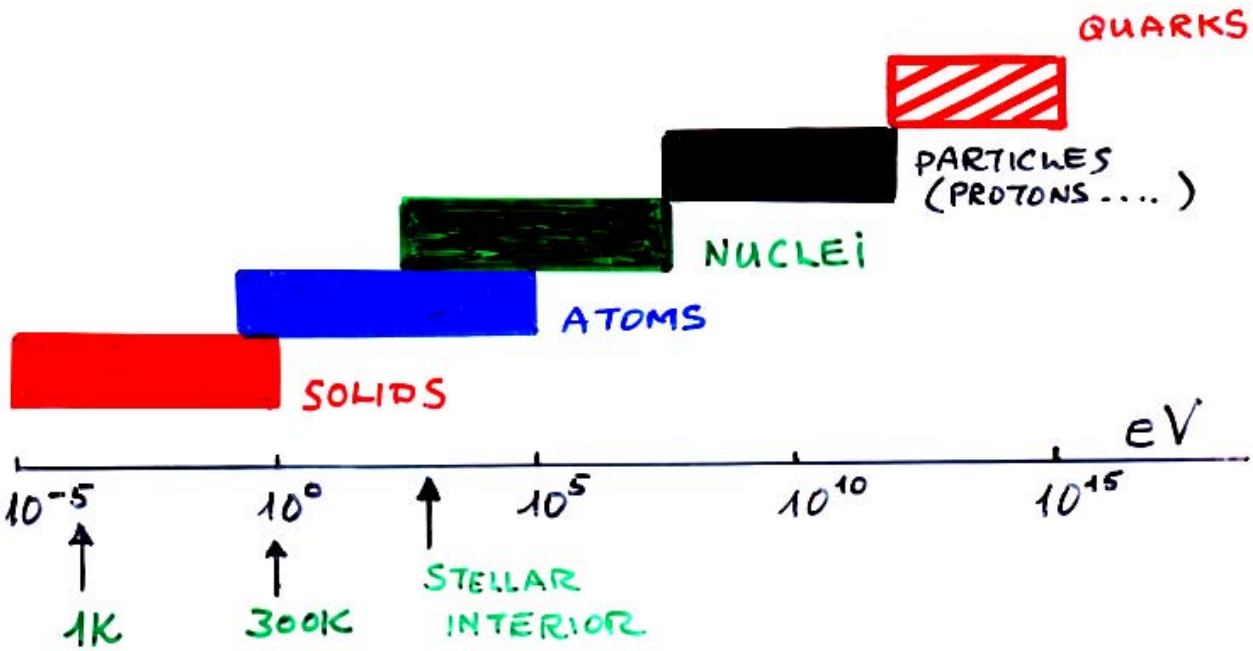
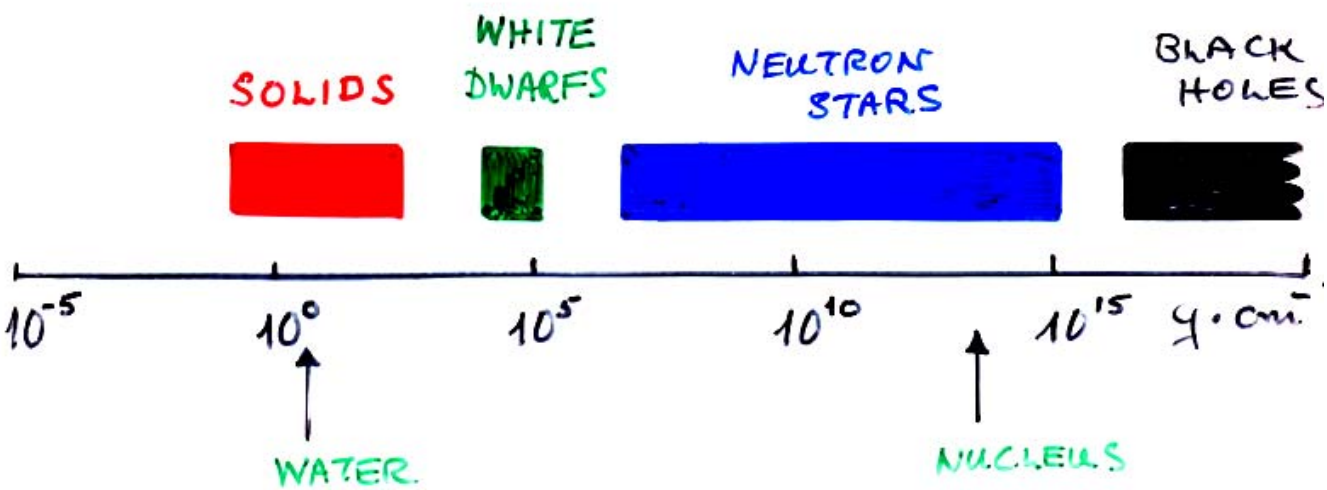
ENERGY DENSITIES



DENSITIES



SPECIAL RELATIVITY

- TO REACH HIGH ENERGY DENSITIES AND SMALL DISTANCES, USE PROBES

$$v \approx c$$

→ RESULTS OF MEASUREMENTS WILL DEPEND ON LORENTZ FRAME

→ ANY SENSIBLE THEORY MUST BE COVARIANT → LORENTZ INVARIANT

→ BASED ON PROPERTIES / QUANTITIES THAT DO NOT DEPEND ON FRAME

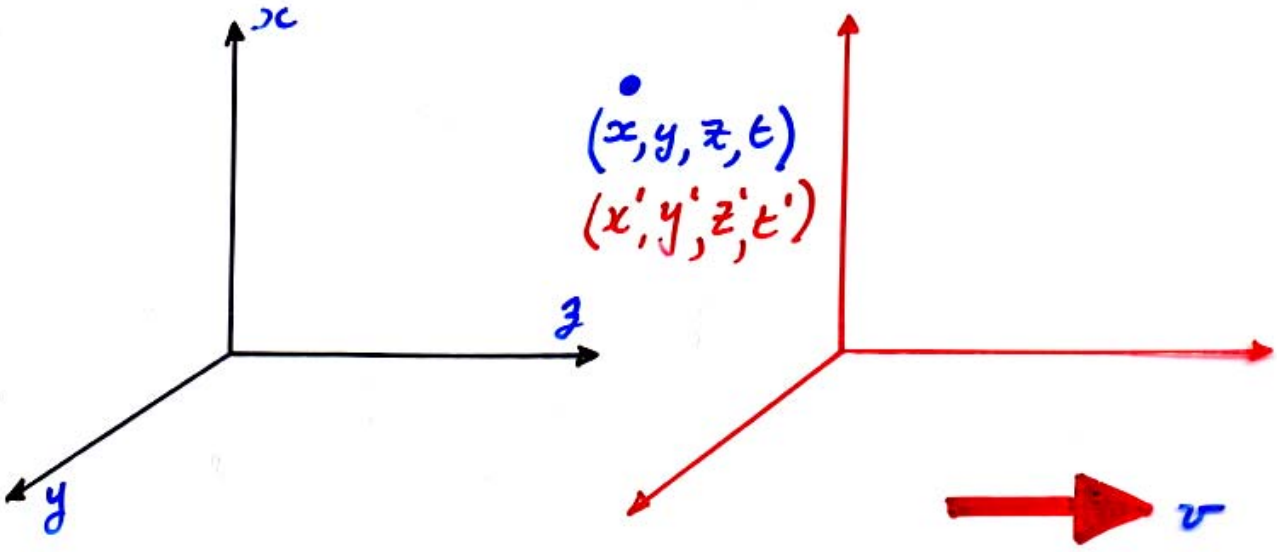
eg: REST MASS
PROPER LIFE TIME

→ FORMULATE IN TERMS OF 4-VECTORS

FIRST LET'S REMEMBER SIMPLE SPECIAL RELATIVITY → THEN MAKE IT A BIT MORE ELEGANT.

SPECIAL RELATIVITY

ELECTRONS, PROTONS, NUCLEI HAVE ELECTRIC CHARGES. MUCH OF WHAT WE TALK ABOUT WILL INVOLVE ELECTROMAGNETIC INTERACTIONS OF PARTICLES AND IT WILL BE RELATIVISTIC



$$x' = x$$

$$y' = y$$

$$z' = \gamma (z - vt)$$

$$t' = \gamma \left(t - \frac{\beta}{c} \cdot z \right)$$

note mixing of z, AND t

$$t' = \gamma\left(t - \frac{\beta}{c}\cdot z\right)$$

γ IS THE LORENTZ BOOST FACTOR

β IS VELOCITY IN UNITS WHERE $c = 1$

$$\gamma = \frac{1}{(1-\beta^2)^{1/2}}$$

$$\beta = v/c$$

$$\bar{p} = m\gamma\bar{v}$$

SO $$v^2 = \frac{p^2}{m^2\gamma^2} = \frac{p^2}{m^2}(1-\beta^2) = \frac{p^2}{m^2}\left(1-\frac{v^2}{c^2}\right)$$

$$v^2 m^2 c^4 = c^4 p^2 - p^2 v^2 c^2$$

$$v^2 \underbrace{(m^2c^4 + p^2c^2)}_{E^2} = c^2 \cdot c^2 p^2$$

$$\frac{v^2}{c^2} E^2 = c^2 p^2$$

$$\beta = \frac{cp}{E}$$

FOUR VECTORS

- 3 DIMENSIONAL VECTOR (x, y, z)

  (p_x, p_y, p_z)

- VECTORS ARE NOT FRAME INVARIANT

- SCALAR PRODUCTS ARE INVARIANT

  $(\text{SPEED})^2 = \bar{v} \cdot \bar{v} \rightarrow$ FRAME INDEP.

- CAN DEFINE 4 DIMENSIONAL VECTOR

  (ct, x, y, z)

  $(ct, \bar{x}) \rightarrow$ SPACE TIME POINT

  $(ct, \bar{x}) \cdot (ct, \bar{x}) \rightarrow$ INVARIANT

ANY THING WHICH TRANSFORMS LIKE

$(ct, \bar{x})$

UNDER A LORENTZ TRANSFORM IS A

4-VECTOR

- 3 DIMENSIONS $\left.\begin{matrix}\bar{x}\\ \bar{p}\end{matrix}\right\}$ VECTORS

- 4 DIMENSIONS $(ct, \bar{x})$

$\left(\frac{E}{c}, \bar{p}\right)$ ENERGY MOMENTUM

$$\left(\frac{E}{c}, \bar{p}\right)\cdot\left(\frac{E}{c}, \bar{p}\right) \leftarrow \text{LORENTZ INVARIANT}$$

WHY?

$$= \frac{E^2}{c^2} - p^2$$

KNOW $E^2 = p^2c^2 + m^2c^4$

$$\frac{E^2}{c^2} - p^2 = m^2c^2$$

$m \rightarrow$ REST MASS

OBVIOUSLY UNITS WHERE $c=1$, CONVENIENT

$$(E, \bar{p})\cdot(E, \bar{p}) = E^2 - p^2 = m^2$$

LORENTZ INVARIANT SCALAR PRODUCT.

4-VECTOR DEFINITIONS

THE PHYSICS IS $E^2 = p^2 + m^2$ [$c=1$]

THIS IS WHAT DEFINES SCALAR PRODUCT MULTIPLICATION RULE

$$\underbrace{(E, \bar{p})(E, \bar{p})}_{m} = E^2 - p^2$$

BETTER NOTATION MATHEMATICALLY

$$A \cdot B = g_{\mu\nu} A^\mu B^\nu = g^{\mu\nu} A_\mu B_\nu$$

ANY 4-VECTORS

$$= A^\mu B_\mu$$

COVARIANT

$$A^\mu = (A^0, \bar{A})$$

$\mu = 0, 1, 2, 3$

CONTRA-VARIANT

$$A_\mu = (A^0, -\bar{A})$$

METRIC

$$g_{\mu\nu} = \begin{pmatrix} +1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & -1 \end{pmatrix}$$

$\Rightarrow$ LORENTZ INVARIANT $\rightarrow$ UPPER or LOWER INDICES BALANCE

$$(E, \bar{p}) = p^\mu = p \quad (\text{SOMETIMES } \vec{p})$$

$$p \cdot p = p^\mu p_\mu = E^2 - p^2 = m^2$$

MINKOWSKI NOTATION

I THINK MINKOWSKI INVENTED ALL THIS AND SOMETIMES (eg PERKINS) YOU WILL SEE HIS NOTATION

3 REAL "SPACE" COMPONENTS

1 IMAGINARY "TIME" COMPONENT

$$p_x \quad p_y \quad p_z \quad E$$

$$p_\mu = 1, 2, 3, 4$$

$$p_1 = p_x, \quad p_2 = p_y, \quad p_3 = p_z, \quad p_4 = iE$$

$$p = (\bar{p}, iE)$$

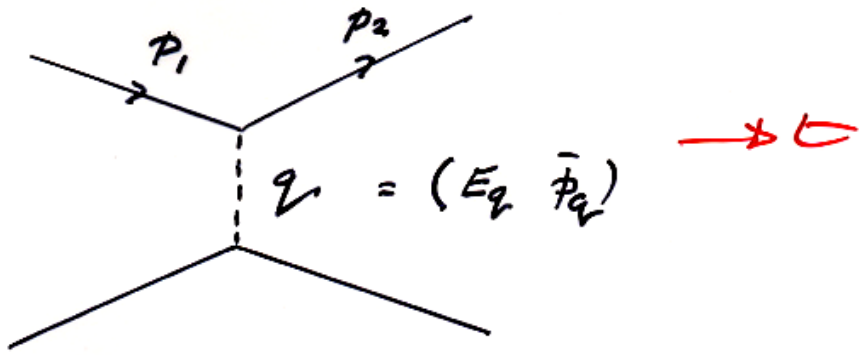
$$p^2 = \sum_\mu p_\mu^2 = p_1^2 + p_2^2 + p_3^2 + p_4^2$$

$$= \bar{p}^2 - E^2 = -m^2$$

$$p^2 = -m^2$$

IT'S JUST A DEFINITION OF p^2. I PERSONALLY DO NOT LIKE IT.

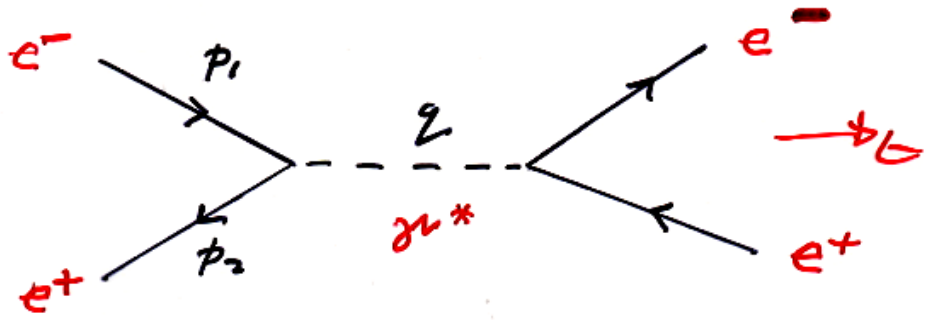
FOR A SCATTERING PROCESS



$$q^2 = (p_1 - p_2)^2 = -ve$$

$$= E_q^2 - \bar{p}_q^2 \rightarrow \text{SPACELIKE}$$

FOR AN ANNIHILATION PROCESS



$$q^2 = (p_1 + p_2)^2 = +ve$$

$$= E_q^2 - p_q^2 = m_{\gamma^*}^2$$

TIMELIKE

SIGNS REVERSED FOR MINKOWSKI

LORENTZ TRANSFORMATION

$$p_\mu' = \sum \alpha_{\mu\nu} p_\nu$$

FOR BOOST ALONG x-AXIS β

MINKOWSKI

$$\begin{pmatrix} p_x' \\ p_y' \\ p_z' \\ iE' \end{pmatrix} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p_x \\ p_y \\ p_z \\ iE \end{pmatrix}$$

"MODERN" METRIC

$$\begin{pmatrix} E' \\ p_x' \\ p_y' \\ p_z' \end{pmatrix} = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$E' = \gamma(E - \beta p_x)$$

$$p_x' = \gamma(p_x - \beta E)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

NB: $c = 1$

UNITS IN SUBATOMIC PHYSICS

- PHYSICS & TECHNOLOGY → SI UNITS USUAL
- RELATIVISTIC PHYSICS cgs / GAUSSIAN
  NATURAL SYSTEM → NO "$4\pi\varepsilon_0$" IN EM.

LENGTH	METER	m
TIME	SECOND	s
ENERGY	ELECTRON VOLT	eV
MASS	"	eV/c^2
MOMENTUM	"	eV/c

$10^6\,eV = MeV$

$10^9\,eV = GeV \approx$ MASS ENERGY OF PROTON

$10^{12}\,eV = TeV \gtrsim$ MASS ENERGY OF HIGGS BOSON

$10^{-15}\,m =$ FEMTOMETER $= 1$ FERMI

$\approx$ DIAMETER OF PROTON

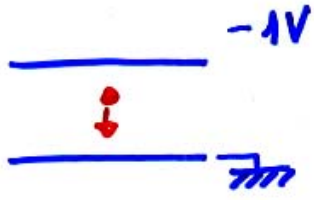
TIME FOR LIGHT SIGNAL TO CROSS PROTON

$$\approx \frac{10^{-15}}{3\times10^8}\,[m]\cdot\frac{[s]}{[m]} \approx 10^{-23}\,s$$

WHY USE ELECTRON VOLTS?

USE OF ELECTRON VOLT VERY CONVENIENT AS ENERGY UNIT. MOST EXPERIMENTAL WORK IN SUBATOMIC PHYSICS IS DONE BY ACCELERATING BEAMS OF PARTICLES IN ELECTROSTATIC FIELDS AND SCATTERING THE BEAM OFF OF A TARGET.

ONE ELECTRON VOLT (eV) IS THE KINETIC ENERGY AN ELECTRON ACQUIRES BY BEING ACCELERATED THRU A POTENTIAL OF ONE VOLT.



$$1\ \text{eV} = e \times 1\ \text{VOLT}$$

$$= 1.60 \times 10^{-19}\ (\text{COULOMB}) \times 1\ \text{VOLT}$$

$$= 1.60 \times 10^{-19}\ \text{JOULES}$$

$$= 1.60 \times 10^{-12}\ \text{ergs}$$

WHY ELECTRON VOLTS FOR MASSES?

WHEN ONE SCATTERS A BEAM OF RELATIVISTIC PARTICLES FROM A TARGET, SOME OF THE KINETIC ENERGY IN THE BEAM CAN APPEAR AS MASS → NEW PARTICLES PRODUCED

SO IT IS CONVENIENT TO MEASURE MASSES IN SAME UNITS YOU USE FOR ENERGIES

$$E^2 = p^2c^2 + m^2c^4$$

TOTAL RELATIVISTIC ENERGY

MOMENTUM

VELOCITY OF LIGHT

MASS SOMETIMES CALLED "REST MASS"

$$E^2 = p^2c^2 + m^2c^4$$

FOR A PARTICLE WITH NO MASS (!)
(PHOTON – γ ; NEUTRINO – ν)

$$E = p \cdot c$$

$$p = \frac{E}{c} = \frac{[eV]}{c}$$

UNIT OF MOMENTUM

A MASSLESS PARTICLE HAVING AN ENERGY OF 1eV, HAS A MOMENTUM OF 1 eV/c

FOR A MASSIVE PARTICLE AT REST
(IN ITS REST FRAME)

$$E = m\,c^2$$

TOTAL RELATIVISTIC ENERGY

MASS

$$m = \frac{E}{c^2} = \frac{[eV]}{c^2}$$

UNIT OF MASS

IF A PARTICLE HAS A MASS OF 1eV/c², WHEN AT REST IT HAS A TOTAL RELATIVISTIC ENERGY OF 1eV

$$E^2 = p^2c^2 + m^2c^4 \qquad (1)$$

SUBATOMIC PHYSICISTS USE UNITS WHERE $c=1$

$$E^2 = p^2 + m^2$$

(→ SOMETIMES CONFUSING. I WILL USE 'c' EXPLICITLY.) SHOULD BE COMFORTABLE WITH EITHER

EACH TERM OF (1) HAS DIMENSIONS $[E]^2$

EXAMPLE, FOR A PARTICLE

$$\text{MASS} = 1000 \ \frac{\text{MeV}}{c^2}$$

$$\text{MOMENTUM} = 1000 \ \frac{\text{MeV}}{c}$$

$$E^2 = 10^6 \ \frac{\text{MeV}^2}{c^2} \cdot c^2 + 10^6 \ \frac{\text{MeV}^2}{c^4} \cdot c^4$$

SO $$E^2 = 2 \cdot 10^6 \ \text{MeV}^2$$

THE PARTICLE HAS A TOTAL RELATIVISTIC ENERGY,

$$E = \sqrt{2} \cdot 10^3 \ \text{MeV}$$

FROM PERKINS

Table 1.1. ***Units in high energy physics***

(a)

Quantity	High energy unit	Value in SI units
length	1 fm	10^{-15} m
energy	1 GeV $= 10^9$ eV	1.602×10^{-10} J
mass, E/c^2	1 GeV/c^2	1.78×10^{-27} kg
$\hbar = h/(2\pi)$	6.588×10^{-25} GeV s	1.055×10^{-34} J s
c	2.998×10^{23} fm s^{-1}	2.998×10^8 m s^{-1}
$\hbar c$	0.1975 GeV fm	3.162×10^{-26} J m

~ PROTON MASS

(b)

natural units, $\hbar = c = 1$	
mass, Mc^2/c^2	**1 GeV**
length, $\hbar c/(Mc^2)$	**1 GeV^{-1}** $= 0.1975$ fm
time, $\hbar c/(Mc^3)$	**1 GeV^{-1}** $= 6.59 \times 10^{-25}$ s
Heaviside–Lorentz units, $\epsilon_0 = \mu_0 = \hbar = c = 1$	
fine structure constant	$\alpha = e^2/(4\pi) = 1/137.06$
Relations between energy units	
1 MeV $= 10^6$ eV	**1 GeV $= 10^3$ MeV** 1 TeV $= 10^3$ GeV

HIGH ENERGY UNITS

CHOOSE

$\hbar c = 1 = 0.1975$ GeV. fm

$\hbar = 1 = 6.59 \times 10^{-25}$ GeV. s

RANDOMNESS OF DECAYS

A PIECE OF RADIOACTIVE MATERIAL CONTAINS A GROUP OF NUCLEI, ANY ONE OF WHICH MAY DECAY AT RANDOM IN ANY SMALL TIME INTERVAL

THO' EACH ONE DECAYS AT RANDOM, THE POPULATION IS CHARACTERIZED BY A

MEAN LIFETIME

THE NUMBER OF PARTICLES DECAYING IN TIME INTERVAL dt, IS:

$$dN = -\omega N(t)\, dt$$

DECAYING

PROBABILITY PER UNIT TIME FOR DECAY

UNDECAYED AT START OF TIME INTERVAL

TIME INTERVAL

$$\frac{dN}{N(t)} = -\omega\, dt$$

DECAY CONSTANT
TRANSITION RATE

$$dN = -\omega N(t)\,dt$$

$$\frac{dN}{N} = -\omega\,dt$$

$$\int_{N(0)}^{N(t)} \frac{dN}{N} = -\omega \int_0^t dt$$

$$\ln N(t) - \ln N(0) = -\omega t$$

$$N(t) = N(0)\,e^{-\omega t}$$

SURVIVAL EQUATION

INTENSITY OF RADIATION = ACTIVITY

$$I(t) = \frac{-dN(t)}{dt} = \underbrace{N(0)\,\omega}\,e^{-\omega t}$$

$$I(t) = I(0)\,e^{-\omega t}$$

INTENSITY OF EMITTED RADIATION

INITIAL ACTIVITY OR INTENSITY

UNITS OF RADIOACTIVITY

CURIE (Ci) AMOUNT OF RADIOACTIVE MATERIAL IN WHICH NUMBER OF DISINTEGRATIONS PER SECOND = 1 g OF RADIUM

$3.7 \times 10^{10}\ s^{-1}$

BECQUEREL (Bq) ONE DISINTEGRATION PER SECOND

STARTED WHOLE SUBJECT BY DISCOVERING RADIOACTIVITY IN 1896

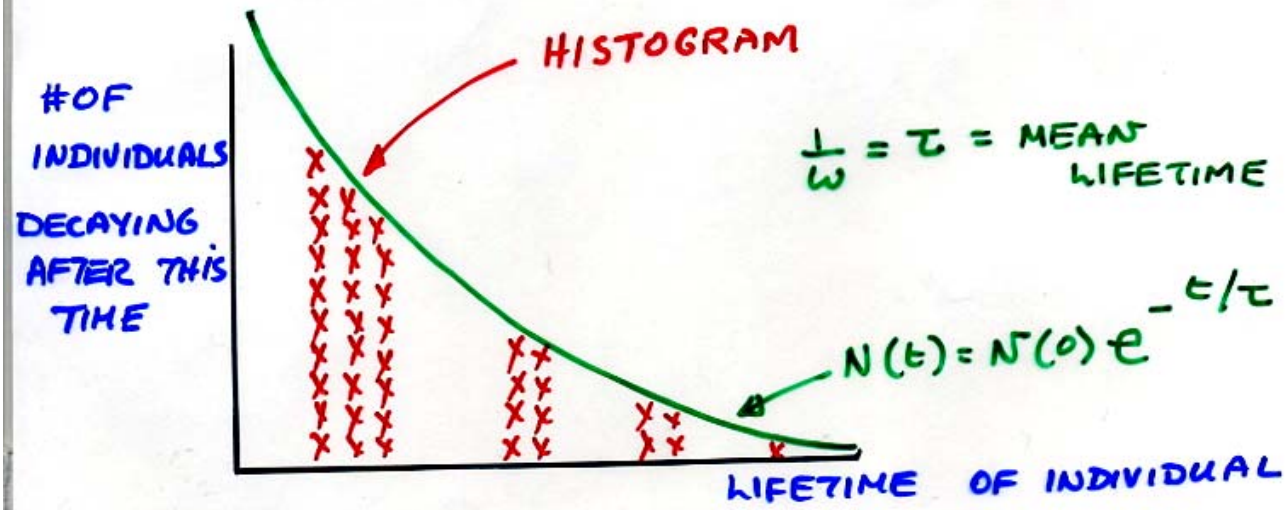
? HOW "MUCH" RADIOACTIVITY DID CHERNOBYL RELEASE?

1000'S OF CURIES !

MEAN LIFETIME

$$N(t) = N(0)\, e^{-\omega t}$$

ALTHOUGH DECAY TIME OF ANY INDIVIDUAL IS RANDOM — POPULATION CHARACTERIZED BY MEAN LIFETIME



ENORMOUS RANGE OF LIFETIMES

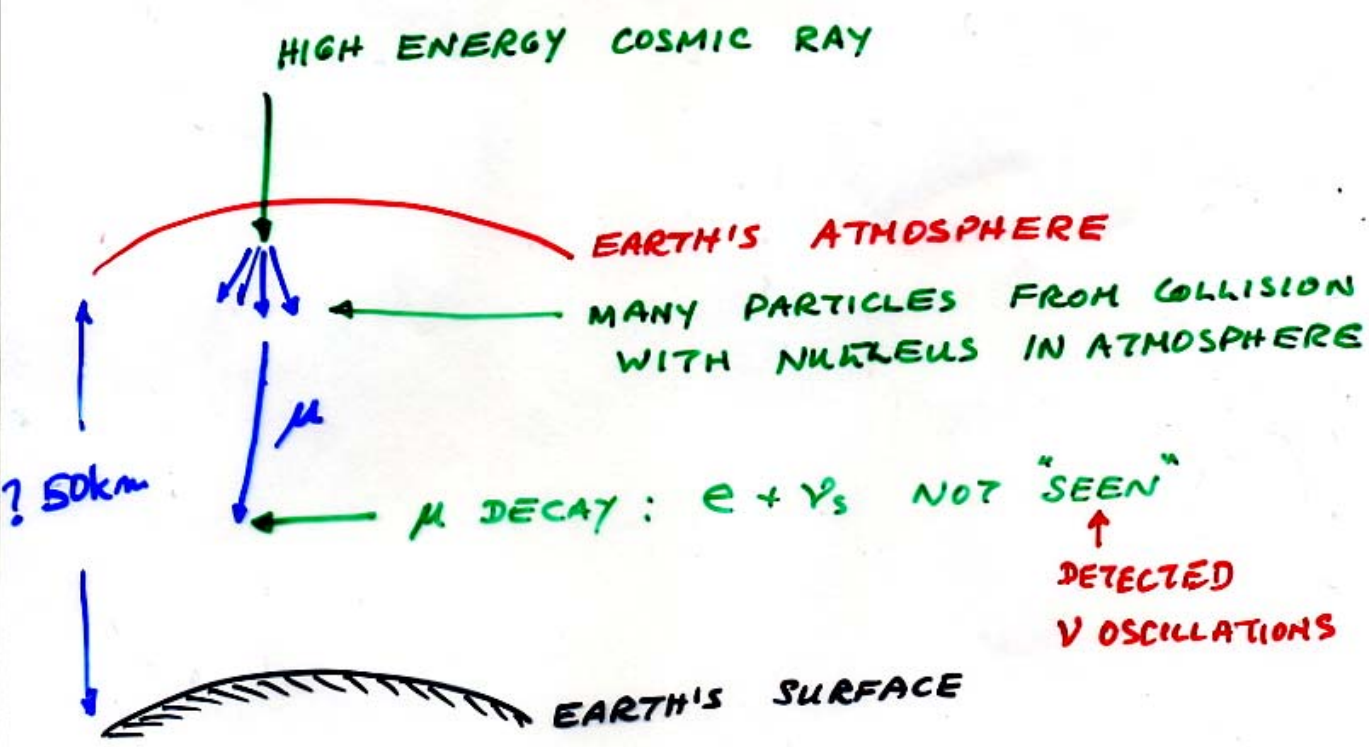
PROTON DECAY	$> 10^{30}$ YEARS
$^{238}U \rightarrow {}^{234}Th + \alpha$	6.5×10^{9} YEARS
$^{215}Po \rightarrow {}^{211}Pb + \alpha$	2.0×10^{3} s
$\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$	2.2×10^{-6} s
$\pi^0 \rightarrow \gamma\gamma$	8.3×10^{-17} s
$\Delta^{++} \rightarrow p\pi^+$	6×10^{-24} s

DECAY RATE GOVERNED BY FORCE (INTERACTION) CAUSING DECAY PROCESS

SPECIAL RELATIVITY & LIFETIME

LIFETIME ONLY MAKES SENSE IN SPECIAL FRAME — REST FRAME OF PARTICLE

EXAMPLE $\mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$; $\tau = 2.2 \times 10^{-6}$ s



ASSUME μ TRAVELLING AT SPEED OF LIGHT.

MEAN DISTANCE TO DECAY = $c \cdot \tau_\mu$

$$= 3 \times 10^8 \times 2 \times 10^{-6} \quad \frac{[m].[s]}{[s]}$$

$$\approx 600 \text{ m.}$$

TIME TO REACH EARTH'S SURFACE AT 50km

$$t_{50} = \frac{50\times10^3}{3\times10^8} = \frac{50}{3}\times10^{-5} \simeq 2\times10^{-4}\,s$$

HOW MANY μ ARE UNDECAYED AFTER THIS TIME ?

$$\text{FRACTION SURVIVING} = \frac{N(t_{50})}{N(0)} = e^{-t_{50}/\tau_{LAB}}$$

$$= \exp-\left\{\frac{2\times10^{-4}}{2\times10^{-6}}\right\} \sim 10^{-44} \sim 0\ !$$

WHY DO WE SEE ANY μ AT SEA LEVEL?
OBVIOUSLY THIS CALCULATION IS INCORRECT.

$$\tau_{LAB} \neq \tau_\mu$$

SAY $\tau = t_2^{REST} - t_1^{REST}$ DEFINES τ IN REST FRAME

SO: $\tau_{LAB} = \gamma\, t_2^{LAB} - \gamma\, t_1^{LAB}$

$$\tau_{LAB} = \gamma\,\tau_\mu$$

THAT'S BETTER!

NOW LET'S HYPOTHESIZE THAT THE COSMIC RAY μ HAVE AN ENERGY OF 100 GeV

$$E = \frac{pc}{\beta} \quad \text{AND} \quad \gamma^2 = \frac{1}{1-\beta^2}$$

$$\gamma^2 = \left(1 - \frac{p^2c^2}{E^2}\right)^{-1} = \frac{E^2}{\underbrace{E^2 - p^2c^2}_{m^2c^4}}$$

$$\gamma = \frac{E}{mc^2}$$ ← WHICH ANY KID KNOWS!

MASS OF μ IS $m_\mu = 106\ \text{MeV}/c^2$

$$\gamma = \frac{10^5\,[\text{MeV}]}{10^2\,\frac{[\text{MeV}]}{[c^2]}\cdot c^2} \approx 10^3$$

$$\tau_{LAB} = 10^3 \times 2.2 \times 10^{-6} = 2.2 \times 10^{-3}$$

FRACTION OF μ LEFT AT SEA LEVEL

$$= \exp - \left\{\frac{t_{50}}{\tau_{LAB}}\right\} = \exp - \left\{\frac{2\times 10^{-4}}{2\times 10^{-3}}\right\} = \exp - \left\{10^{-1}\right\}$$

FRACTION UNDECAYED AT SEA LEVEL

$$\sim 90\%$$