ELECTRIC CHARGE & DIPOLE MOMENT

AN ELECTRICALLY CHARGED PARTICLE MAY BE ELEMENTARY OR COMPOSITE

ALL CHARGES OCCUR IN INTEGRAL MULTIPLES OF SOME ELEMENTARY UNIT. THE CHARGE ON A PARTICLE DOES NOT TELL ONE ANYTHING ABOUT ITS STRUCTURE

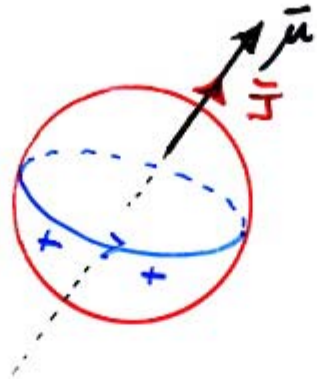
PROTON — CHARGE $+e$ — COMPOSITE

MUON — CHARGE $+e$ — POINTLIKE

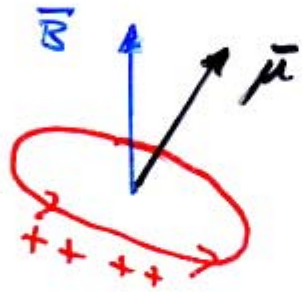
HOWEVER, MAGNETIC MULTIPOLE MOMENTS DO GIVE INFORMATION ON PARTICLE STRUCTURE.

AN EXAMPLE IS THE MAGNETIC DIPOLE MOMENT

LET'S JUST THINK IN SEMI-CLASSICAL TERMS



SPINNING CHARGED SPHERE IS LIKE A CURRENT LOOP



THERE WILL BE AN INTERACTION ENERGY BETWEEN THE CURRENT LOOP, AND AN APPLIED EXTERNAL FIELD

$$E_{mag} = -\bar{\mu} \cdot \bar{B}$$

$$\mu = \frac{1}{c} \text{ CURRENT} \times \text{AREA}$$

CONNECTION BETWEEN MAGNETIC MOMENT AND ANGULAR MOMENTUM.

PARTICLE CHARGE q MOVING WITH VELOCITY v IN A CIRCULAR ORBIT

PARTICLE REVOLVES $\frac{v}{2\pi r}$ TIMES / SEC

SITTING IN LAB, OBSERVER SEES A CHARGE q PASSING $\frac{v}{2\pi r}$ TIMES IN A SEC

$$\text{CURRENT} = \frac{qv}{2\pi r}$$

$$\mu = \frac{1}{c} \cdot \frac{qv}{2\pi r} \cdot \pi r^2$$

$$\mu = \frac{1}{c} \, q \frac{v \cdot r}{2}$$

$$\bar{L} = \bar{r} \times \bar{p} \rightarrow m \bar{r} \times \bar{v}$$

$$\bar{\mu} = \frac{q}{2mc} \cdot \bar{L}$$

$$\bar{\mu} = \frac{q}{2mc} \cdot \bar{L}$$

EVEN THO' THIS IS A CLASSICAL RESULT

$$\frac{\bar{\mu}}{\bar{L}} = \frac{q}{2mc}$$

QUANTUM MECHANICALLY, FOR CHARGE e

$$\bar{\mu} = g \frac{e}{2mc} \bar{J}$$

g FACTOR IS A MEASURE OF DEVIATION OF MAGNETIC MOMENT FROM $\frac{e}{2mc}$

$[J] = [\hbar] \rightarrow \frac{J}{\hbar}$ IS DIMENSIONLESS

SO:

$$\bar{\mu} = g \mu_0 \frac{\bar{J}}{\hbar}$$

WHERE $\mu_0 = \frac{e\hbar}{2mc}$

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μ_0 IS KNOWN AS A MAGNETON

IF m IS THE ELECTRON MASS

$$\mu_B = \frac{e\hbar}{2m_e c} = 0.5788 \times 10^{-14} \text{ MeV/GAUSS}$$

IS THE BOHR MAGNETON

IF m IS THE PROTON MASS

$$\mu_N = \frac{e\hbar}{2m_p c} = 3.1525 \times 10^{-18} \text{ MeV/G}$$

IS THE NUCLEAR MAGNETON

NOTE THAT

$$\mu_N \sim 2 \times 10^{-3} \mu_B$$

INFORMATION ABOUT STRUCTURE OF PARTICLE IS CONTAINED IN g-FACTOR

$$\bar{\mu} = g \cdot \frac{e}{2mc} \bar{J}$$

$g = 1$ FOR ORBITAL ANGULAR MOMENTUM

FOR A POINT PARTICLE $g = 2$ $\mu_e = 2\mu_B$

INTRINSIC → DIRAC EQUATION

FOR A PARTICLE WITH INTERNAL STRUCTURE $g \neq 2$

IF A PROTON WAS A POINT ONE WOULD EXPECT

$$\mu_p = 2\mu_N$$

NUCLEAR MAGNETON

IN FACT $\mu_p = 2.79\mu_N$ ← INTERNAL STRUCTURE

ALSO, EVEN THOUGH THE NEUTRON IS ELECTRICALLY NEUTRAL

$$\mu_n = -1.91\mu_N$$

SO THE NEUTRON MUST HAVE SOME CHARGED INTERNAL STRUCTURE

cf REMARK ON TOTAL CHARGE NOT CARRYING INFORMATION ON STRUCTURE

HISTORICALLY COMPARISON OF MEASURED NUCLEAR MAGNETIC MOMENTS WITH PREDICTIONS OF MODELS HAS BEEN A STRINGENT TEST OF MODELS

A SIMPLE EXAMPLE – β DECAY

$$ {}^{A}_{Z}X \rightarrow {}^{A}_{Z+1}Y \; e^- \; (\bar{\nu}_e) $$

UNSEEN

A SIMPLE MODEL MIGHT SAY THAT A NUCLEUS CONSISTED OF A PROTONS, PLUS $(A-Z)$ ELECTRONS TO GET CHARGE CORRECT

$\Rightarrow$ MAYBE THAT IS WHERE β DECAY COMES FROM?

<u>NO</u>! IF THERE WERE ELECTRONS INSIDE NUCLEI, IN ${}^{14}_{7}N$

$$ \mu \sim 14\mu_p + 7\mu_e \qquad \mu_e \sim 2000\mu_p $$

BUT THIS PREDICTION IS THOUSANDS OF TIMES THE OBSERVED VALUE.