SYSTEMATICS OF NUCLEAR PROPERTIES

ATOMIC NUMBER Z CORRESPONDS TO POSITION IN PERIODIC TABLE, HENCE CHARGE ON THE NUCLEUS

ATOMIC MASS NUMBER A NEAREST INTEGER TO THE RATIO OF MASS OF ATOM, TO MASS OF ATOM OF LIGHTEST ISOTOPE OF H

ATOMIC MASS RATIO OF MASS OF ATOM, TO $\frac{1}{12}$ OF MASS OF ATOM OF CARBON $A=12$

OR $\frac{1}{16}$ OF MASS OF ATOM OF OXYGEN $A=16$

(SLIGHTLY MORE COMPLEX) →

ATOMIC WEIGHT ATOMIC MASS AVERAGED OVER TERRESTRIAL ISOTOPE ABUNDANCE FOR ELEMENTS WITH MORE THAN ONE STABLE ISOTOPE

ASSUMING THAT A NUCLEUS CONTAINS PROTONS & NEUTRONS

A GIVEN SPECIES (NUCLIDE) IS CHARACTERIZED:

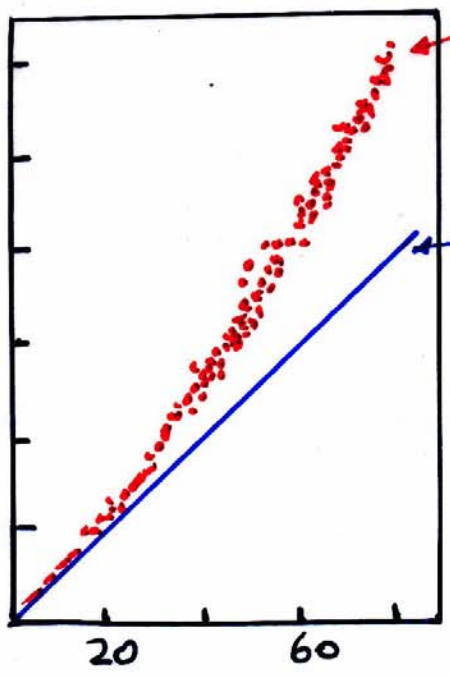
$^{A}_{Z}X$ ← CHEMICAL SYMBOL

Z = ATOMIC NUMBER = NO. OF PROTONS = # e^- (ATOM)

N = NUMBER OF NEUTRONS

A = MASS NUMBER = $Z + N$ = # p + # n

$^{1}_{1}H$ $^{14}_{7}N$ $^{75}_{33}As$ $^{214}_{82}Pb$ $^{238}_{92}U$



- N INCREASES FASTER THAN Z
- WHAT CAUSES VALLEY OF STABILITY?

ISOTOPES - SAME Z

ISOBARS - SAME A

ISOTONES - SAME N

IF ONE KNEW NOTHING ABOUT SPECIAL RELATIVITY — ONE MIGHT EXPECT

$$M(A,Z) = Z m_p + (A-Z) m_n$$

MASS OF NUCLEUS

PROTON MASS $938.27 \frac{MeV}{c^2}$

NEUTRON MASS $939.56 \frac{MeV}{c^2}$

- NUCLEUS IS STABLE, HAVE TO DO WORK TO PULL IT APART → BOUND STATE

$$M(A,Z) < Z m_p + (A-Z) m_n$$

$$\Delta M(A,Z) = M(A,Z) - Z m_p - (A-Z) m_n$$

MASS DEFICIT

MASS OF BOUND STATE

MASS OF CONSTITUENTS AT INFINITY

$$-\frac{BE}{A} = -\frac{\Delta M(A,Z) c^2}{A} =$$

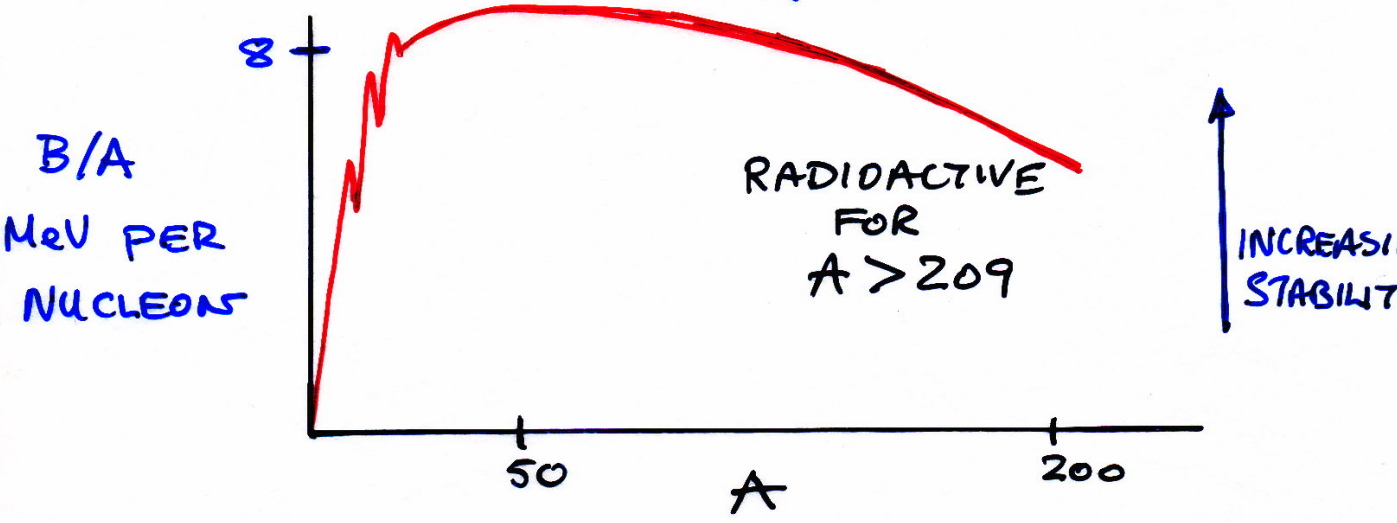
BINDING ENERGY PER NUCLEON

BINDING ENERGY IS ENERGY ONE HAS TO PUT INTO SYSTEM TO REMOVE CONSTITUENTS TO INFINITY.

TOTAL BINDING ENERGY WILL DEPEND ON TOTAL NUMBER OF NUCLEONS

$\frac{BE}{A}$ ⇒ EFFECT ON INDIVIDUAL NUCLEON

$$\frac{B}{A} = \frac{(Zm_p + (A-Z)m_n - M(A,Z))c^2}{A}$$



- INCREASES RAPIDLY → SPIKES ← VERY STABLE
- PEAKS AT 9 MeV/NUCLEON
- DROPS SLOWLY ← ROUGHLY CONSTANT AT 8 MeV/NUCLEON

SATURATION IS AN IMPORTANT INDICATOR OF BEHAVIOUR OF FORCE BETWEEN TWO NUCLEONS.

- FISSION / FUSION ⇒ ENERGY RELEASE

MARGINAL NOTE ON MASSES

- NUCLEAR MASS ≈ ATOMIC MASS − Zm_e

  NEGLECT ELECTRON BINDING ENERGY

- 16 amu = MASS OF $^{16}_{8}O$ ⇐ OLD SCALE

  12 u = MASS OF $^{12}_{6}C$ ⇐ NEW SI MASS SCALE

- ONE amu = MASS IN GRAMS OF ONE "ATOM OF ELEMENT WITH ATOMIC WEIGHT = 1.0000 "

AVOGADRO = # OF ATOMS IN MOLE

MOLE = ATOMIC WT IN GRAMS

A_0 NUCLEI IN A MOLE

MOLE HAS MASS A

$\therefore$ MASS OF NUCLEUS = $\frac{A}{A_0}$

- MASS OF FICTIOUS ATOM WITH ATOMIC WEIGHT OF 1.0000 gm = A_0^{-1}

AVOGADRO SETS SCALE OF amu.

$m_p = 1.00728$ amu

$$1 \text{ amu} = \frac{938.27}{1.00728} \frac{\text{MeV}}{c^2}$$

$$1 \text{ amu} = 1.6726 \times 10^{-24} \text{ gm}$$

THIS IS ON THE OLD ^{16}O BASED SCALE

ON THE SI – "UNIFIED CARBON SCALE"

$$1u = 931.4932 \text{ MeV}/c^2$$
$$= 1.660540 \times 10^{-27} \text{ kg}$$

$$m_p = 938.27 \frac{\text{MeV}}{c^2} \cdot \frac{1}{931.494} \frac{c^2}{\text{MeV}} \cdot u$$

$$m_p = 1.00727 \, u$$

m_p IS THE SAME PHYSICAL VALUE !
JUST SLIGHTLY DIFFERENT STANDARD FOR UNITS

DENSITY OF NUCLEAR MATTER

MEASUREMENTS OF NUCLEAR SIZES IN SCATTERING EXPERIMENTS SHOW

$$R = R_0 A^{\frac{1}{3}} \sim 1.2 \times 10^{-13} A^{1/3} \text{ cm}$$

R_0 ← CONSTANT

$$= 1.2 A^{1/3} \text{ fm}$$

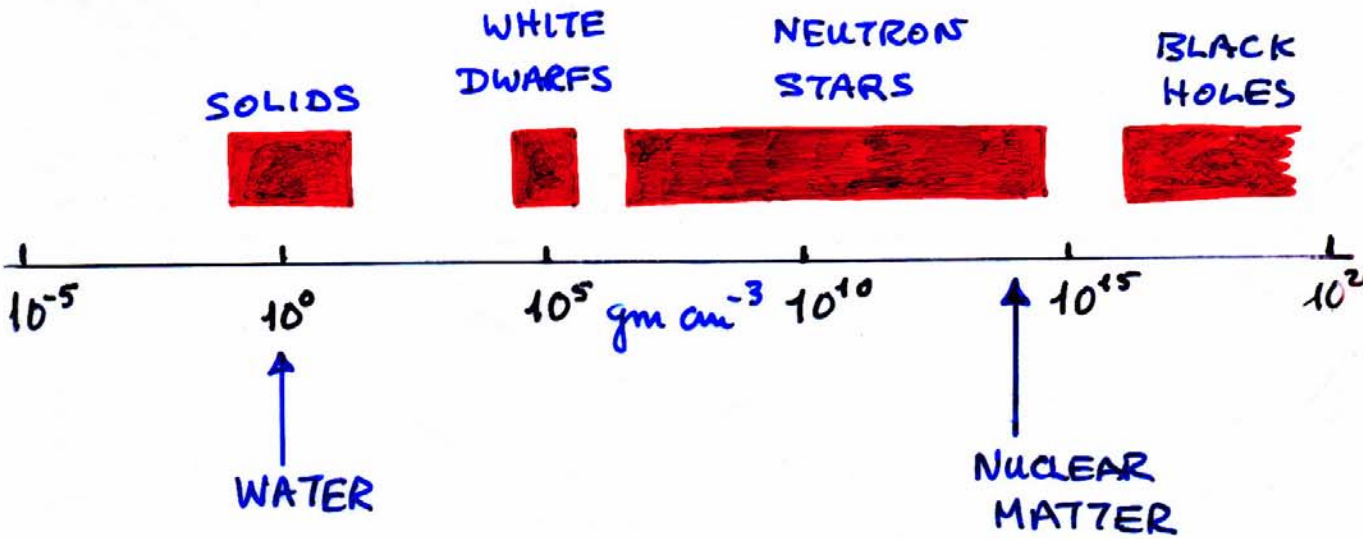
SINCE VOLUME $= \frac{4}{3}\pi R^3 = \frac{4}{3}\pi R_0^3 A$

VOLUME $\propto$ NUMBER OF NUCLEONS

→ NUCLEONS TIGHTLY PACKED IN NUCLEUS

$$\rho = \frac{\text{MASS}}{\text{VOLUME}} = \frac{A \cdot 1.660 \times 10^{-24} \text{ gm}}{\frac{4}{3} \cdot \pi \cdot R_0^3 \cdot A}$$

$$\approx 2 \times 10^{14} \text{ gm. cm}^{-3}$$ { FOR ALL A



NUCLEAR INSTABILITY

AWAY FROM THE VALLEY OF STABILITY NUCLEI ARE UNSTABLE

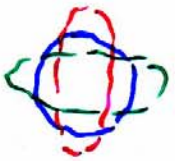
$$ {}^{A}_{Z}X \longrightarrow {}^{A'}_{Z'}Y + \begin{matrix} \alpha \\ \beta \\ \gamma \end{matrix} $$

α : BOUND SYSTEM OF ${}^{4}_{2}He$ "TUNNELS" OUT OF NUCLEUS

β : NUCLEUS EMITS AN ELECTRON

$$ n \longrightarrow p + e^{-} + \bar{\nu}_e $$

γ : NUCLEUS EMITS A PHOTON



CHARGED BODY OF OSCILLATING SHAPE CAN EMIT ELECTROMAGNETIC RADIATION

NUCLEAR MAGNETIC MOMENT

SPINNING ELECTRIC CHARGE

MAGNETIC DIPOLE MOMENT

$$\vec{\mu} = g \cdot \frac{e}{2mc} \cdot \vec{s}$$

LANDÉ g-FACTOR — MASS — SPIN

BOHR MAGNETON

DIRAC

FOR A POINT PARTICLE $g = 2$ $\mu_e = \mu_B$

FOR A PARTICLE WITH SUBSTRUCTURE $g \neq 2$ $\mu_p = \frac{\mu_B}{2000}$

$$\mu_B = e\hbar/2m_e c \qquad \mu_N = e\hbar/2m_p c$$

$\mu_p = 2.79\mu_N$; $\mu_n = -1.91\mu_N$

OBSERVE MAGNETIC MOMENTS OF NUCLEI $\sim \mu_N \neq 0.3\mu_p$

- IF ELECTRONS IN NUCLEI

NITROGEN $\mu \sim 14\mu_p + 7\mu_e$

THOUSANDS OF TIMES LARGER THAN OBSERVED

NATURE OF THE NUCLEAR FORCE

CAVEAT – BY NUCLEAR FORCE, MEAN FORCE BETWEEN NUCLEONS

– WILL LEARN LATER THAT IT IS A RESIDUAL EFFECT OF THE FORCE BETWEEN QUARKS

FORCE THAT BINDS NUCLEONS INTO NUCLEI

– CANNOT BE A KNOWN CLASSICAL FORCE

- GRAVITY TOO WEAK BY MANY ORDERS OF MAGNITUDE
- EM FORCE COULD ONLY BIND PROTONS & NEUTRONS THROUGH INTERACTION OF MAGNETIC MOMENTS → AGAIN TOO WEAK
- COULOMB FORCE TENDS TO DISRUPT NUCLEUS

SHORT RANGE FORCE

- FACT THAT ATOMIC STRUCTURE CAN BE VERY ACCURATELY CALCULATED ASSUMING ONLY THE COULOMB FORCE
  - NUCLEAR FORCE DOES NOT EXTEND BEYOND NUCLEI

- BINDING ENERGY PER NUCLEON IS ROUGHLY CONSTANT AS A INCREASES
  - → BINDING ENERGY DOES NOT DEPEND UPON SIZE OF NUCLEUS
  - → IF NUCLEAR FORCE WAS LONG RANGE LIKE COULOMB

$\frac{1}{2}(A-1)A$ PAIRWISE COMBINATIONS

BINDING ENERGY ~ TOTAL POTENTIAL ENERGY OF ALL COMBINATIONS

$$B \propto A(A-1)$$

FOR $A \gg 1$ → $B \propto A^2$

BINDING ENERGY PER NUCLEON $= \frac{B}{A} \propto A$ ← LONG RANGE FORCE

LONG RANGE FORCE $\frac{B}{A} \propto A$

EXPERIMENTALLY $\frac{B}{A}$ ROUGHLY CONSTANT

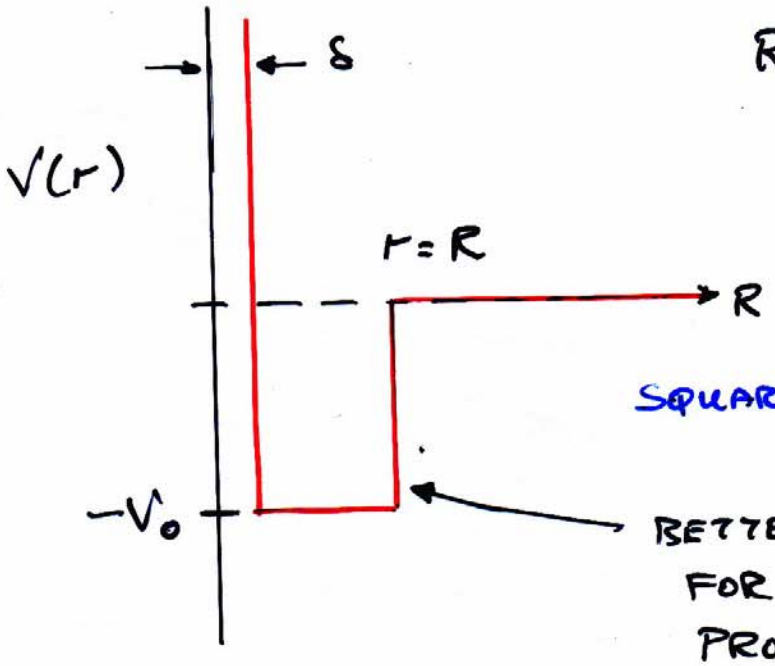
→ FORCE SATURATES

- A GIVEN NUCLEON CAN ONLY INTERACT WITH ITS NEAREST NEIGHBOURS
- ADDING MORE NUCLEONS INCREASES SIZE OF NUCLEUS – BUT NOT BINDING ENERGY/NUCLEON
- ATTRACTIVE FORCE, BUT MUST HAVE A REPULSIVE CORE IN ORDER TO PREVENT COLLAPSE – INTERNAL STRUCTURE OF NUCLEONS

$\delta \ll R$

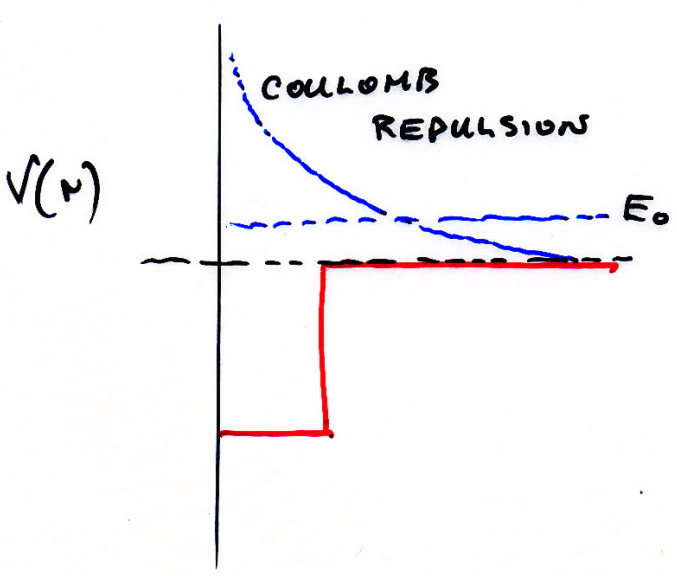
RANGE $\sim R$

$\frac{B}{A} \sim V_0$

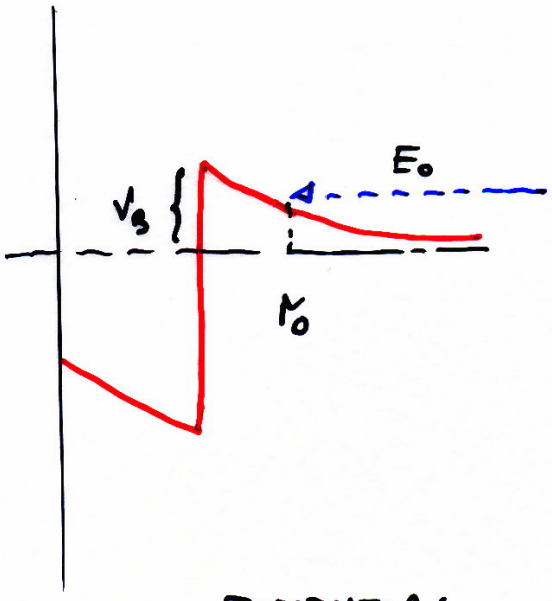


SQUARE WELL

BETTER APPROXIMATION FOR NEUTRONS THAN PROTONS



POTENTIAL SEEN BY NEUTRON



POTENTIAL SEEN BY PROTON

- A NEUTRON JUST SNEAKS INTO THE NUCLEUS, AND FALLS INTO THE POTENTIAL WELL
- A PROTON SEES A REPULSIVE FORCE. MINIMUM DISTANCE OF PENETRATION
- SCATTERING EXPERIMENTS INSENSITIVE TO EXACT FORM OF NUCLEAR POTENTIAL
- FROM ELEMENTARY QUANTUM MECHANICS WE EXPECT THE SQUARE WELL TO HEAD TO DISCRETE ENERGY LEVELS
  $\Rightarrow$ LIKE ATOMS

NON CLASSICAL FEATURES OF NUCLEAR FORCE

FIND "MIRROR NUCLEI", WHERE

$$ {}^{A}_{Z}X \qquad {}^{A}_{A-Z}Y $$

$$ {}^{15}_{8}O \qquad {}^{15}_{7}N $$

- HAVE SAME NUMBER OF n-p INTERACTIONS BUT DIFFERENT NUMBER OF pp nn

- AFTER CORRECTING FOR COULOMB EFFECTS

  FORCE BETWEEN nn = FORCE BETWEEN pp

  → CHARGE INDEPENDENCE OF NUCLEAR FOR

- NEUTRONS & PROTONS ARE TWO "CHARGE" STATES OF ONE PARTICLE → NUCLEON

- NUCLEON CAN HAVE TWO ORIENTATIONS IN "CHARGE SPACE"

THIS LEADS TO CONCEPT OF ISOTOPIC SPIN SYMMETRY

→ NUCLEAR INTERACTIONS INVARIANT UNDER "ROTATIONS IN ISOSPIN SPACE"

PAIRING OF NUCLEONS

EASY TO SEE (SINCE $\vec{n}, \vec{p} = \frac{\hbar}{2}$)

EVEN A → INTEGRAL NUCLEAR SPIN

ODD A → $\frac{1}{2}$ INTEGRAL NUCLEAR SPIN

- STRIKING EXPERIMENTAL FACT IS THAT ALL ~~NUCLEONS~~ NUCLEI

  EVEN PROTONS
  EVEN NEUTRONS
  } ZERO NUCLEAR SPIN

  EVEN - EVEN

- ALSO LARGE NUCLEI (MANY $\frac{\hbar}{2}$) GENERALLY HAVE VERY SMALL NUCLEAR SPINS

QUANTUM MECHANICAL PHENOMENON
→ NUCLEONS PAIR UP

PROTONS

NEUTRONS

↑ $\frac{\hbar}{2}$ TWO LIKE NUCLEONS BIND MORE TIGHTLY THAN TWO UNLIKE NUCLEONS

- THIS IS CONSISTENT WITH FACT THAT NUCLEAR MAGNETIC MOMENTS ARE SMALL

$$-3\mu_N \rightarrow +10\mu_N$$