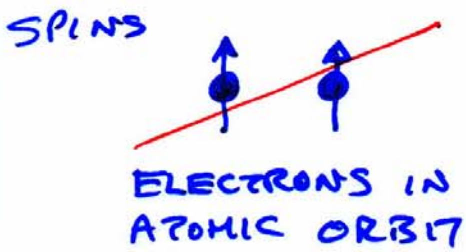
HAVE IGNORED QUANTUM MECHANICS

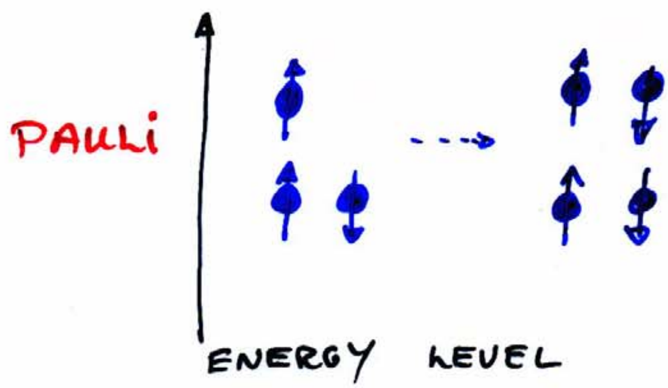
IMPLICIT ASSUMPTION:~

IF IGNORE COULOMB FORCE, THEN BINDING ENERGY OF PROTONS – PROTONS, NEUTRON-NEUTRON IS SAME

THIS IGNORES THE ACTION OF THE PAULI EXCLUSION PRINCIPLE

TWO IDENTICAL SPIN $\frac{1}{2}$ PARTICLES CANNOT OCCUPY THE SAME QUANTUM STATE



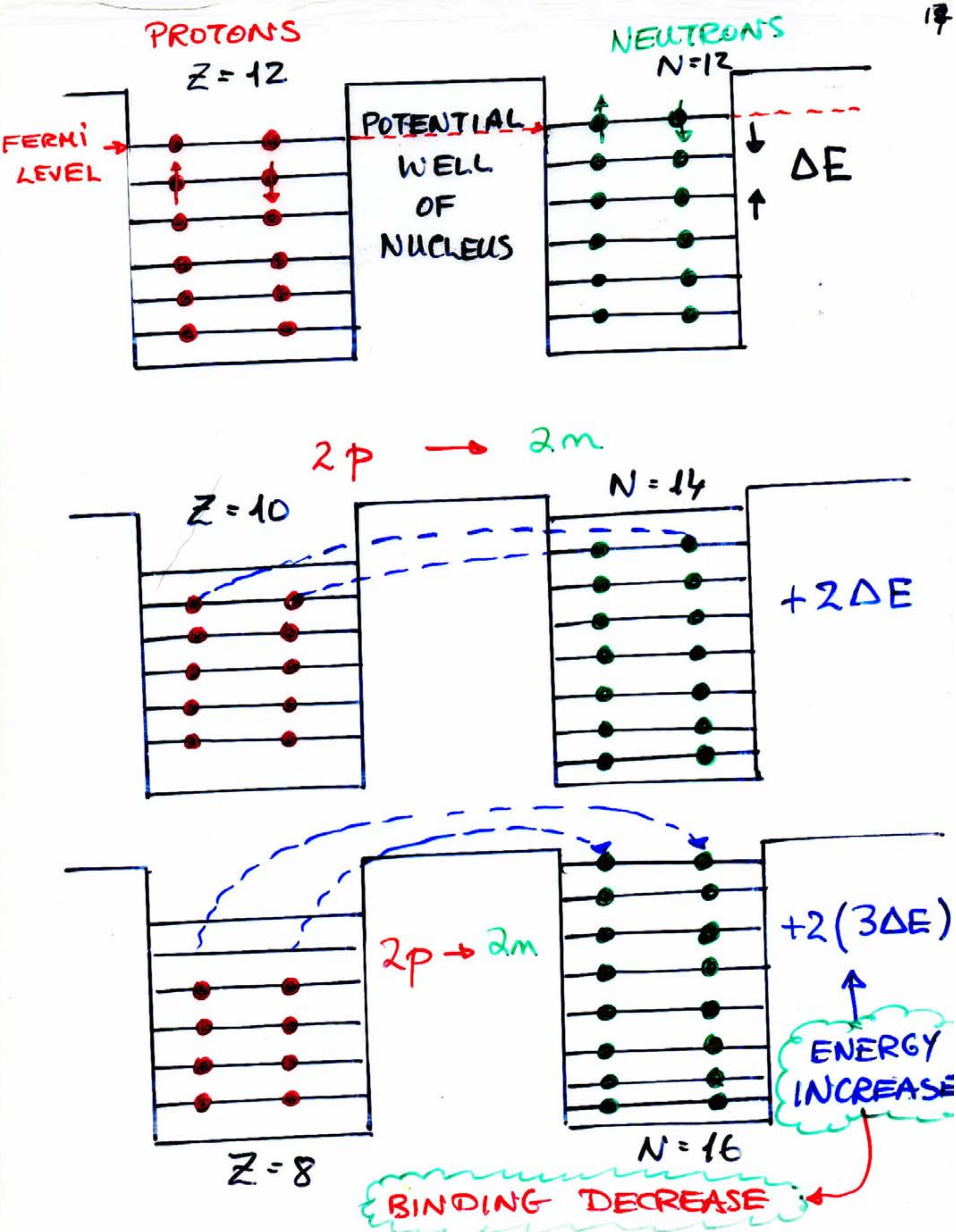


- FERMI – DIRAC QUANTUM STATISTICS LEADS TO ELECTRONS IN ATOMS SITTING IN SUCCESSIVE SHELLS → CHEMICAL PERIODIC TABLE

- n + p ARE SPIN $\frac{1}{2}$ → SAME BEHAVIOUR

  BUT $n + p$ NOT IDENTICAL, SO PAULI WILL MAKE n + p SEE DIFFERENT POTENTIALS

$\boxed{\text{FERMI GAS}}$


PROTONS
Z = 12
NEUTRONS
N = 12
FERMI LEVEL
POTENTIAL WELL OF NUCLEUS
ΔE
2p → 2n
Z = 10
N = 14
+2ΔE
2p → 2n
+2(3ΔE)
ENERGY INCREASE
Z = 8
N = 16
BINDING DECREASE

BECAUSE OF THIS INTRINSICALLY QUANTUM MECHANICAL EFFECT, MOVING AWAY FROM

$$Z = N$$

MAKES THE NUCLEUS LESS STABLE, SINCE

BINDING ENERGY REDUCED

AS ONE STARTS TO HAVE A LARGER EXCESS OF, FOR EXAMPLE, PROTONS OVER NEUTRONS (ASYMMETRY), THE BINDING ENERGY SHOW CORRESPONDINGLY FASTER DECREASE, BECAUSE:

$$\left.\begin{array}{c}\text{PROTON FERMI}\\ \text{LEVEL}\end{array}\right\} \xleftrightarrow{\text{DIVERGE}} \left\{\begin{array}{c}\text{NEUTRON}\\ \text{FERMI}\\ \text{LEVEL}\end{array}\right.$$

IN OUR PICTORIAL EXAMPLE, FOR SUCCESSIVE PROTON → NEUTRON SWAPS THERE ARE ENERGY CHANGES

ΔE 1, 1, 3, 3, 5, 5, 7

LEADING TO CUMMULATIVE DECREASES IN THE BINDING ENERGY OF

$\Delta E \times$	1	2	5	8	13	18	27
$N-Z$	2	4	6	8	10	12	14

AS THE NEUTRON - PROTON ASYMMETRY GROWS BY ONE NUCLEON AT A TIME, BINDING ENERGY DECREASES BY

$\Delta E \times$	1	2	5	8	13	18	27

FOR NEUTRON PROTON ASYMMETRIES OF

$(N-Z)$	2	4	6	8	10	12	14

SO, IN THIS SIMPLE EXAMPLE, TO GO FROM A NUCLEUS WITH EQUAL NUMBER OF PROTONS & NEUTRONS $N-Z=0$ TO ONE WITH $N>Z$; $A=N+Z=\text{CONST}$

THE BINDING ENERGY WILL BE DECREASED BY:

$$\delta E_{ASYM} \sim (N-Z)^2 \cdot \frac{\Delta E}{8}$$

THIS IS TRUE WHETHER N OR Z IS INCREASED

SO, WE EXPECT THAT NUCLEI WITH UNEQUAL NUMBERS OF PROTONS AND NEUTRONS WILL BE LESS STABLE THAN

SYMMETRIC NUCLEI

OUR SIMPLE FORMULA, FOR A SIMPLE MODEL

$$\delta E_{ASYM} \sim (N-Z)^2 \cdot \frac{\Delta E}{8}$$

THIS IS TREATED IN THE SPIRIT OF THE SEMI-EMPIRICAL MASS FORMULA, BY ADDING AN ASYMMETRY TERM

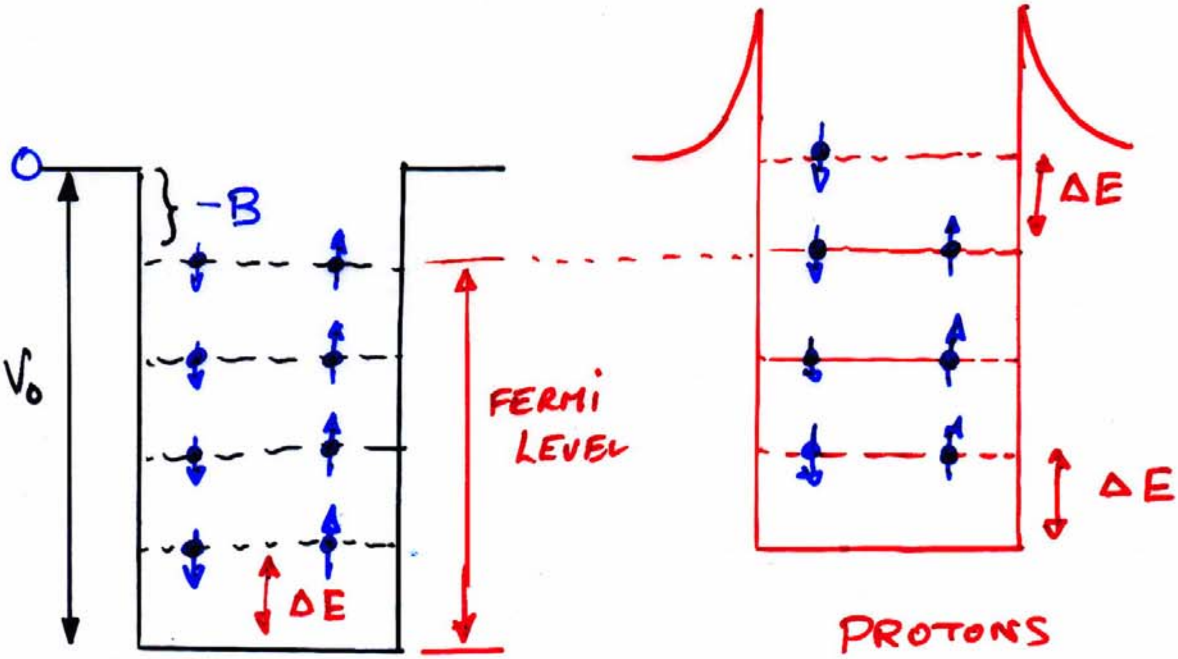
$$(a_4) \qquad - a_s \frac{(Z-N)^2}{A}$$

THE $\frac{1}{A}$ COMES FROM THE NUMBER OF AVAILABLE ENERGY LEVELS

$N_{E.L.} \propto$ VOLUME OF POTENTIAL WELL $\propto A$

$$\text{SO } \Delta E \propto \frac{1}{A}$$

THIS CAN ACTUALLY BE MORE CLEARLY UNDERSTOOD BY TREATING THE NUCLEUS AS A FERMI-GAS



$N > Z$

→ WELL DEPTHS ARE DIFFERENT

IF THEY WERE THE SAME IN HEAVY NUCLEI WHERE $N > Z$
E_F FOR NEUTRONS $>$ E_F PROTONS
THEN BINDING ENERGY OF LAST WOULD BE CHARGE DEPENDENT

→ THIS IS NOT SO

⇒ CHARGE INDEPENDENCE

- ALSO IF $E_F(n) > E_F(p)$ NEUTRONS WOULD TRANSFORM TO PROTONS BY β-DECAY & DROP INTO PROTON LEVELS
→ $N > Z$ UNSTABLE (NOT TRUE)

NON-RELATIVISTICALLY, THE FERMI ENERGY

$$E_F = \frac{p_F^2}{2m} \leftarrow \text{NUCLEON MASS}$$

IN MOMENTUM SPACE

$$V_{p_F} = \frac{4\pi}{3} p_F^3$$

PHASE SPACE = CONFIGURATION SPACE × MOMENTUM SPACE

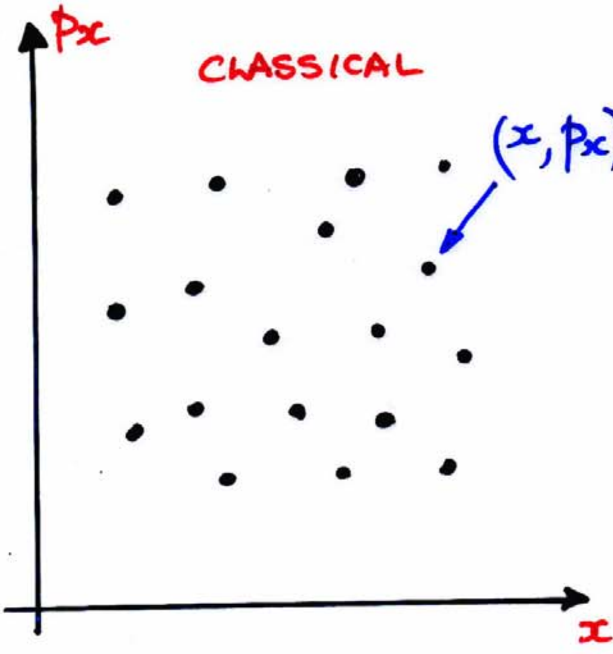
$$\downarrow$$

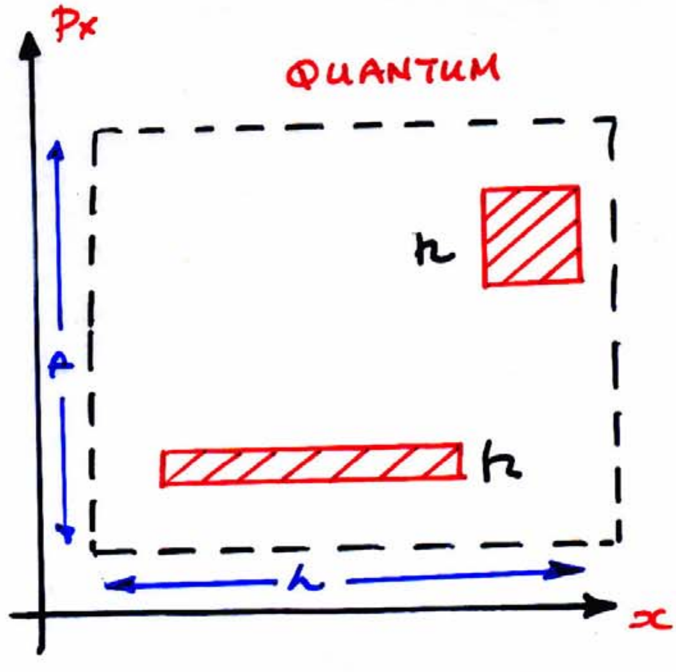
$$V_{TOT} = V \cdot V_{p_F} = \frac{4\pi}{3} r_0^3 A \cdot \frac{4\pi}{3} p_F^3$$

$$V_{TOT} = \left(\frac{4\pi}{3}\right)^2 A \cdot (r_0 p_F)^3$$

THIS IS KNOWN AS p^3 PHASE SPACE AND IS

$$\propto \begin{pmatrix} \text{NUMBER OF QUANTUM} \\ \text{STATES OF SYSTEM} \end{pmatrix}$$





WE KNOW FROM $\Delta x \cdot \Delta p \sim \hbar$ THAT A ONE DIMENSIONAL PARTICLE CANNOT BE REPRESENTED BY A POINT IN PHASE SPACE → CELL AREA $\hbar$.
IN QUANTUM MECHANICS COURSE SAW THAT

NO. OF ENERGY LEVELS = NO. OF STATES = $\dfrac{L \cdot p}{2\pi\hbar}$

GENERALIZING TO 3-DIMENSIONS

$$n_F = \frac{1}{(2\pi\hbar)^3} \int d^3x \int d^3p$$

$$n_F = \frac{\text{PHASE SPACE VOLUME}}{\text{CELL SIZE}}$$

FOLLOW DETAILS THROUGH IN PROBLEM SET

NUMBER OF FERMIONS THAT CAN FILL STATES UP TO THE FERMI LEVEL

$$n_F = 2\,\frac{V_{TOT}}{(2\pi\hbar)^3} = \frac{4}{9\pi} A \left(\frac{r_0 p_F}{\hbar}\right)^3$$

SPIN UP/DOWN

FOR $N = Z = A/2$; ALL STATES TO E_F FILLED

$$N = Z = \frac{A}{2} = \frac{4}{9\pi}\cdot A \cdot \left(\frac{r_0 p_F}{\hbar}\right)^3$$

$$p_F = \frac{\hbar}{r_0}\left(\frac{9\pi}{8}\right)^{\frac{1}{3}}$$

DOES NOT DEPEND ON NUMBER OF NUCLEONS

$$E_F = \frac{p_F^2}{2m} = \frac{1}{2m}\left(\frac{\hbar}{r_0}\right)^2\left(\frac{9\pi}{8}\right)^{2/3} \sim 33\,\text{MeV}$$

$B/A \sim -8\,\text{MeV}$ → ASSUME REPRESENTS LAST NUCLEON

$$\boxed{V_0 = E_F + B \approx 40\,\text{MeV}}$$

DEPTH OF SQUARE WELL

THE FINAL TERM ADDED TO THE SEMI-EMPIRICAL MASS FORMULA, IS TO TAKE ACCOUNT OF THE EXPERIMENTAL OBSERVATION THAT

$$\begin{matrix} pp \\ nn \end{matrix}$$

PAIRS ARE MORE TIGHTLY BOUND THAN

$$np$$

THIS LEADS TO THE ADDITION OF A

PAIRING TERM

FOR A_{ODD} CAN HAVE Z EVEN N ODD / Z ODD N EVEN

AND THERE IS NO EFFECT

FOR A EVEN

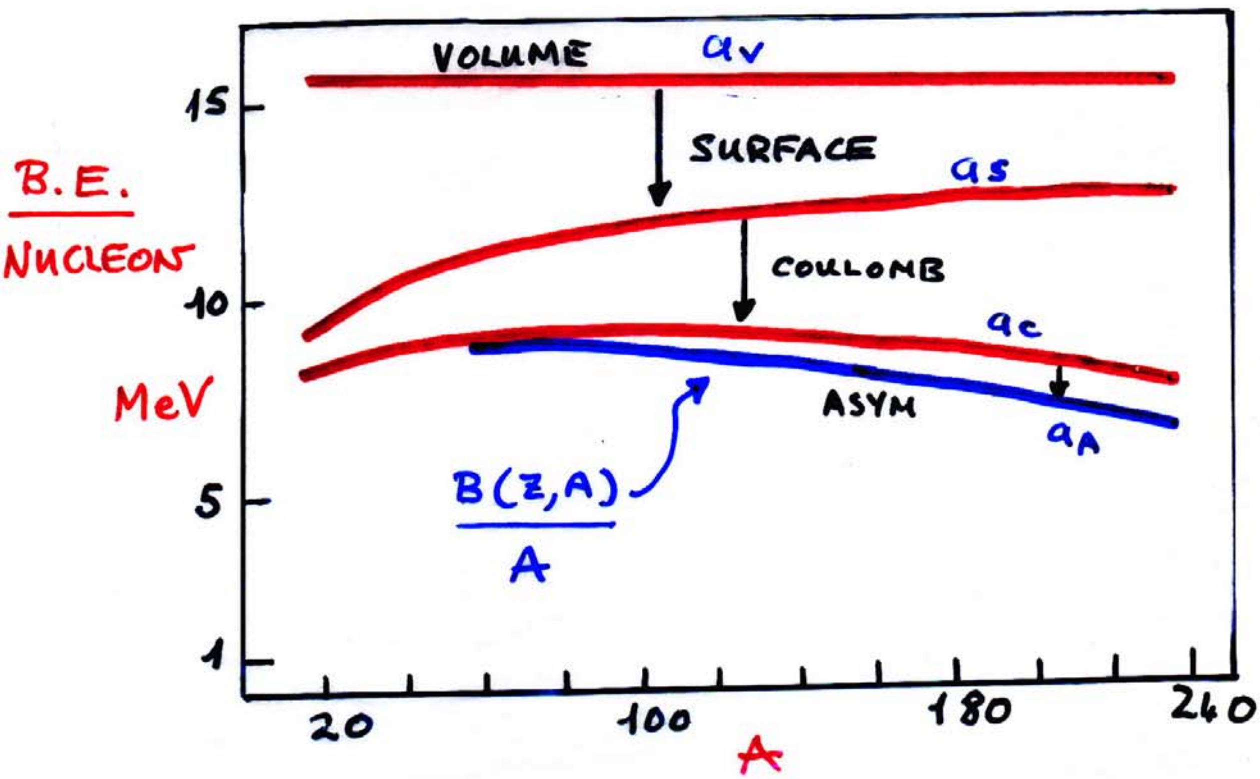
Z EVEN, N EVEN { MORE TIGHTLY BOUND THAN } Z ODD N ODD

$$\delta(Z,A) = \frac{a_p}{A^{1/2}} \quad \leftarrow \sim 12\,\mathrm{MeV}$$

- THIS TERM IS +VE FOR EVEN-EVN AND INCREASES THE BINDING ENERGY IT IS −VE FOR ODD-ODD AND THUS DECREASES THE BINDING ENERGY

SEMI-EMPIRICAL MASS FORMULA FOR NUCLEI

(BETHE - WEIZSÄCKER)



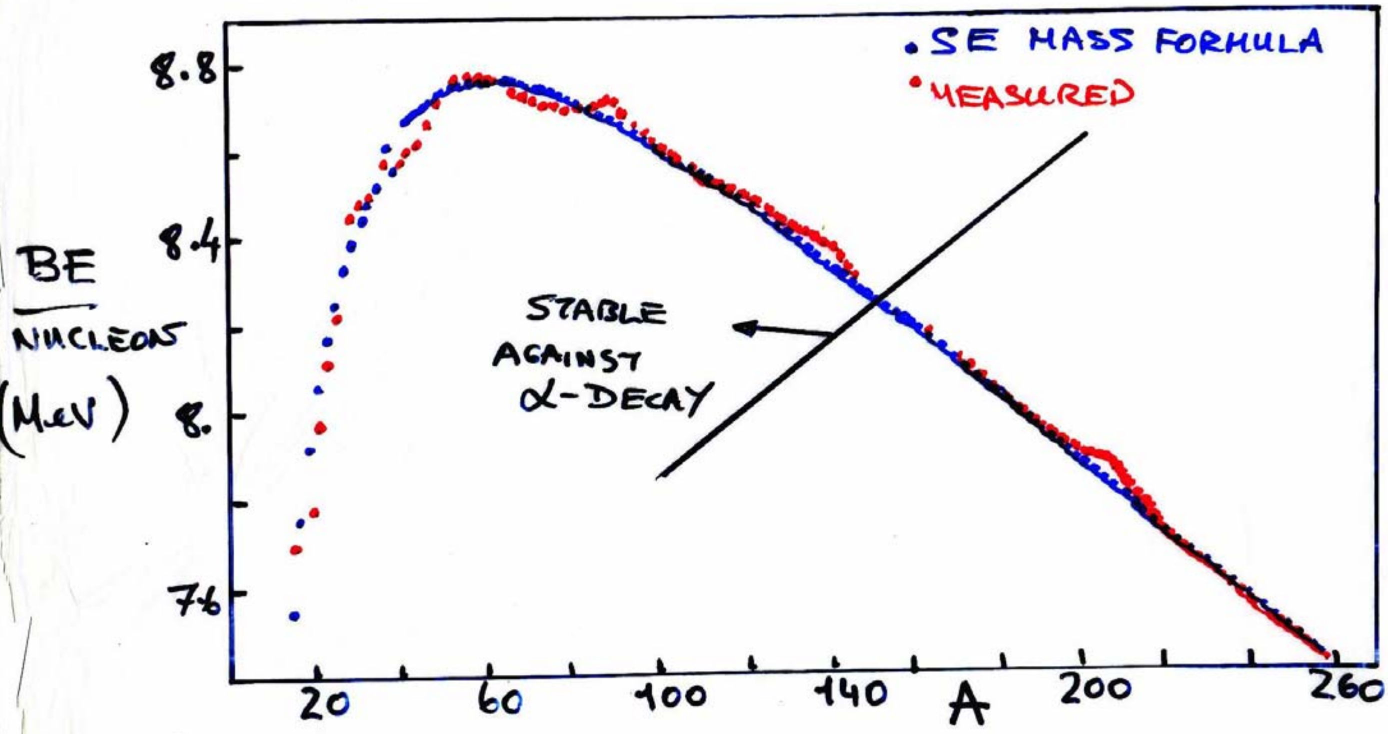
$$B = a_V A - a_S A^{2/3} - \frac{a_C Z^2}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} \pm \delta(Z,A)$$

VOLUME — SURFACE — COULOMB — ASYMMETRY — PAIRING

$a_i =$ 15 MeV, 17, .7, 23, 12

$\underbrace{15\,\text{MeV} \quad 17 \quad .7}$ LIQUID DROP AVERAGE PROPERTIES OF STRONG + COULOMB FORCE

$\underbrace{23 \quad 12}$ QUANTUM EFFECTS



SE MASS FORMULA
MEASURED
STABLE AGAINST α-DECAY
BE / NUCLEON (MeV)
8.8
8.4
8.
7.6
20
60
100
140
200
260
A