SCATTERING EXPERIMENTS & STRUCTURE OF MATTER

OUR METHODOLOGY FOR INVESTIGATING SMALLER & SMALLER DISTANCE SCALES IS EPITOMISED BY RUTHERFORD SCATTERING – GEIGER & MARSDEN

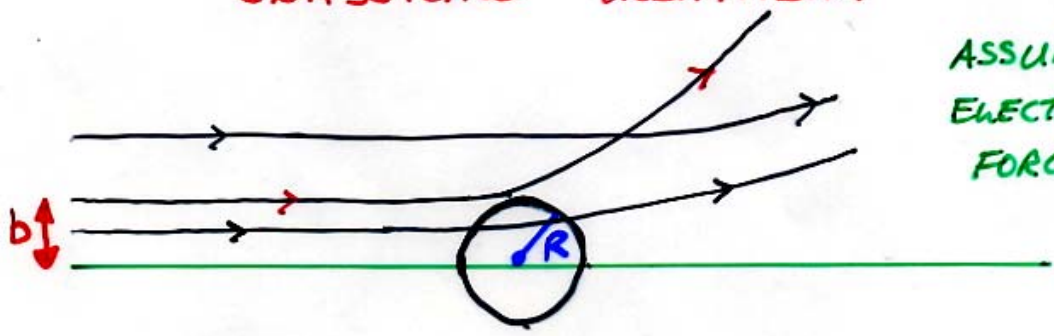
YOU MUST HAVE ALREADY SEEN THIS. HERE WE JUST REMEMBER WHAT IT IS ABOUT.

WILL COME BACK TO A QUANTUM MECHANICAL TREATMENT IN A BIT.

- ARCHETYPAL EXPERIMENT
- PROBES STRUCTURE OF TARGET
- REVEALS NATURE OF FORCE ACTING BETWEEN BEAM PARTICLE & TARGET.

RUTHERFORD SCATTERING – SIMPLE PICTURE

CLASSICAL TREATMENT



ASSUME ELECTROSTATIC FORCE

GEIGER & MARSDEN FIRED 7.7 MeV α PARTICLES AT A THIN GOLD FOIL

WANTED TO UNDERSTAND WHETHER DISTRIBUTION OF +VE CHARGE IN NUCLEUS EXTENDED BUT TO "ATOMIC SIZE"

GREATEST DEFLECTION WHEN INCOMING PROJECTILE JUST GRAZES PERIPHERY OF NUCLEUS

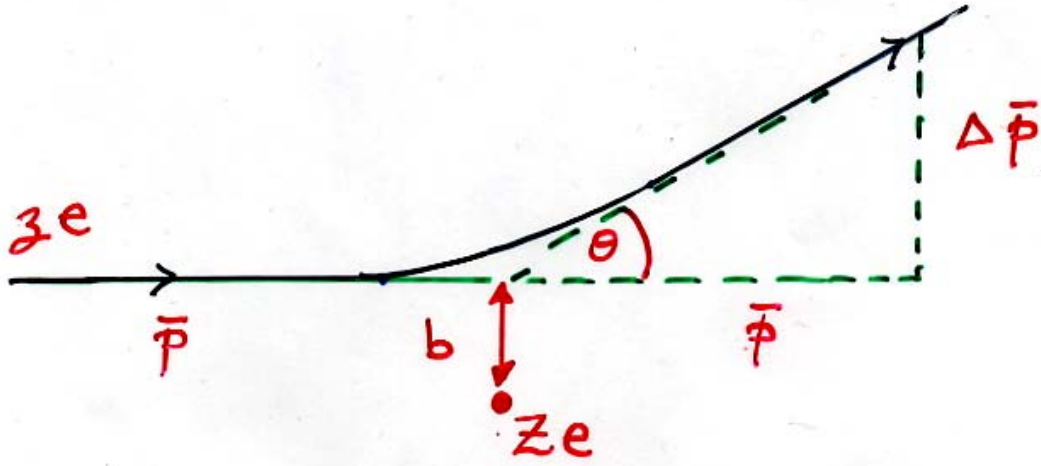
FOR $b > R$ ELECTROSTATIC REPULSION FALLS OFF

FOR $b < R$ α PASSES THROUGH CHARGE DISTRIBUTION. SOME CHARGE REPELS α ONE WAY, SOME THE OTHER

? WHAT IS b CORRESPONDING TO MAXIMUM DEFLECTION

↳ i.e. WHAT IS SIZE OF NUCLEUS

IDEALIZED MODEL FOR SIMPLE CALCULATION



- ASSUME THAT θ IS A SMALL ANGLE

ALONG ORIGINAL LINE OF FLIGHT α-PARTICLE IS SLOWED DOWN BY COULOMB REPULSION AFTER IT IS SCATTERED IT IS ACCELERATED BY SAME AMOUNT

→ THIS ASSUMES $M_{NUCLEUS} \gg m_\alpha$

ELASTIC SCATTERING

APPROXIMATE (WE'LL DO IT BETTER, LATER!) ALL DEFLECTION AS AN IMPULSE, TRANSVERSE TO ORIGINAL MOTION, AS α PASSES BY THE NUCLEUS

$$\text{POTENTIAL} = \frac{zZe^2}{b}$$

ASSUME ONLY COULOMB FORCE ACTING

FORCE ACTING UPON α WHEN IT IS b AWAY FROM NUCLEUS IS :-

$$F = \frac{z Z e^2}{b^2}$$

ANOTHER APPROXIMATION, ASSUME FORCE ACTS FOR DISTANCE b ON EITHER SIDE OF NUCLEUS, AND IS ALWAYS NORMAL TO ORIGINAL DIRECTION OF THE α

TIME FORCE ACTS FOR ; $\Delta t = \frac{2b}{v}$ ← VELOCITY

$$\frac{\Delta p}{\Delta t} = F \; ; \qquad \Delta p = F \cdot \Delta t$$

↑ DEFINITION OF A FORCE

$$\Delta p = \frac{Z z e^2}{b^2} \cdot \frac{2b}{v}$$

APPROXIMATELY, THE DEFLECTION (SCATTERING) ANGLE, IS GIVEN BY :-

$$\sin\theta \approx \theta \approx \frac{\Delta p}{p}$$

NON RELATIVISTIC

$$\theta \approx \frac{Z z e^2}{b^2} \cdot \frac{2b}{v} \cdot \frac{1}{mv}$$

$$\theta \approx \frac{Z z e^2}{b} \cdot \frac{1}{m v^2}$$

"CORRECT" TREATMENT

$$b \approx \frac{z Z e^2}{m v^2 \theta} \cdot 2$$

$$\frac{z Z e^2}{2E} \cdot \cot \frac{\theta}{2}$$

GEIGER & MARSDEN OBSERVED MAXIMUM DEFLECTIONS OF ~1 RADIAN (NOT SMALL, BUT ---)

PUT $b = R$ ← RADIUS OF NUCLEUS

$\theta = 1$ RADIAN

$$R = 2 \cdot \frac{z Z e^2}{m v^2}$$

$$= \frac{2 \cdot z Z e^2}{2E}$$

NON RELATIVISTIC

$$R = \frac{z Z e^2}{E}$$

$$R = \frac{z Z e^2}{E}$$

FOR α on GOLD FOIL

$$z = 2$$
$$Z = 79$$

and $E = 7.7$ MeV ← NOTE THIS IS NONRELATIVISTIC

$$R = \frac{2 \times 79 \times e^2}{7.7 \text{ MeV}}$$

FORTUNATELY WE DO NOT HAVE TO FOOL AROUND WITH e^2 IN COULOMBS OR WHAT EVER

"FINE STRUCTURE CONSTANT" $\alpha = \frac{e^2}{\hbar c} = 1/137$

$$e^2 = \frac{\hbar c}{137}$$ ← $\hbar c = 197.3$ MeV. fm.

SO

$$e^2 = \frac{197.3}{137} \text{ MeV} \cdot \text{fm}$$

$$= 1.44 \text{ MeV} \cdot \text{fm}$$

$$R = \frac{2 \times 79}{7.7} \times 1.44 \frac{[\text{MeV}]}{[\text{MeV}]} \cdot [\text{m}] \times 10^{-15}$$

$$R \sim 3 \times 10^{-14} \text{ m}$$

SEEING THESE LARGE SCATTERING ANGLES DEMONSTRATES THAT THE NUCLEAR RADIUS IS 10^4 TIMES SMALLER THAN THE BOHR RADIUS OF AN ATOM

MACROSCOPIC PHENOMENA REVEAL THE MICROCOSM.

IF YOU THINK THIS IS AN APPROXIMATE CALCULATION → ACTUALLY RUTHERFORD DID A <u>MUCH</u> SIMPLER ONE!

QUANTUM FORCES & FEYNMAN DIAGRAMS

IN OUR SIMPLE MODEL OF RUTHERFORD SCATTERING, WE USED THE CLASSICAL IDEA OF A FORCE.

WHAT IS A FORCE? IT'S SOMETHING WHICH CHANGES THE MOMENTUM OF A PARTICLE. NEWTON INVENTED THE IDEA

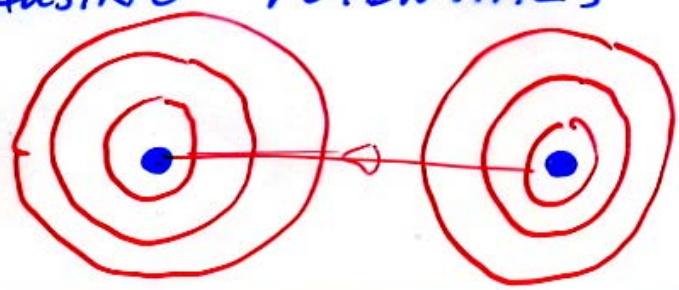
$$\text{FORCE} = \text{MASS} \times \text{ACCELERATION}$$

$$= m \cdot \frac{\Delta v}{\Delta t} = \frac{\Delta p}{\Delta t}$$

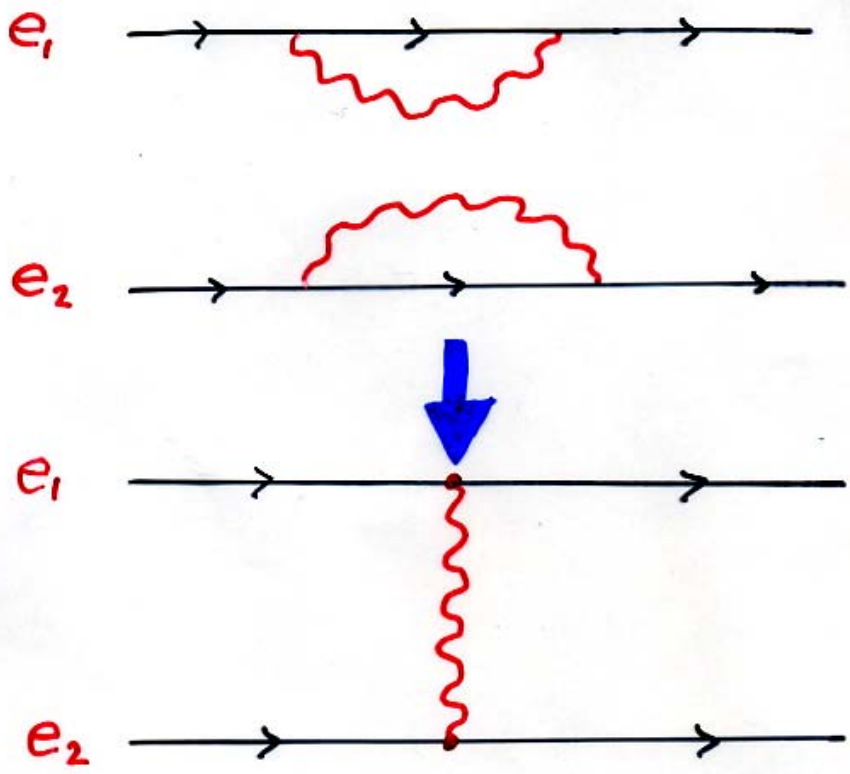
WHAT ONE MEASURES PHYSICALLY IS A MOMENTUM CHANGE.

ONE CAN HAVE DIFFERENT ABSTRACT MODELS OF WHAT THE FORCE IS THAT "CAUSES" THE MOMENTUM CHANGE

CLASSICALLY WE THINK OF FIELDS CAUSING POTENTIALS



INTERACTION – FORCE BETWEEN PARTICLES



4 MOMENTUM TRANSFER BETWEEN e_1 & e_2

- NOTE THAT THE 4-MOMENTUM TRANSFER, TRANSFERS BOTH ENERGY AND MOMENTUM
- THE TWO ELECTRONS IN INITIAL & FINAL STATE ENSURE THAT ENERGY & MOMENTUM ARE CONSERVED

THEY REMAIN "ON MASS SHELL"

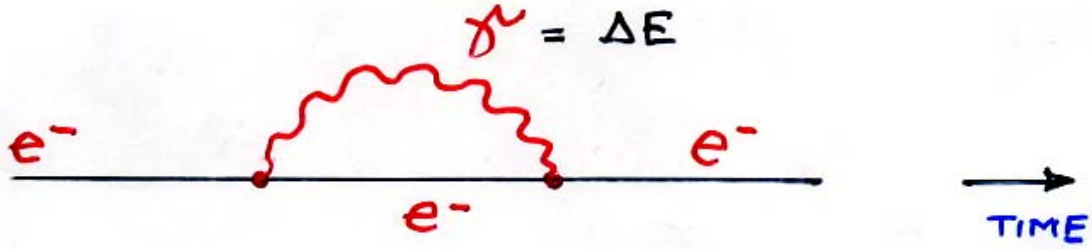
$$E^2 = p^2 + m^2$$

$$(E^2 = p^2c^2 + m^2c^4)$$

IN RELATIVISTIC QUANTUM MECHANICS, WE THINK OF FORCES BEING CAUSED BY THE EXCHANGE OF VIRTUAL PARTICLES WHICH ARE GAUGE BOSONS

? HOW CAN EXCHANGING A PARTICLE BE A "FORCE" ANSWER THIS ON TWO LEVELS

SIMPLE ANSWER IS THAT IT CAUSES A MOMENTUM CHANGE. THAT IS WHAT WE HAVE DEFINED A FORCE TO BE.

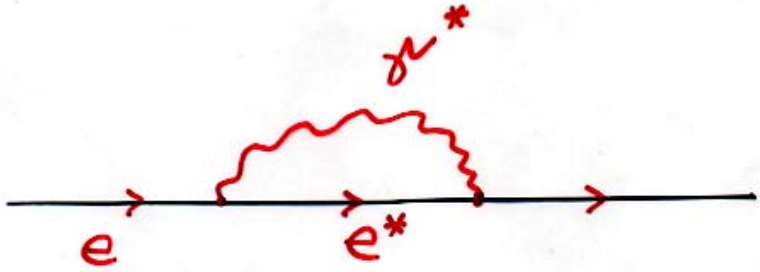


ACCORDING TO HEISENBERG'S UNCERTAINTY PRINCIPLE. A FREELY PROPAGATING ELECTRON CAN EMIT + RE-ABSORB A PHOTON.

NOT TRUE; BECAUSE OFF MASS SHELL

~~SINCE THE EMISSION VIOLATES ENERGY AND MOMENTUM CONSERVATION~~, THE RE-ABSORPTION HAS TO OCCUR WITHIN A TIME

$$\Delta t \lesssim \frac{\hbar}{\Delta E}$$ HEISENBERG



THE γ WILL CARRY 4-MOMENTUM AWAY FROM THE ELECTRON

IF $$E_e^2 = p_e^2 c^2 + m_e^2 c^4$$

$$m_e^2 c^4 = E_e^2 - p_e^2 c^2$$

BEFORE THE EMISSION OF THE γ, THEN

$$m_e^2 c^4 \neq E_e'^2 - p_e'^2 c^2 = m_e^{*2} c^4$$

AFTER THE EMISSION

FOR A REAL γ

MASSLESS

$$E_\gamma^2 - p_\gamma^2 c^2 = 0$$

FOR VIRTUAL γ

$$E_\gamma'^2 - p_\gamma'^2 c^2 = m_\gamma^{*2} c^4$$

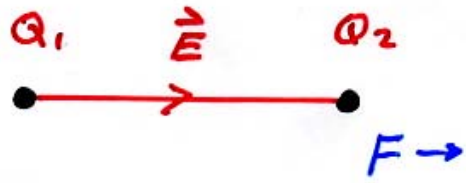
BOTH γ AND e WILL HAVE NONPHYSICAL MASSES FOR SHORT TIME ΔE $\rightarrow$ "OFF MASS SHELL"

VIRTUAL PARTICLES $\Big\}$ TRANSFER MOMENTUM

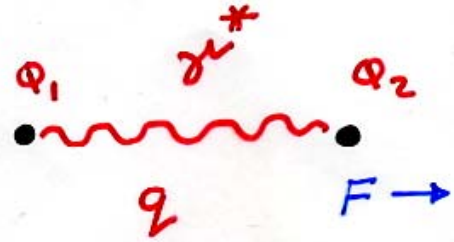
$\downarrow$

TRANSMIT FORCES

CLASSICAL ⟷ QUANTUM ANALOGY



CLASSICAL FIELD



EXCHANGE OF VIRTUAL QUANTUM.

SINCE $\Delta E \Delta t \lesssim \hbar$

PROPAGATION TIME OF VIRTUAL PARTICLE IS:

$$t \sim \Delta t \lesssim \frac{\hbar}{mc^2}$$

$\sim \Delta E$

"MASS" OF VIRTUAL PARTICLE

ASSUME VIRTUAL PARTICLE PROPAGATES AT VELOCITY OF LIGHT

RANGE OF FORCE $R = c \cdot \Delta t \lesssim \frac{\hbar}{mc}$

FOR $m \rightarrow 0$; $R \rightarrow \infty$ COULOMB $m_\gamma = 0$

FOR $M \rightarrow$ LARGE ; $R \rightarrow$ SMALL

THE MASS OF THE EXCHANGED VIRTUAL PARTICLE, DETERMINES THE RANGE OF THE FORCE

CLASSICALLY

$$\vec{F}_2 = \vec{E}(\vec{r}) \cdot Q_2 = \hat{r} \, \frac{Q_1 Q_2}{r^2}$$

QUANTUM MECHANICALLY, FORCE IS DUE TO ABSORPTION OF γ WITH MOMENTUM q

$$\Delta p \cdot \Delta x \sim \hbar$$

$$q \cdot r \sim \hbar$$

THE TRANSFER OF THIS QUANTUM OF MOMENTUM TAKES A TIME r/c

$$\frac{dq}{dt} \sim \frac{\hbar}{r} \cdot \frac{c}{r} = \frac{\hbar c}{r^2} \Rightarrow \text{FORCE}$$

THIS "GIVES" THE "CORRECT" $\frac{1}{r^2}$ BEHAVIOUR

- NO OF γ EMITTED/ABSORBED $\sim Q_1 Q_2$

$$F \sim \frac{Q_1 Q_2}{r^2}$$

VIRTUAL PARTICLE	CLASSICAL FIELD
QUANTUM FIELD	SPOOKY ACTION AT A DISTANCE

EQUALLY ABSTRACT (CONCRETE?) CONCEPTS

FEYNMAN DIAGRAMS

- FEYNMAN DIAGRAMS ARE GRAPHICAL REPRESENTATIONS OF:

  QUANTUM MECHANICAL AMPLITUDES

PROBABILITY OF PROCESS $\propto (\text{AMP})^2$

- YOU KNOW THAT IF A PROCESS CAN OCCUR IN 2 DIFFERENT WAYS

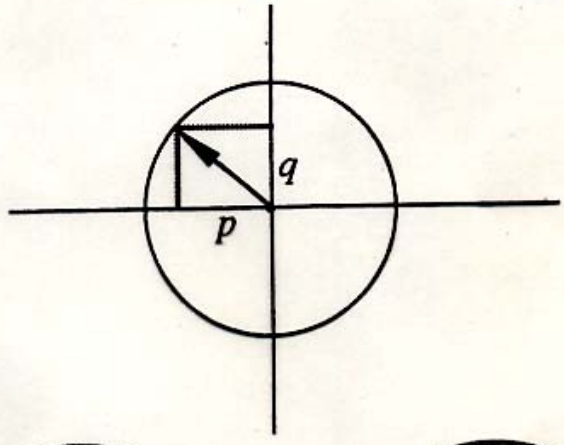
$$\text{PROB} \propto (A_1 + A_2)(A_1 + A_2)$$

A_1 A_2 → COMPLEX NUMBERS

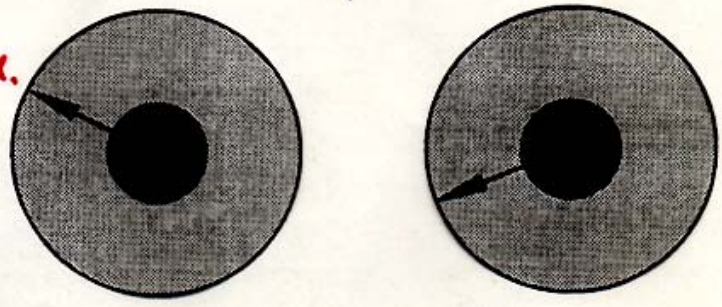
∴ INTERFERENCE

cf → TWO SLIT EXPERIMENT WITH ELECTRONS

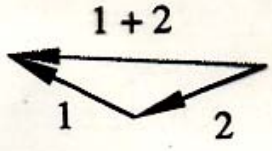
Q.M. AMPLITUDE $\Longleftrightarrow$ SPACE TIME DIAGRAM



AMPLITUE 1.
PHASE 1.



AMPLITUDE 2
PHASE 2



ADDITION OF AMPLITUDES

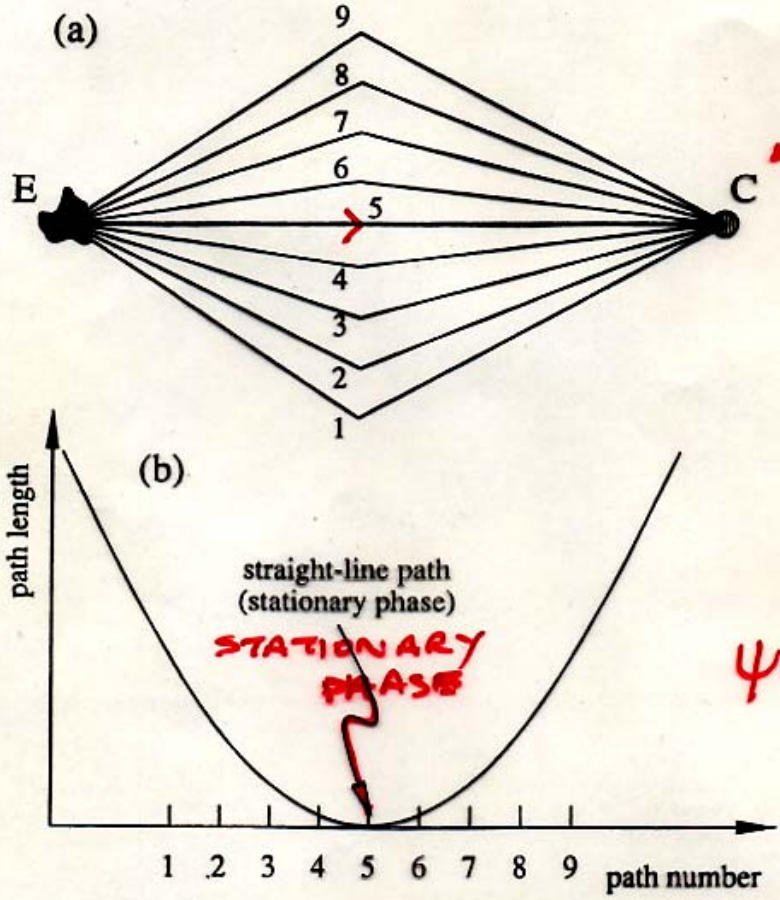
FREE PARTICLE $\psi = A e^{-i(Et - px)/\hbar}$

{PLANE WAVE STATE}

PHASE VARIES ALONG PATH

FREE PARTICLE $E \to C$

POSITIONS IN SPACE !



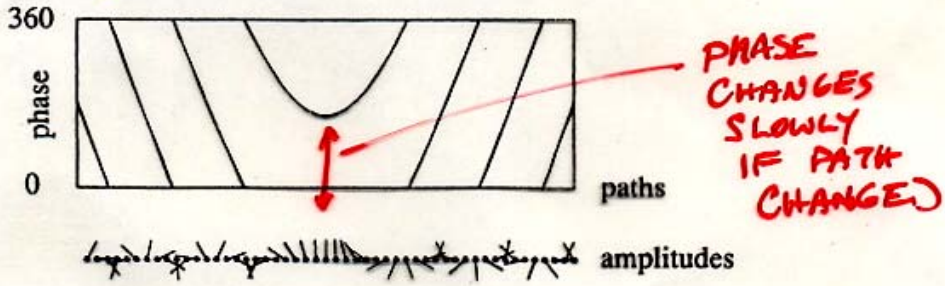
$$\psi \sim e^{ipx/\hbar}$$

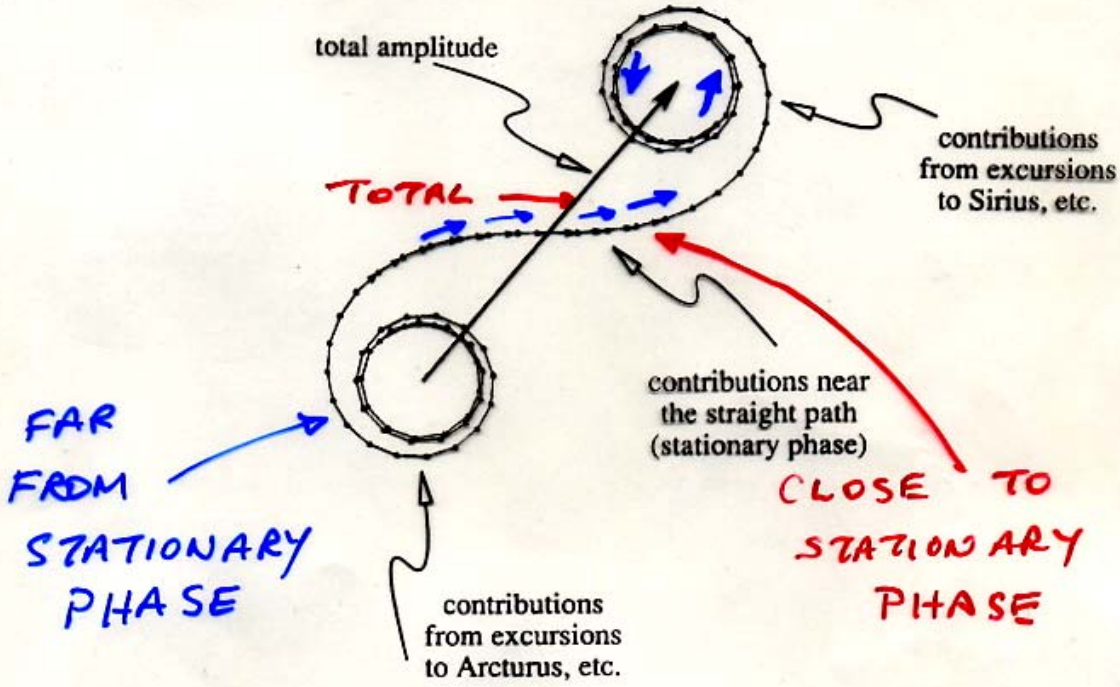
PHASES FAR FROM STRAIGHT LINE INTERFERE DESTRUCTIVELY

PHASES CLOSE TO STRAIGHT LINE INTERFERE CONSTRUCTIVELY

ADD AMPLITUDES FROM ALL POSSIBLE PATHS

SQUARE SUM

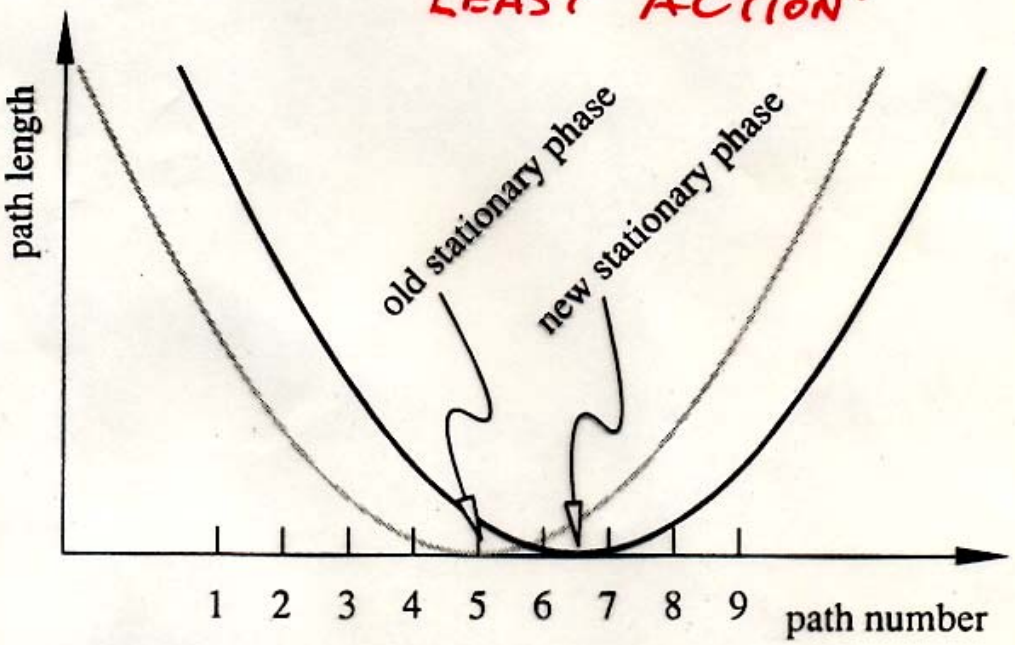




"FORCE OF SYMMETRY"

VINCENT ICKE

PARTICLE FOLLOWS PATH OF LEAST ACTION

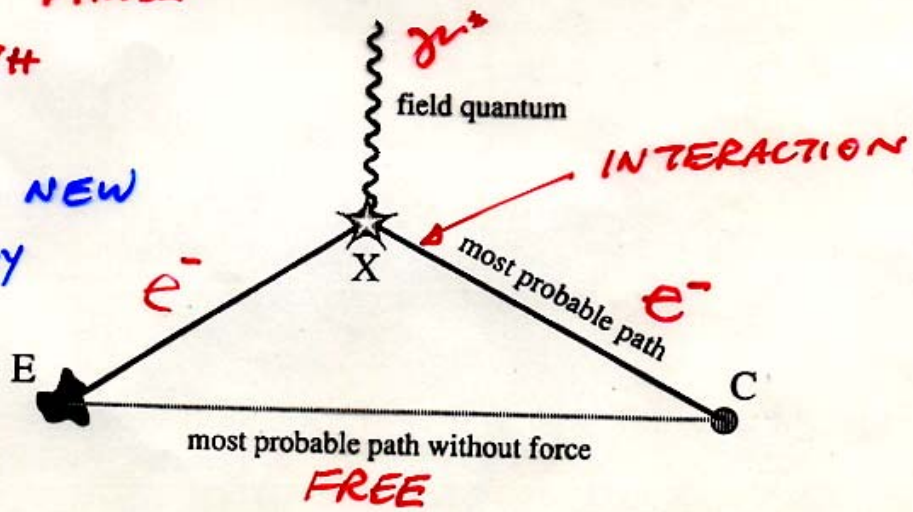


$$\text{PHASE} = \frac{\text{ACTION}}{\hbar}$$

ABSORBING γ^* CHANGES PHASE OF PATH

↓

BECOMES NEW STATIONARY PATH



- IN QM

$$\delta(\text{PHASE}) = \frac{\delta(\text{ACTION ALONG PATH})}{\hbar \ (=1)}$$

$$\delta\phi \propto \delta S$$

$$\delta\phi = F\,\delta S$$

PHASE CHANGE IN STEP

FIELD DENSITY = "PROPORTIONALITY CONSTANT"

ACTION CHANGE IN A STEP ALONG PATH

$$P_{E\to C} = \left[\sum_{\text{PATH}} \delta\phi\right]^2$$

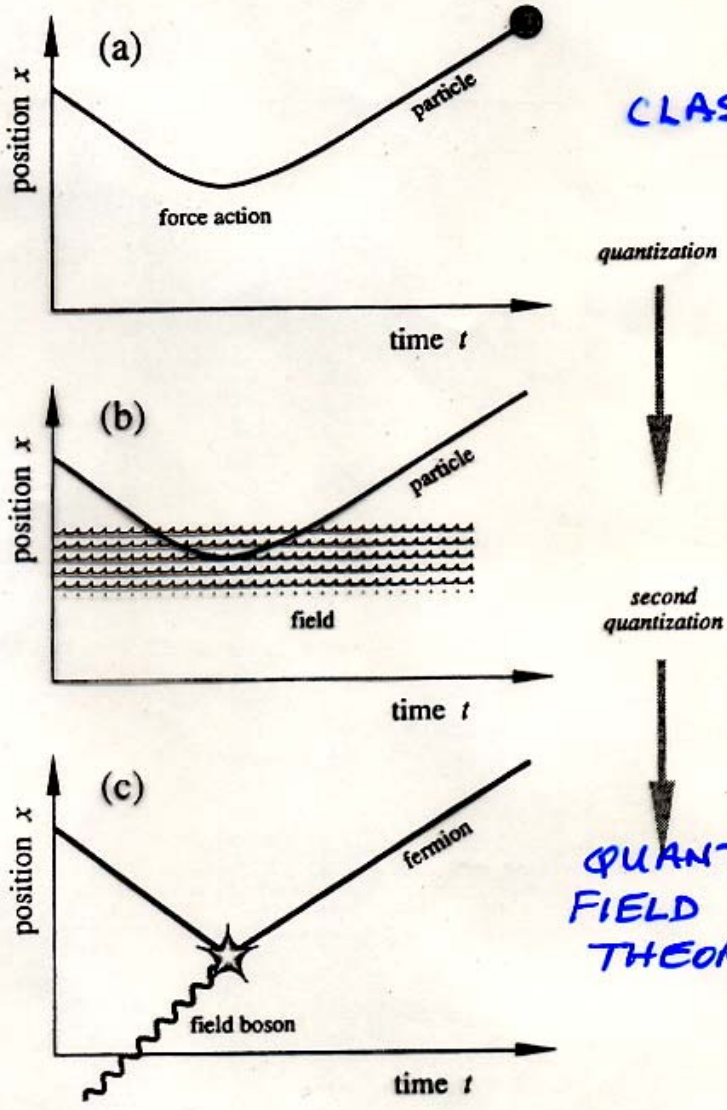
TOTAL PROBABILITY

SUM OVER ALLOWED PATHS

THIS is SCHEMATIC!

SPACE TIME DIAGRAM

CLASSICAL

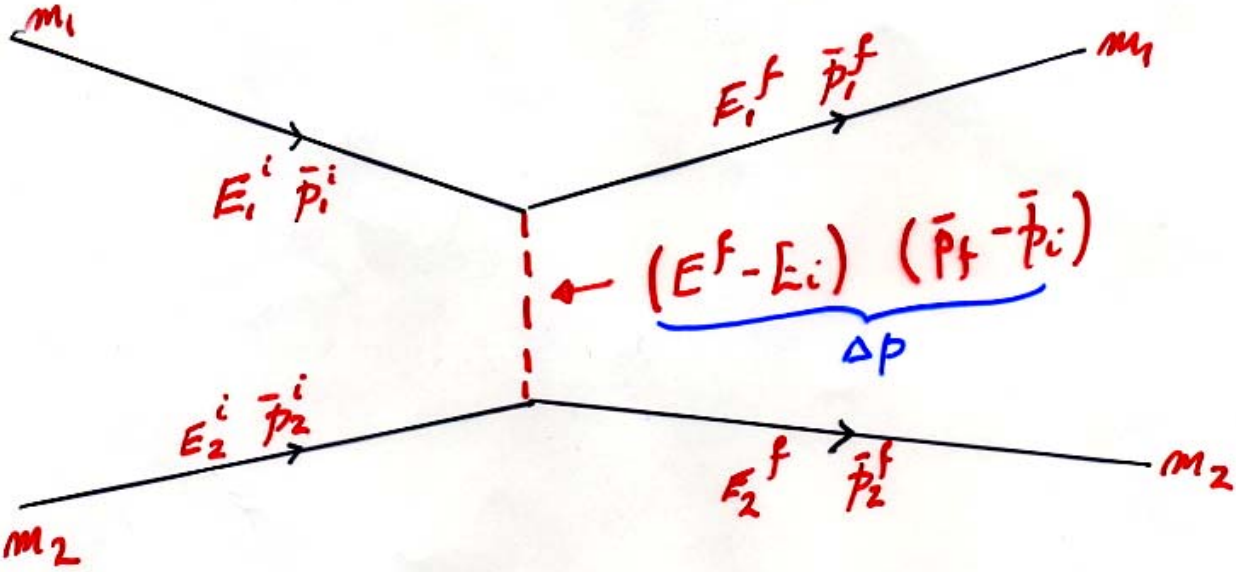


QUANTUM AMPLITUDE

QUANTUM FIELD THEORY

SPACE TIME DIAGRAM = QUANTUM AMPLITUDE

SCATTERING ANGLE ~ 4 MOMENTUM TRANSFER



USE 4-VECTORS, PUT $c = 1$

$$(\Delta p)^2 = (p_1^i - p_1^f)^2$$

4 VECTORS

$$= p_1^{i\,2} + p_1^{f\,2} - 2\, p_1^i \cdot p_1^f$$

$$p^2 = (E, \bar{p})^2$$
$$= E^2 - \bar{p}^2$$
$$= m^2$$

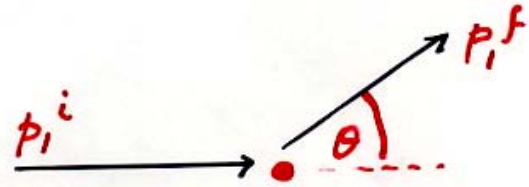
$$(E_1^i \ \bar{p}_1^i)(E_1^f, \bar{p}_1^f)$$
$$= 2 E_1^i E_1^f - 2 \bar{p}_1^i \cdot \bar{p}_1^f$$
$$= 2 E_1^i E_1^f - 2 |\bar{p}_1^i| |\bar{p}_1^f| \cos\theta$$

SCALAR

3 VECTORS

$$(\Delta p)^2 = m_1^2 + m_1^2 - 2E_1^i E_1^f + 2|\bar{p}_1^i||\bar{p}_1^f|\cos\theta$$

IN LAB



IF $E_1 \gg m_1$

$$|\bar{p}_1|^2 = E_1^2 - m_1^2 \approx E_1^2$$

$$(\Delta p)^2 = 2E^i E^f(\cos\theta - 1)$$

$$q^2 \sim \cos\theta$$

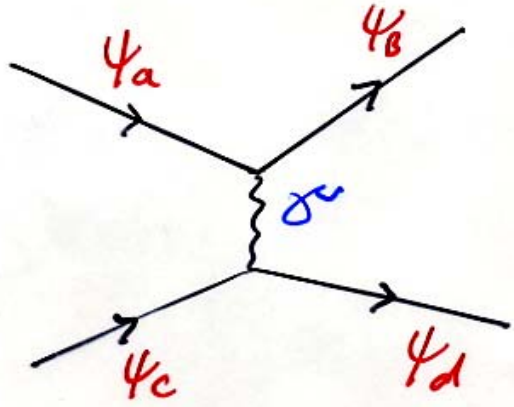
4-MOMENTUM TRANSFER GIVEN BY FUNCTION OF LAB KINEMATICS

E^i INCOMING ENERGY

E^f OUTGOING (SCATTERED) ENERGY

θ SCATTERING ANGLE

FIELD THEORY



$$\text{AMP} \sim \bar{\psi}_B \psi_A \frac{1}{q^2} \bar{\psi}_d \psi_c$$

CREATE B

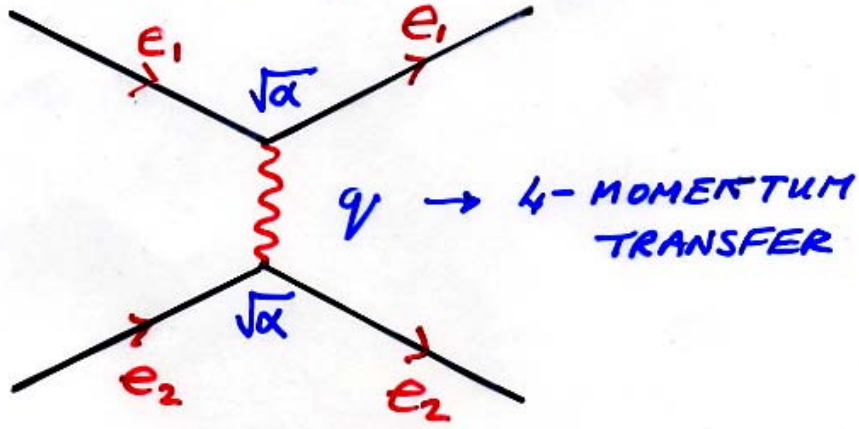
ANNIHILATE A

PROPAGATE γ^μ

- ABOVE IS VERY SCHEMATIC!

FEYNMAN DIAGRAMS FOR RELATIVISTIC QUANTUM SCATTERING

DIAGRAM REPRESENTS Q.M. AMPLITUDE



PROBABILITY OF SCATTERING THRU θ $P(\theta)$

$$P(\theta) = P(q^2) = |A(q^2)|^2$$

QUANTUM MECHANICAL SCATTERING AMPLITUDE

SIMPLE FEYMAN RULE

$$A_{ee \to ee} \sim \frac{\sqrt{\alpha} \cdot \sqrt{\alpha}}{q^2} \qquad \alpha = \frac{e^2}{\hbar c}$$

$$\sim e^2/q^2$$

$$P \propto |A|^2 \sim e^4/q^4$$

SIMPLE FEYMAN RULE

$$P \propto |A|^2 \sim e^4/q^4$$

SCATTERING DEPENDS ON STRENGTH OF INTERACTION $\sqrt{\alpha} \sim e$

HARD TO SCATTER THROUGH LARGE ANGLES

→ HAVE TO TRANSFER A LOT OF 4-MOMENTU

→ NEED A VERY VIRTUAL PHOTON

→ HARDER

IN OUR SIMPLE MODEL OF RUTHERFORD SCATTERING, WE HAD

$$\Delta p^2 \sim q^2 \sim p^2 \theta^2$$

SO, WE WOULD PREDICT THAT

PROB OF SCATTERING THROUGH θ; $P(\theta)$

$$P(\theta) \sim \frac{e^4}{p^4 \theta^4}$$

WE WILL COME BACK TO THIS

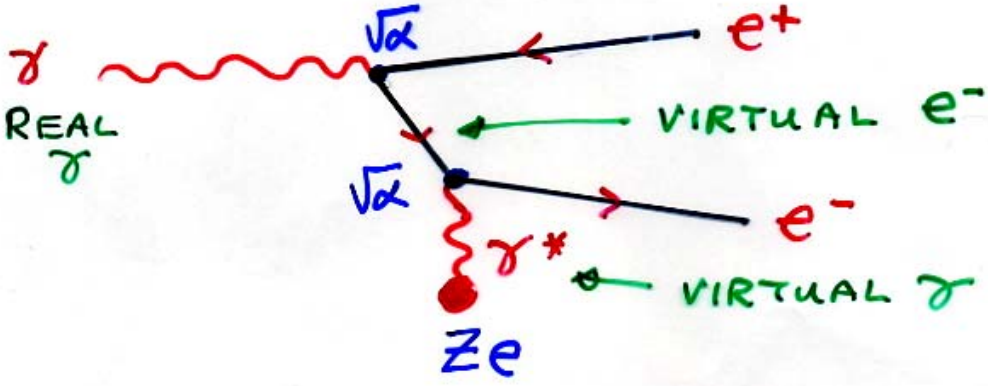
APPLY FEYNMAN RULE TO OTHER SIMPLE PROCESSES

IN FREE SPACE ONE CANNOT HAVE

$$\gamma \rightarrow e^+e^-$$

THIS DOES NOT CONSERVE MOMENTUM.

HOWEVER, A RECOILING NUCLEUS CAN PROVIDE MOMENTUM RECOIL



$$\text{PROBABILITY} \sim \left| \sqrt{\alpha} \cdot \sqrt{\alpha} \cdot Z\sqrt{\alpha} \right|^2$$

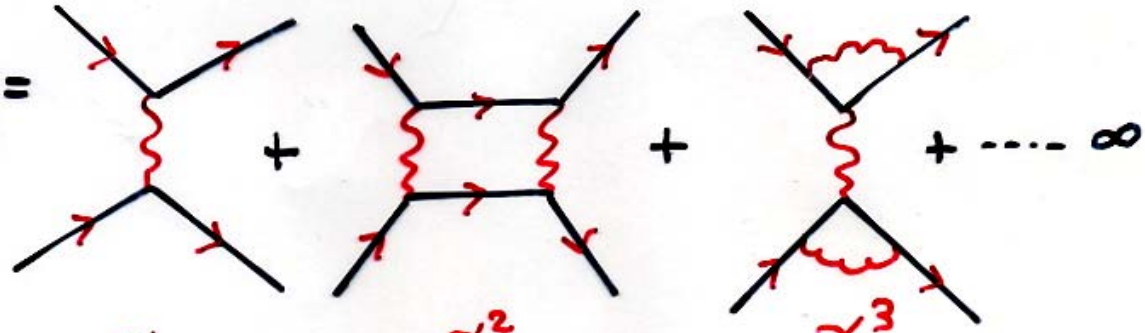
$$\sim \alpha^3 Z^2$$

$$\sim e^6 Z^2$$

PERTURBATION EXPANSION

WHY HAVE I ONLY DRAWN ONE EXCHANGED PHOTON? IF YOU HAVE READ FEYNMAN'S BOOK ON "QED", YOU KNOW THAT IN QUANTUM MECHANICS YOU HAVE TO TAKE ACCOUNT OF ALL POSSIBLE WAYS A PROCESS CAN OCCUR. ALSO INTERFERENCE TERMS

$A_{TOTAL} =$



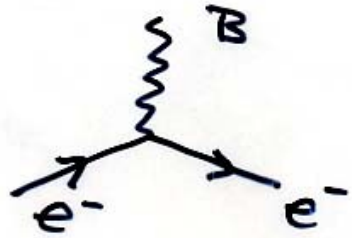
α	α^2	α^3
$\frac{1}{137}$	$\left(\frac{1}{137}\right)^2$	$\left(\frac{1}{137}\right)^3$
7×10^{-3}	5×10^{-5}	4×10^{-7}

- YOU HAVE TO JOIN INCOMING & OUTGOING "LINES" BY ALL POSSIBLE INTERNAL "VIRTUAL" LINES
- IN ELECTROMAGNETISM A GOOD APPROXIMATION IS JUST FIRST TERM: BORN APPROXIMATION

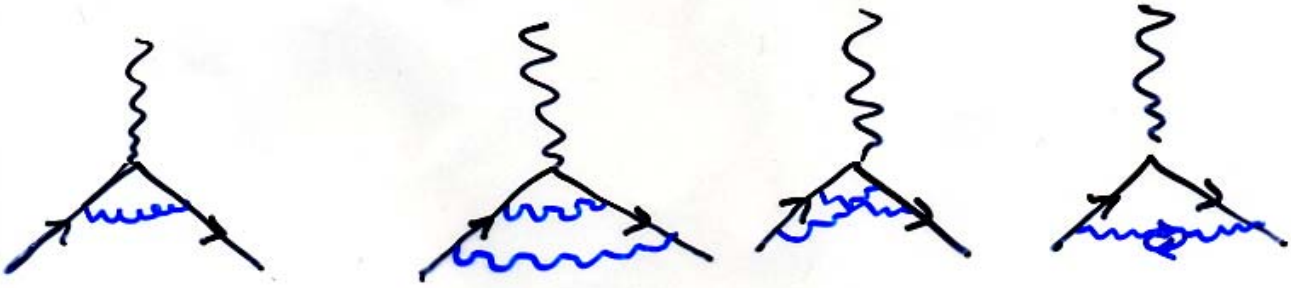
ONLY WORKS BECAUSE $\alpha \ll 1$. IF A FORCE HAS "STRONG COUPLING" $\alpha_S \gg 1$ AND "CORRECTIONS" DOMINATE $\sum$

PRECISION OF QUANTUM ELECTRODYNAMICS

→ CORRECTIONS TO MAGNETIC MOMENT OF ELECTRON



← LEADING ORDER



HIGHER ORDER CORRECTIONS

PERTURBATION EXPANSION

$$1 + \frac{\alpha}{2\pi} - 0.328478966\left(\frac{\alpha}{\pi}\right)^2 + 1.1765\left(\frac{\alpha}{\pi}\right)^3 - 0.8\left(\frac{\alpha}{\pi}\right)^4$$

$$= 1.001\,159\,652\,307\ (110)$$

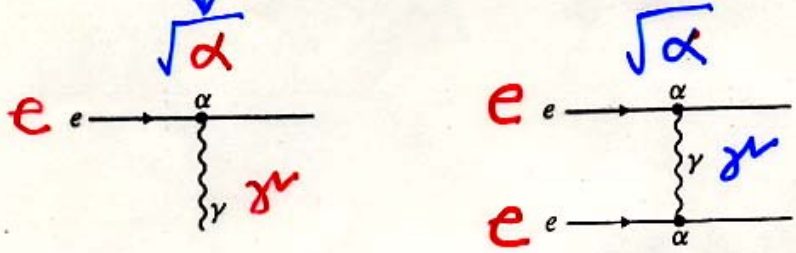
$$1.001\,159\,652\,193\ (10)$$

↑ UNCERTAINTY

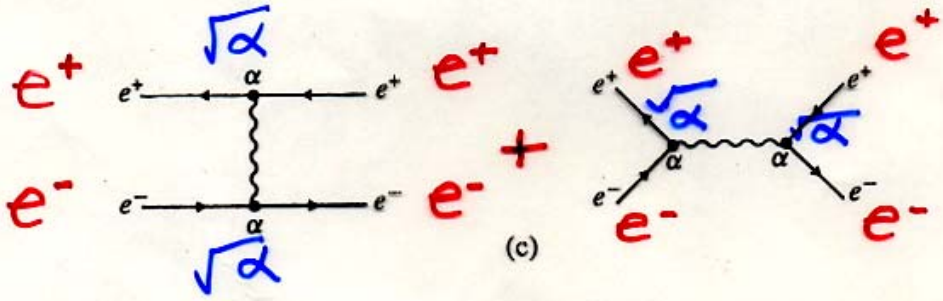
EXPERIMENTAL RESULT

E. M.

IN AMPLITUDE!



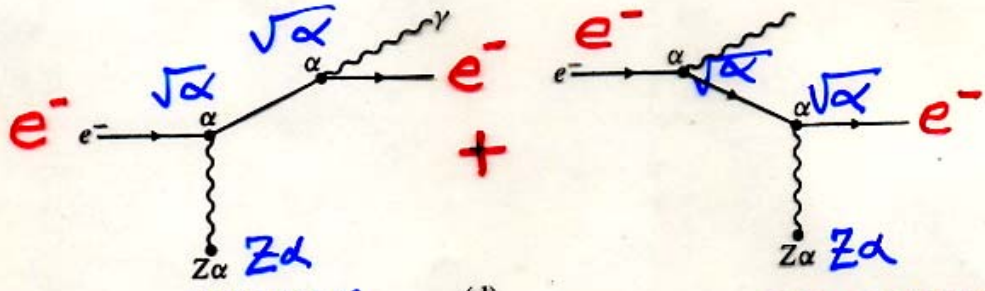
(a) (b)



(c)

TWO WAYS IT CAN HAPPEN

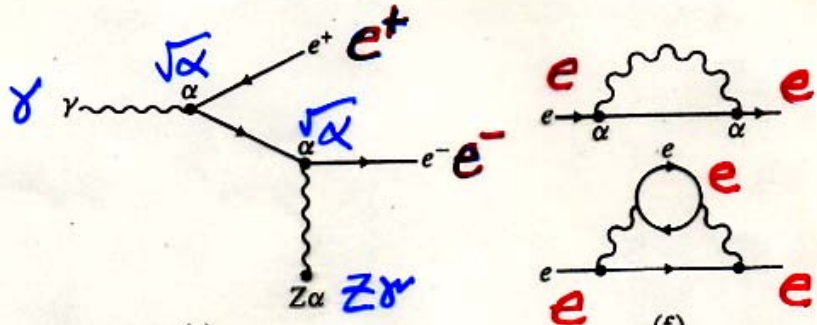
$A = (A_1 + A_2)$



NUCLEUS

(d)

BREMSTRAHLUNG



(e)

PAIR PRODUCTION

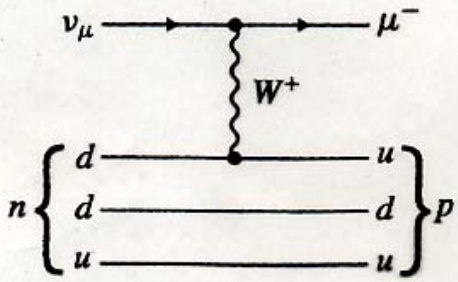
(f)

1ST ORDER α^2

2ND ORDER α^4

CORRECTIONS TO BARE ELECTRON MASS

WEAK INTERACTION



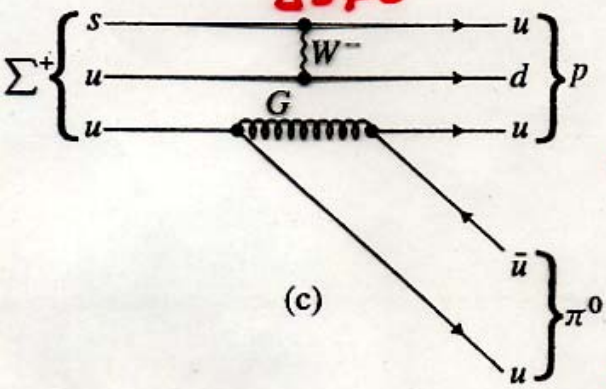
(a)

CLOSELY RELATED TO EM

EM	WEAK
	W^+
$\gamma^0 \longleftrightarrow$	Z_0
$m=0$	W^-

$M \sim 90\,\mathrm{GeV}$

$\Delta S \neq 0$

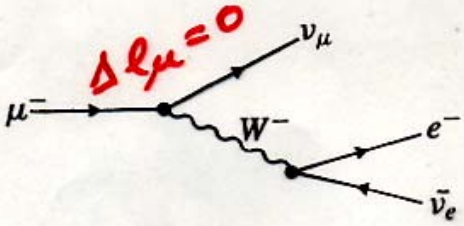


(c)

- $\dfrac{e^2}{q^2} \longrightarrow \dfrac{g^2}{q^2 - M^2_{W,Z}}$

- $q^2 \ll M^2_{WZ}$

$\Delta l_\mu = 0$



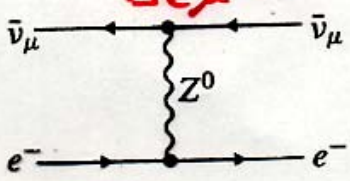
(b)

POINTLIKE 4-FERMI

$G \equiv \dfrac{g^2}{M^2} \approx 10^{-5}\,\mathrm{GeV}^{-2}$

$\Rightarrow$ "REASON FOR WEAKNESS"

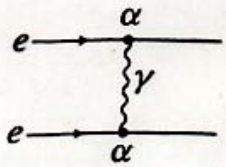
$\Delta l_\mu = 0$



$g \approx e$

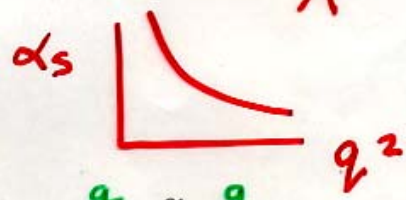
• QCD — ASSYMPTOTICALLY FREE

QED

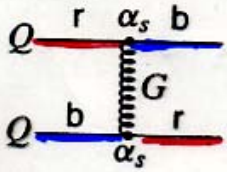


(a)

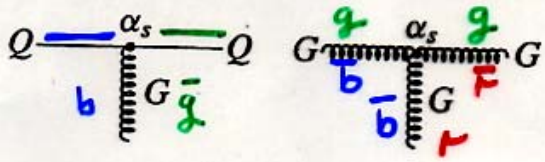
$$\alpha_s(q^2) = \frac{1}{B \ln \frac{q^2}{\Lambda^2}}$$



QCD



(b)

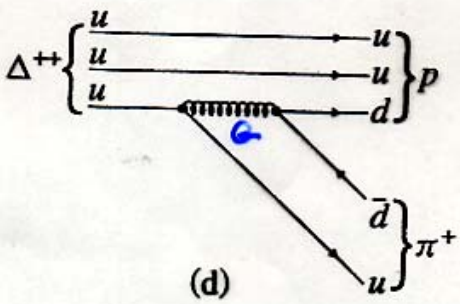


(c)

BICOLORED GLUONS

• FOR $q^2 \gg \Lambda^2$

→ PERTURBATION THEORY



(d)

• GLUONS CARRY COLOUR

• CHANGE COLOUR OF QUARKS

• cf

	1	3	8
	γ	$W^\pm Z^0$	gluons

Table 2.2. *Fundamental interactions* ($Mc^2 = 1$ *GeV*)

	Gravitational	Electromagnetic	Weak	Strong
field boson	graviton	photon	$W^{\pm}, Z$	gluon
spin-parity	2^+	1^-	$1^-, 1^+$	1^-
mass, GeV	0	0	$M_W = 80.2$ $M_Z = 91.2$	0
range, m	∞	∞	10^{-18}	$\leq 10^{-15}$
source	mass	electric charge	'weak charge'	'colour charge'
coupling constant	$\frac{G_N M^2}{4\pi\hbar c}$ $= 5 \times 10^{-40}$	$\alpha = \frac{e^2}{4\pi\hbar c}$ $= \frac{1}{137}$	$\frac{G(Mc^2)^2}{(\hbar c)^3}$ $= 1.17 \times 10^{-5}$	$\alpha_s \leq 1$
typical cross-section, m^2		10^{-33}	10^{-39}	10^{-30}
typical lifetime, s		10^{-20}	10^{-10}	10^{-23}