MODELS OF NUCLEAR STRUCTURE

LIQUID DROP THE PROPERTIES OF SATURATION OF THE STRONG NUCLEAR FORCE AND THE INCOMPRESSIBILITY OF NUCLEAR MATTER LEAD TO THIS EARLY SEMI CLASSICAL MODEL.

THESE PROPERTIES ARE SIMILAR TO THOSE OF A CLASSICAL FLUID. HOWEVER, THE NUCLEONS ARE QUANTUM PARTICLES

SO, THIS FLUID MUST BE A QUANTUM FLUID IN THE LIQUID DROP MODEL, THESE QUANTUM EFFECTS ARE INTRODUCED IN AN ad hoc FASHION.

THE LIQUID DROP MODEL GIVES A GOOD DESCRIPTION OF

AVERAGE PROPERTIES

IT GIVES NO REAL UNDERSTANDING OF

INDIVIDUAL NUCLIDE PROPERTIES

E.G. WHY ARE PARTICULAR NUCLIDES MUCH MORE STABLE THAN OTHERS VERY CLOSE TO THEM IN MASS?

SHELL MODEL

A TRULY QUANTUM MECHANICAL MODEL
IN MANY RESPECTS IT IS A REPLAY OF
OUR MODEL OF ATOMIC STRUCTURE

- NUCLEONS ORBIT IN A COMMON POTENTIAL DUE TO THE OTHER NUCLEONS IN THE NUCLIDE
- PAULI EXCLUSION EFFECTS LEAD TO A SHELL STRUCTURE, JUST AS IN ATOM
- IN ADDITION TO COMMON POTENTIAL THERE IS A SPIN-ORBIT POTENTIAL

$$V_{TOT} = V(r) - f(r)\,\vec{L}\cdot\vec{S}$$

GIVES AN EXCELLENT DESCRIPTION!

- DETAILS OF ENERGY LEVELS
- NUCLEAR SPINS
- NUCLEAR MAGNETIC MOMENTS
- CLOSED SHELLS LEAD TO PARTICULARLY STABLE NUCLIDES → JUST LIKE ATOMS

COLLECTIVE MODEL

WHILE THE SHELL MODEL DOES AN EXCELLENT JOB OVERALL, THERE ARE SOME DETAILED NUMERICAL DISAGREEMENTS BETWEEN EXPERIMENT & PREDICTIONS

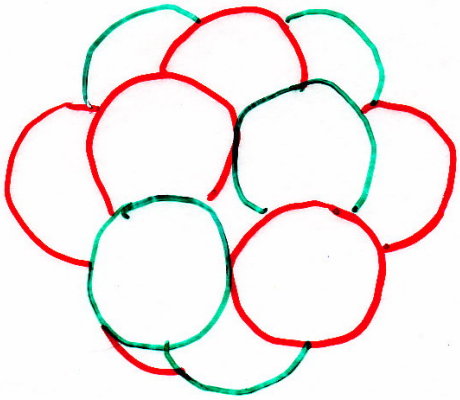
- DIPOLE MOMENTS
- QUADRUPOLE MOMENTS SHOULD VANISH FOR CLOSED SHELLS — SPHERICAL NUCLEI

EVIDENCE POINTS TO NUCLEI, ESPECIALLY VERY HEAVY ONES, BEING ASPHERICAL

- HARD NUCLEAR CORE CONSISTING OF FILLED SHELLS — LIQUID DROP
- VALENCE NUCLEONS — DOMINATE SPIN MAGNETIC MOMENT
- ROTATION OF VALENCE SHELL AROUND CORE

→ NON CENTRAL POTENTIAL

$$V(\vec{r}) \neq V(|\vec{r}|)$$



PROTONS

NEUTRONS

- BINDING ENERGY / NUCLEON ~ CONSTANT
- DENSITY OF NUCLEAR MATTER ~ CONSTANT

ANALOGY TO A LIQUID

- CONSTANT BINDING ENERGY / MOLECULE
  → SHORT RANGE VAN DER WAALS FORCE
- CONSTANT DENSITY $V \propto A$
  → INCOMPRESSIBLE LIQUID

- SAY ENERGY ASSOCIATED WITH EACH NUCLEON – NUCLEON BOND IS u ← SHOULD BE ABLE TO PREDICT!

EACH BOND SHARED BY 2 NUCLEONS

$\therefore$ BINDING ENERGY PER NUCLEON $= \frac{1}{2} u$

- GEOMETRICAL FACT THAT FOR CLOSELY PACKED SPHERES, EACH INTERIOR SPHERE IS IN CONTACT WITH 12 OTHERS

BINDING ENERGY OF INTERIOR NUCLEON $= 12 \times \frac{u}{2} = 6u$

↑ CONSTANT

- IF ALL A NUCLEONS WERE INTERIOR TOTAL BINDING ENERGY

$$E_V = 6Au$$

DEFINE VOLUME ENERGY OF NUCLEUS

$$\boxed{E_V = a_1 A \propto A}$$

a_V

- NOW SURFACE NUCLEONS HAVE FEWER THAN 12 NEIGHBOURS

- CLEARLY THE NUMBER OF SURFACE NUCLEONS DEPENDS UPON THE SURFACE AREA OF A PARTICULAR NUCLEUS

$$\text{AREA} = 4\pi r^2 = 4\pi r_0^2 A^{2/3}$$

$\Rightarrow$ NUMBER OF SURFACE NUCLEONS

$$\propto A^{2/3}$$

THIS REDUCES THE BINDING ENERGY FROM AN "∞" NUCLEUS BY

SURFACE ENERGY OF NUCLEUS

$$\boxed{E_S = -a_2 A^{2/3}} \qquad a_S$$

THIS IS MOST IMPORTANT FOR LIGHT NUCLEI, AS THEY HAVE A LARGER SURFACE: VOLUME RATIO.

-ve - REDUCES MAGNITUDE OF BINDING ENERGY

THESE CONSIDERATIONS LEAD US TO THE IDEA THAT NUCLEI WILL BE SPHERICAL

→ SAME ARGUMENT AS THAT FOR THE SPHERICITY OF A LIQUID DROP

→ NUCLEI WILL "MOVE" INTO MOST STABLE CONFIGURATION

↓

|BINDING ENERGY| MAXIMIZED

MINIMIZE NUMBER OF "MISSING" BONDS

↓

"SURFACE TENSION" → REDUCES |B/A|

↓

REDUCE SURFACE AREA

↓

SPHERE HAS LEAST SURFACE AREA FOR A GIVEN VOLUME.

SO FAR, HAVE NEGLECTED ELECTROSTATIC REPULSION BETWEEN PROTONS

→ THIS MUST DECREASE THE BINDING ENERGY

COULOMB ENERGY E_C IS THE WORK THAT MUST BE DONE TO BRING TOGETHER Z PROTONS FROM ∞ — COULOMB IS LONG RANGE FORCE AND PUT THEM IN A VOLUME EQUAL TO THAT OF THE NUCLEUS

LONG RANGE

$$E_C \propto \frac{Z(Z-1)}{2}$$ ← NO OF PAIRWISE PROTON COMBINATIONS

PROBLEM SET

$$E_C \propto \frac{1}{r} \propto \frac{1}{r_0 A^{1/3}}$$ ← NUCLEAR RADIUS

$$\boxed{E_C = -a_3 \frac{Z(Z-1)}{A^{1/3}}}$$ a_c

AGAIN −ve SINCE IT ACTS AGAINST NUCLEAR STABILITY BY REDUCING THE MAGNITUDE OF THE BINDING ENERGY

TOTAL NUCLEAR BINDING ENERGY

$$E_b = E_v + E_s + E_c \rightarrow B_N$$

$$= \underset{a_v}{a_1 A} - \underset{a_s}{a_2 A^{2/3}} - \underset{a_c}{a_3 \frac{Z(Z-1)}{A^{1/3}}}$$

BINDING ENERGY / NUCLEON

$$\frac{E_b}{A} = a_1 - \frac{a_2}{A^{1/3}} - a_3 \frac{Z(Z-1)}{A^{4/3}} \rightarrow Z^2$$

SEMI-EMPIRICAL FORMULE.