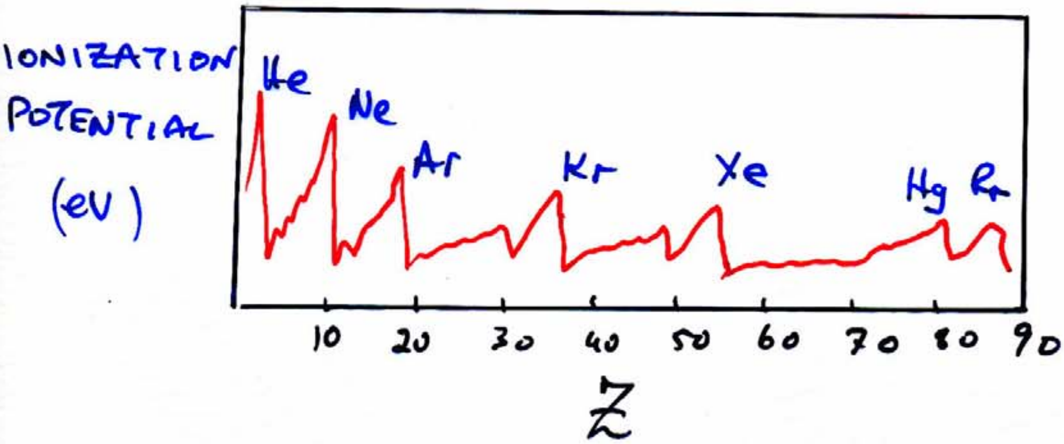
SHELL MODEL OF NUCLEI

IN ATOMS THERE ARE MAGIC NUMBERS



Z = 2, 10, 18, 36, 54

→ CHEMICALLY INERT → NOBLE GASES

→ LARGE IONIZATIONS POTENTIALS

↓

VERY STABLE → WE KNOW THIS CORRESPONDS TO CLOSED ELECTRON SHELLS

IN PHYSICS ONE OFTEN SOLVES NEW PROBLEMS BY ANALOGY WITH OLD

→ MIGHT HOPE THAT ONE CAN UNDERSTAND NUCLEUS BY ANALOGY WITH ATOM.

AIDE MEMOIRE ON ATOM

→ CENTRAL COULOMB POTENTIAL

- ORBITS + HENCE ENERY LEVELS LABBELED BY PRINCIPAL QUANTUM NUMBER

$$n = 1, 2, 3 \cdots$$

- IN ABSENCE OF MAGNETIC FIELD, ENERGY DEGENERATE LEVELS WITH DIFFERENT ORBITAL ANGULAR MOMENTUM

$$l = 0, 1, 2, \cdots (n-1)$$

- FOR ANY VALUE OF l THERE ARE $(2l+1)$ VALUES OF l_z

$$m_l = -l,\ -l+1,\ \cdots 0, 1 \cdots\ l-1,\ l$$

- ELECTRON SPIN

$$m_s = \pm \frac{1}{2}$$

ALL ENERGY DEGENERATE DUE TO ROT. SYMMETRY OF POTENTIAL

ANY ENERGY EIGENSTATE LABELED BY

$$(n, \ell, m_\ell, m_s)$$

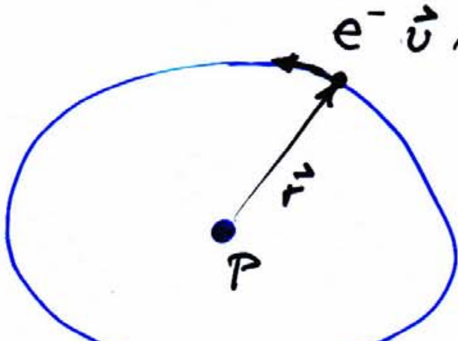
$\Rightarrow$ $2n^2$ DEGENERATE ENERGY LEVELS

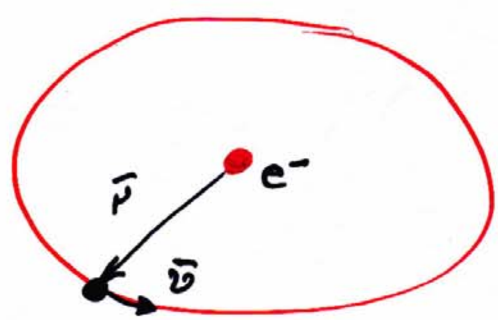
DEGENERACY BROKEN BY MAGNETIC FIELD

- EXTERNAL eg. $\vec{\mu}_\ell \cdot \vec{B}$
- SPIN-ORBIT

SPIN ORBT IS DUE TO FACT THAT IN THE REST FRAME OF AN ATOMIC ELECTRON, IT SEES NUCLEAR COULOMB FIELD AS MAGNETIC FIELD

$\Rightarrow$ LORENTZ TRANSFOR





$\vec{L}_e = m\vec{r}\times\vec{v}$

PROTON REST FRAME

$\vec{B}_p \sim \vec{r}\times\vec{v}$

ELECTRON REST FRAME

ANY ATOMIC SHELL $2n^2$ ELECTRONS

$\Rightarrow$ PAULI

$$\left.\begin{array}{l}\sum m_s = 0 \\ \sum m_\ell = 0\end{array}\right\} \text{ PAIRING } \uparrow\downarrow \cdots \uparrow\downarrow$$

FILLED SHELLS

$\rightarrow$ FOR FILLED SHELLS $\rightarrow$ SPHERICALLY SYMMETRICAL

$\Downarrow$

$\vec{J} = 0$

$$\vec{L} = \vec{S} = 0$$

TOTAL ORBITAL ANGULAR MOMENTUM

TOTAL SPIN ANGULAR MOMENTUM

$$\vec{J} = \vec{L} + \vec{S} = 0$$

TOTAL ANGULAR MOMENTUM

$\rightarrow$ CLOSED SHELLS LEAD TO MAGIC NUMBERS

$\rightarrow$ NUCLEI ALSO HAVE MAGIC NUMBERS

NUCLEAR MAGIC NUMBERS

AND OTHER EVIDENCE FOR SHELL STRUCTURE

PEAKS IN BINDING ENERGY AT

$N = 2, 8, 20, 28, 50, 82, 126$

$Z : 2, 8, 20, 28, 50, 82$

STABLE → MAGIC NUCLEI

${}^{4}_{2}He \quad {}^{16}_{8}O \quad {}^{208}_{82}Pb \longrightarrow$ DOUBLY MAGIC

ALSO

MAGIC NUCLEI HAVE MORE STABLE NUCLEAR SPECIES → ISOTOPES, ISOTONES

eg FOR $N = 20$ → 5 STABLE ISOTONES

$= 19$ → NONE

$= 21$ → ONE, UNSTABLE

MAGNETIC MOMENTS VANISH FOR MAGIC NUCLEI

LIST OF PHENOMENA TO EXPLAIN

LIQUID DROP GIVES NO INSIGHT

- VALUES OF COEFFICIENTS IN SEMI-EMPIRICAL MASS FORMULA
- NUCLEAR DENSITY
- MAGIC NUMBERS
- MAGNETIC MOMENTS
- SPINS & PARITIES
- EXCITED STATE SPINS & PARITIES

THIS IS ACTUALLY
A PRETTY LONG
LIST
⋮

ASSUME NUCLEONS ORBIT IN MEAN POTENTIAL CAUSED BY ALL OF THEM

? HOW CAN THIS BE? THEY ARE SUPPOSED TO BE CLOSELY PACKED

→ PAULI EXCLUSION PREVENTS THEM FROM HAVING ENERGY LOSING COLLISIONS

ASSUME POTENTIAL IS SOME UNKNOWN CENTRAL POTENTIAL. ENERGY EIGENEQUATION

$$\mathcal{H}\psi(\vec{r}) = E\psi(\vec{r})$$

NON RELATIVISTIC SCHRÖDINGER

$$\left[\vec{\nabla}^2 + \frac{2m}{\hbar}(E - V(r))\right]\psi(\vec{r}) = 0$$

$$[\mathcal{H}, J] = 0$$

ENERGY + ANGULAR MOMENTUM OPERATORS COMMUTE FOR A CENTRAL POTENTIAL

ENERGY EIGENSTATES ARE ALSO ANGULAR MOMENTUM EIGENSTATES

WRITE SCHRÖDINGER IN SPHERICAL COORDS ⑧
& USE FACT THAT IT IS SEPARABLE
→ JUST LIKE HYDROGEN ATOM!

ANG MOM } → $\vec{L}^2 \, Y_{\ell, m_\ell}(\theta,\phi) = \hbar^2 \ell(\ell+1) \, Y_{\ell, m_\ell}(\theta,\phi)$

Z COMP } → $L_z \, Y_{\ell, m_\ell}(\theta,\phi) = \hbar \, m_\ell \, Y_{\ell, m_\ell}(\theta,\phi)$

→ $[\vec{L}^2, L_z] = 0$

$$\psi_{n \ell m_\ell}(\vec{r}) = \frac{u_{n\ell}(r)}{r} \, Y_{\ell, m_\ell}(\theta,\phi)$$

RADIAL WAVE FUNCTION

$$\left(\frac{d^2}{dr^2} + \frac{2m}{\hbar^2}\left(E_{n\ell} - V(r) - \frac{\hbar^2 \ell(\ell+1)}{2mr^2}\right)\right) u_{n\ell}(r) = 0$$

1-d SCHRÖDINGER

n - RADIAL QUANTUM NUMBER

⇓

LABELS ENERGY EIGENSTATES

$$\left(\frac{d^2}{dr^2} + \frac{2m}{\hbar^2}\left(E_{n\ell} - V(r) - \frac{\hbar^2 \ell(\ell+1)}{2mr^2}\right)\right) U_{n\ell}(r) = 0$$

CENTRIFUGAL BARRIER
ACTS AS EFFECTIVE
POTENTIAL

$U_{n\ell}(r)$ MUST
VANISH AT ORIGIN
+ ALSO ∞ SEPARATION

$$V'(r) = V(r) + \frac{\hbar^2 \ell(\ell+1)}{2mr^2}$$

ACTS AS REPULSIVE
POTENTIAL

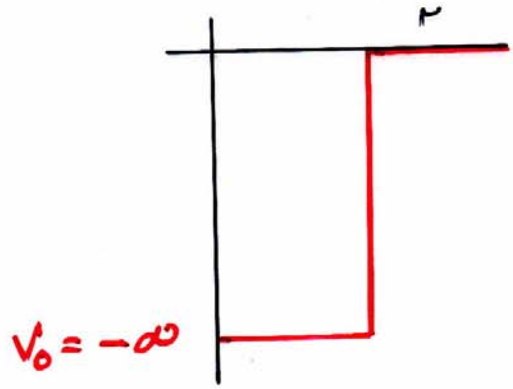
DIFFERENT BOUNDARY CONDITION FROM ATOM

MAIN DIFFICULTY IS THAT HAVE NO DEFINITE KNOWLEDGE OF WHAT $V(r)$ LOOKS LIKE

TRY TO DEDUCE NUCLEAR POTENTIAL
- MAKE ASSUMPTION
- CALCULATE
- COMPARE EXPERIMENT

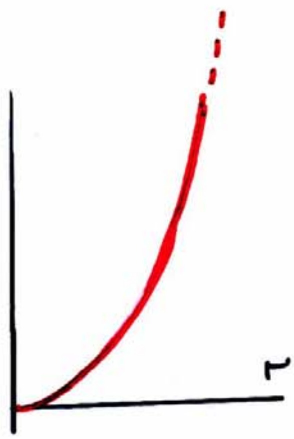
MODIFY

POSSIBLE "GUESSED" POTENTIALS



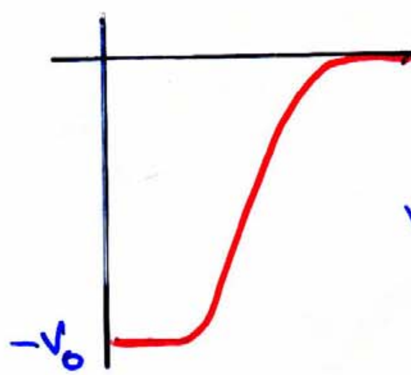
INFINITE SQUARE WELL

$$V(r) = \begin{cases} \infty, & r \geq R \\ 0, & R \geq r \geq 0 \end{cases}$$



HARMONIC OSCILLATOR

$$V(r) = \frac{1}{2} m \omega^2 r^2$$



SAXON - WOODS

$$V(r) = \frac{-V_0}{1 + \exp\left(\frac{r-R}{d}\right)}$$

INFINITE SQUARE WELL

"TOY" MODEL SINCE WE KNOW BINDING IS NOT ∞.

CAN SOLVE RADIAL SCHRÖDINGER

$$u_{n\ell}(r) = j_\ell(k_{n\ell} r)$$

↑ SPHERICAL BESSEL eg $j_0(x) = \frac{\sin x}{x}$

$$k_{n\ell} = \left(\frac{2mE_{n\ell}}{\hbar^2}\right)$$

⇒ ∞ WELL, NUCLEONS BOUND, $u_{n\ell}$ vanishes @ $\underline{r=R}$

$$u_{n\ell}(R) = j_\ell(\underbrace{k_{n\ell} R}_{\text{q. nos}}) = 0$$

$\ell = 0, 1, 2, 3$

for ANY ℓ

$n = 1, 2, 3 \ldots$ FOR ANY ℓ

⇒ SOLUTION USING BESSEL FUNCTIONS REPRODUCES SOME OF MAGIC NUMBERS

ie CLOSED SHELLS FOR $n = 1$

$$2,\ \underset{2+6}{8},\ \underset{8+10}{18},\ \underset{18+14}{32},\ \underset{32+18}{50} \ldots$$

⇓

EACH SHELL HAS $2(2\ell+1)$ NUCLEONS

BUT NOT ALL

ENLARGE A LITTLE UPON THIS!-

$$k_{n\ell} = \sqrt{\frac{2mE_{n\ell}}{\hbar^2}}$$

- ENERGY EIGENVALUE is

n^{th} ZERO OF ℓ^{th} SPHERICAL BESSEL FN.

$$j_\ell(x) = \sqrt{\frac{\pi}{2x}}\, J_{\ell+\frac{1}{2}}(x)$$

$$\longrightarrow J_N(x) = \sum_{\lambda}^{\infty} \frac{(-1)^\lambda \left(\frac{x}{2}\right)^{n+2\lambda}}{\Gamma(\lambda+1)\Gamma(\lambda+n+1)}$$

- DIFFERENT n, ℓ ARE NON DEGENERATE

- ROTATIONAL INVARIANCE STILL LEADS TO

$(2\ell+1)$  DIFFERENT m_ℓ

FOLD DEGENERACY

EACH STATE $\longrightarrow$ TWO NUCLEONS

FOR ∞ WELL $2(2\ell+1)$ IN EACH SHELL

FOR $n=1$; CLOSED SHELLS ARE

$\ell=0$	$\ell=1$	$\ell=2$
2	6	10

$$\begin{array}{r} 6 \\ +\,2 \\ \hline \underline{\underline{8}} \end{array} \qquad \begin{array}{r} 10 \\ +\,8 \\ \hline \underline{18} \end{array} \;\ldots$$

MAGIC NUMBERS $\Big\}\Rightarrow$ 8, 18 ....

HARMONIC OSCILLATOR REPRODUCES SOME, BUT NOT ALL, OF MAGIC NUMBERS

→ IT HAS CLOSED SHELLS FOR

2, 8, 20, 40, 70 ...

- A SIMPLE CENTRAL POTENTIAL CANNOT REPRODUCE ALL THE MAGIC NUMBERS — IE CANNOT REPRODUCE SHELL STRUCTURE

USE ANALOGY OF ATOMIC PHYSICS

⇒ SPIN · ORBIT INTERACTION

$$V_{TOT} = V(r) - f(r)\,\vec{L}\cdot\vec{S}$$

ORBITAL ANGULAR MOMENTUM

SPIN ANGULAR MOMENTUM

SOME UNKNOWN FUNCTION

- SPIN – ORBIT SPLITS $j = \ell \pm \frac{1}{2}$ ENERGY LEVELS

IN ATOMS ⟹ FINE STRUCTURE IN ENERGY LEVELS

→ SPECTRAL LINES

SPIN ORBIT WILL MOVE ENERGY LEVELS & ALSO SPLIT DEGENERACY

$$j = \ell + \tfrac{1}{2}$$

$$j = \ell - \tfrac{1}{2}$$

ENERGY

STILL RATHER EMPIRICAL, SINCE SIGN AND MAGNITUDE OF $f(r)$ WILL BE "FITTED" TO DATA

$$\vec{J} = \vec{L} + \vec{S}$$

$$\vec{J}^2 = (\vec{L} + \vec{S}) \cdot (\vec{L} + \vec{S})$$

$$= \vec{L}^2 + \vec{S}^2 + \underbrace{\vec{S} \cdot \vec{L} + \vec{L} \cdot \vec{S}}_{[\vec{L}, \vec{S}] = 0}$$

$$= \vec{L}^2 + \vec{S}^2 + 2 \vec{L} \cdot \vec{S}$$

$$\underbrace{\vec{L} \cdot \vec{S}}_{\text{SPIN ORBIT TERM}} = \frac{1}{2} \left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2 \right)$$

TOTAL ORBIT SPIN

EXPRESSED AS

$$\vec{L}\cdot\vec{S} = \frac{1}{2}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)$$

- ALLOWS US TO SEE THAT THE EXPECTATION VALUE OF SPIN ORBIT CAN BE WRITTEN IT TERMS OF EXPECTATION VALUES OF $\vec{J}^2$ $\vec{L}^2$ $\vec{S}^2$

- A DEFINITE STATE OF $\vec{J}^2$ $\vec{L}^2$ $\vec{S}^2$ CORRESPONDS TO AN EIGENSTATE, AND THE EXPECTATION VALUES CORRESPOND TO EIGENVALUES OF OPERATORS

$$\langle \vec{L}\cdot\vec{S} \rangle = \left\langle \frac{1}{2}\left(\vec{J}^2 - \vec{L}^2 - \vec{S}^2\right)\right\rangle$$

$$= \frac{\hbar^2}{2}\left[j(j+1) - \ell(\ell+1) - s(s+1)\right] \qquad s = \frac{1}{2}$$

$$= \frac{\hbar^2}{2}\left[j(j+1) - \ell(\ell+1) - \frac{3}{4}\right]$$

$\ell+\frac{1}{2}\left(\ell+\frac{1}{2}+1\right)$ $\quad \ell^2 + \frac{\ell}{2} + \ell + \frac{\ell}{2} + \frac{1}{4} + \frac{1}{2}$ $\quad \ell^2 + \ell$ $\quad \ell + \frac{3}{4} - \frac{3}{4} = \ell$

EXPECTATION VALUE OF SPIN ORBIT OPERATOR

$$\langle \vec{L}\cdot\vec{S} \rangle = \begin{cases} \dfrac{\hbar^2}{2}\ell & \rightarrow j = \ell + \dfrac{1}{2} \\[2ex] -\dfrac{\hbar^2}{2}(\ell+1) & \rightarrow j = \ell - \dfrac{1}{2} \end{cases}$$

SAY WE HAVE THE EIGEN EQUATION

$$\mathcal{H}\,\psi_n = E_n\,\psi_n$$

HAMILTONIAN

ENERGY EIGENSTATE

ENERGY EIGENVALUE

ENERGY EIGENSTATE

THE EXPECTATION VALUE OF THE OPERATOR $\mathcal{H}$ IS:-

$$\int d^3r\, \mathcal{H}\,\psi_n^*\psi_n = \int d^3r\, E_n\,\psi_n^*\,\psi_n$$

$$= E_n \int d^3r\, |\psi|^2$$

THIS IS MORE OR LESS A DEFINITION OF EXPECTATION VALUE.

TO GET THE ENERGY SHIFT DUE TO SPIN-ORBIT INTERACTION, WE TAKE THE EXPECTATION VALUE

$$f(r)\left(\vec{L}\cdot\vec{S}\right)\psi_{nl}(\vec{r}) = \left\langle\left(\vec{L}\cdot\vec{S}\right)\right\rangle \psi_{nl}(r)\, f(r)$$

ENERGY SHIFT DUE TO SPIN-ORBIT

$$\Delta E_{n\ell}\left(j=\ell+\tfrac{1}{2}\right)=\int d^3r\, f(r)\langle \vec{L}\cdot\vec{S}\rangle |\psi_{n\ell}(\vec{r})|^2$$

$$=\frac{\hbar^2 \ell}{2}\int d^3r\, |\psi_{n\ell}(\vec{r})|^2 f(r)$$

$$\Delta E_{n\ell}\left(j=\ell-\tfrac{1}{2}\right)=-\frac{\hbar^2(\ell+1)}{2}\int d^3r\, |\psi_{n\ell}(\vec{r})|^2 f(r)$$

THE TOTAL SPLITTING DUE TO THE SPIN-ORBIT POTENTIAL BREAKING THE DEGENERAGY OF THE SPIN UP/DOWN STATES IS THE DIFFERENCE BETWEEN THESE TWO TERM

$$\Delta = \hbar^2\left(\frac{\ell}{2}+\frac{\ell}{2}+\frac{1}{2}\right)\int \cdots\cdots$$

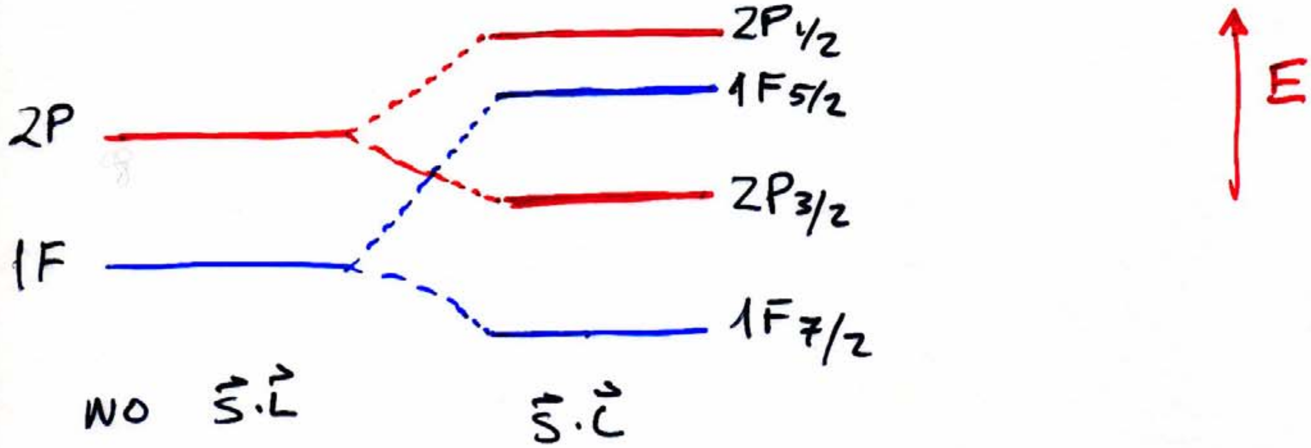
$$\boxed{\Delta = \hbar^2\left(\ell+\tfrac{1}{2}\right)\int d^3r\, |\psi_{n\ell}(\vec{r})|^2 f(r)}$$

$f(r)$ IS A PRIORI UNKNOWN.

$$\Delta = \hbar^2 \left(l + \tfrac{1}{2}\right) \int d^3r \, |\psi_{nl}(\vec{r})|^2 f(r)$$

SPLITTING GETS LARGER FOR LARGE ORBITAL ANGULAR MOMENTUM

FOR l LARGE CAN HAVE "LEVEL CROSSING"



CAN REPRODUCE ALL THE MAGIC NUMBERS

→ CONFIRMATION OF SHELL STRUCTURE AND STRONG SPIN-ORBIT

SPECTROSCOPIC NOTATION $\;{}_nL_j$

TOTAL ANGULAR MOMENTUM

n ENERGY Q.N.

ORBITAL ANGULAR MOMENTUM

AGAIN RECALL HYDROGEN ATOM

WE LABELED THE STATES BY

	s	p	d	f	g	h
ORBITAL ANG. MOM	$\ell = 0$	1	2	3	4	5

PRINCIPAL Q. NO (ENERGY)

$n = 1$	1s		
2	2s	2p	
3	3s	3p	3d
4	4s	4p	4d
5			
6		etc.	

19TH CENTURY OPTICAL SPECTROSCOPY

- **S**HARP
- **P**RINCIPAL
- **D**IFFUSE
- **F**UNDAMENTAL

$n\,L_j$

ENERGY LEVEL → n

ORBITAL ANGULAR MOMENTUM → L

TOTAL ANGULAR MOMENTUM → j

$\vec{J} = \vec{L} + \vec{S}$

SPIN → $\vec{S}$

- FOR A GIVEN ENERGY LEVEL THE SPIN-ORBIT COUPLING WILL SPLIT OUT THE DEGENERATE ENERGY LEVELS
- NUMBER OF LEVELS YOU GET FOR EACH STATE IS GIVEN BY NUMBER OF QUANTIZED VALUES OF J_z

$$(2j+1)$$

REMEMBER → FOR A GIVEN VALUE OF THE ANGULAR MOMENTUM THERE ARE $(2j+1)$ VALUES OF Z-COMPONENT

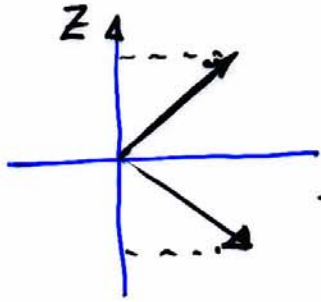
→ THESE ARE DISTINCT LEVELS (STATE, QUANTUM NUMBERS) AND THE NUMBER OF THESE STATES CORRESPONDS TO NUMBER OF NUCLEONS THAT CAN BE PUT IN THAT LEVEL (SHELL)

→ WILL DETERMINE CLOSED SHELLS → MAGIC NUMBERS

NUMBER OF DISTINCT STATES (LEVELS)

$1S_{\frac{1}{2}}$ $\ell = 0$; 2 SPIN STATES

in hydrogen atom.

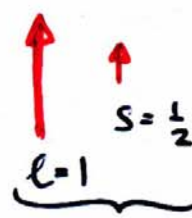


$+\frac{1}{2}\hbar$ $m_s = 1/2$

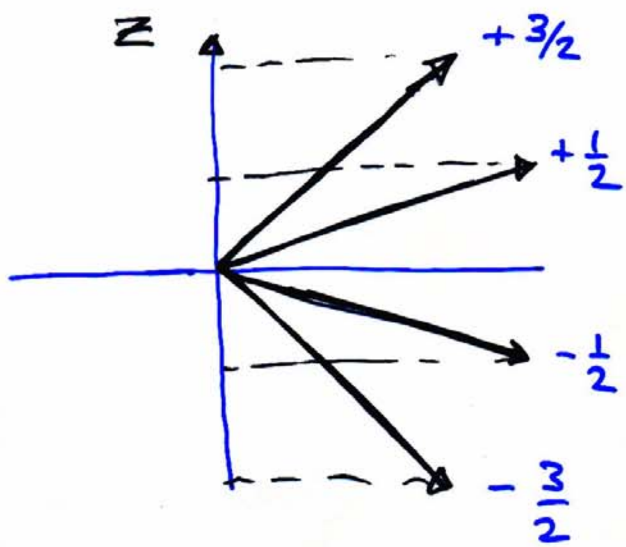
$-\frac{1}{2}\hbar$ $m_s = -1/2$

$\boxed{\text{2 STATES}}$

$1P_{3/2}$ $\ell = 1$; SPIN AND ORBITAL ALIGNED



$\underbrace{\ell = 1 \quad s = \tfrac{1}{2}}_{j = 3/2}$



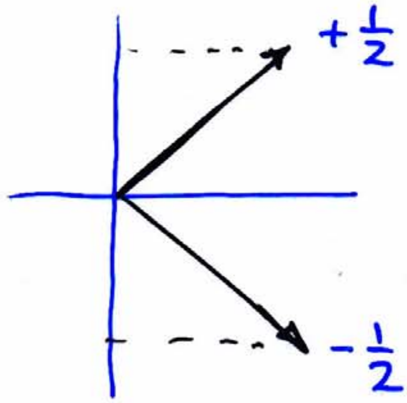
$\boxed{\text{4 STATES}}$

$1P_{\frac{1}{2}}$ $\quad l = 1$

SPIN + ORBITAL ANTI-~~PAR~~PARALLEL

$\underbrace{l=1 \qquad s=-\frac{1}{2}}_{j = +\frac{1}{2}}$

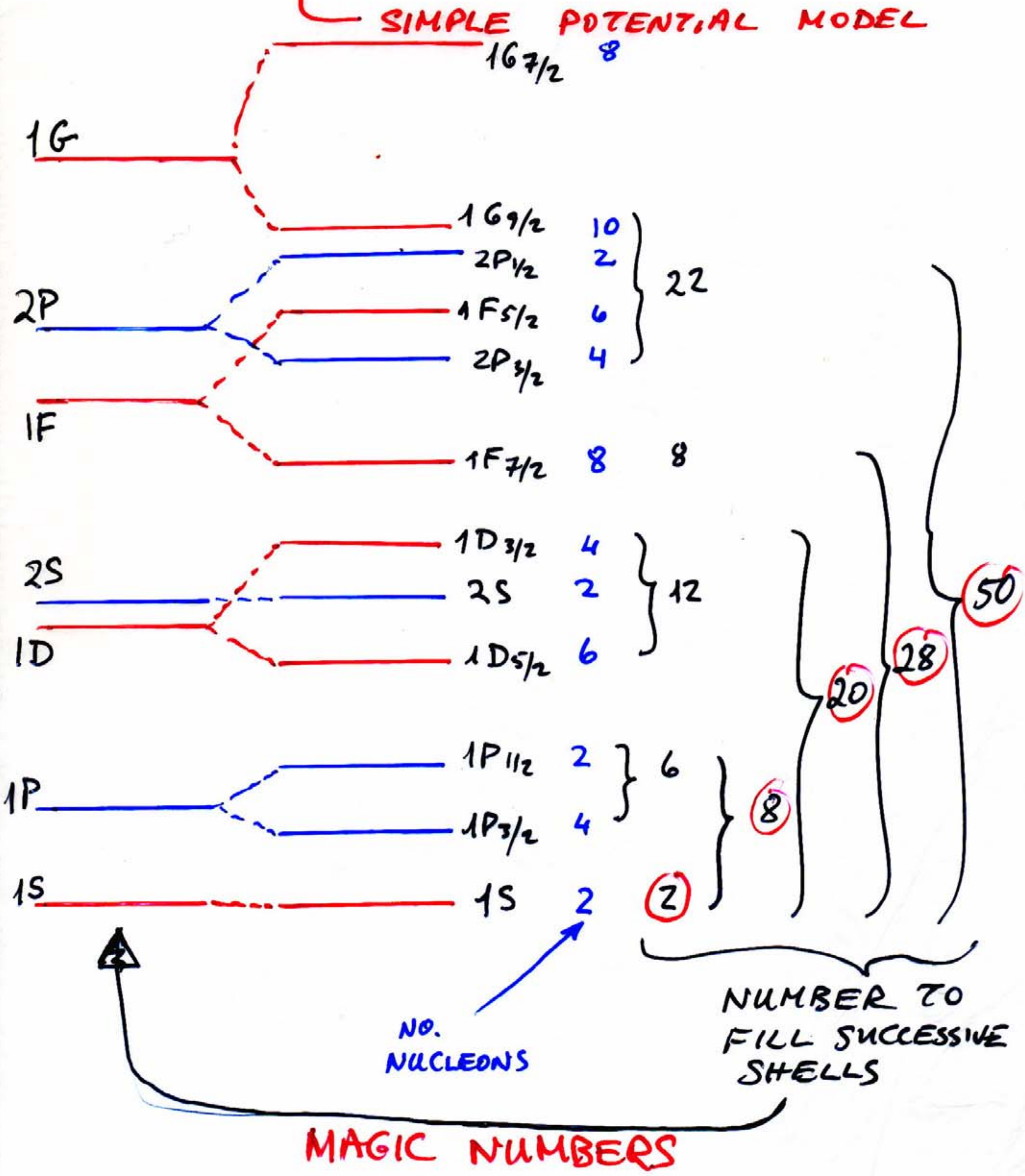
NOTE THAT THESE ARE Z-COMPONENTS



$\boxed{\text{2 STATES}}$

EVEN FINITE SQUARE WELL CAN REPRODUCE ALL MAGIC NUMBERS

SIMPLE POTENTIAL MODEL



SHELL MODEL SHOULD ALSO PREDICT MAGNETIC MOMENTS CORRECTLY

PROTON $\mu_p = 2.79\,\mu_N$ ← NUCLEAR MAGNETON

↑

NEUTRON $\mu_n = -1.91\,\mu_N$

DUE TO SPIN INTRINSIC ANGULAR MOMENTUM

⇒ ANY UNPAIRED NUCLEON SHOULD CONTRIBUTE TO THE MAGNETIC MOMENT

⇒ UNPAIRED PROTONS WILL HAVE A CONTRIBUTION DUE TO ORBITAL ANGULAR MOMENTUM SINCE THEY ARE CHARGED

→ DEUTERON

ASSUME n, p ARE IN $1S_{\frac{1}{2}}$ STATES

SO $\ell = 0$; JUST ADD INTRINSIC MAGNETIC MOMENTS

$$\mu_d = 2.79\mu_N - 1.91\mu_N = 0.88\mu_N$$

EXPERIMENT FINDS $0.86\mu_N$

FOR TRITIUM $^{3}_{1}H$

AGAIN GROUND STATE IS $1S_{1/2}$

THE NEUTRONS ARE PAIRED AND FORM A CLOSED SHELL

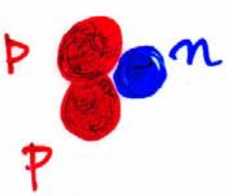
↳ ONLY THE PROTON CONTRIBUTES TO THE MAGNETIC MOMENT

$\ell = 0$

$$\mu_{^{3}_{1}H} = \mu_p = 2.79\mu_N$$

✓ OK

$2.98\mu_N$



$^{3}_{2}He$

THE UNPAIRED NUCLEON IS A NEUTRON

$\therefore$ EXPECT $-1.91\mu_N$

$-2.13\mu_N$ IS EXPERIMENTAL VALUE.

α- PARTICLE

$${}^{4}_{2}He$$

⇒ CLOSED SHELL ; DOUBLY MAGIC
NO SPIN ; NO MAGNETIC MOMENT ✓

${}^{10}_{5}B$ 5 PROTONS ; 5 NEUTRONS

FILLED STATES

$$\left(1S_{\frac{1}{2}}\right)^{2}\left(1P_{3/2}\right)^{3}$$

2 NUCLEONS PAIR

{ ONE UNPAIRED NEUTRON
ONE UNPAIRED PROTON }

PROTON IS IN $\ell = 1$ STATE ⇐ MAGNETIC MOMENT

ORBITAL MOTION $\mu = \left(\frac{e\hbar}{2m_p \cdot c}\right) \cdot \underset{=1}{\underset{\downarrow}{\ell}} = \mu_N$

SO FOR BERYLIUM :-

$$2.79\mu_N + \mu_N - 1.91\mu_N = 1.88\mu_N$$

PROTON INTRINSIC

PROTON ORBITAL

NEUTRON INTRINSIC

$1.80\mu_N$ EXPERIMENT

COLLECTIVE MODEL

VERY HEAVY NUCLEI DO NOT FOLLOW PREDICTIONS OF SINGLE PARTICLE SHELL MODEL

→ WRONG MAGNETIC QUADRUPOLE

→ HAVE PERMANENT ELECTRIC DIPOLE

↓

NON SPHERICAL CHARGE DISTRIBUTION

BOHR + RAINWATER

↳ MANY PROPERTIES OF HEAVY NUCLEI COULD BE EXPLAINED BY NON-SPHERICAL LIQUID DROP.

LIQUID DROP ← DILEMMA → SINGLE PARTICLE SHELL

VERY DIFFERENT POINTS OF VIEW

NOT WILLING TO THROW AWAY SINGLE PARTICLE PREDICTIONS OF SHELL-MODEL

J^P ; MAGIC NUMBERS

ASSUME NUCLEUS CONSISTS OF

- CORE ; CLOSED SHELLS OF SHELL MODEL
- OUTER NUCLEONS BEHAVING LIKE SURFACE MOLECULES IN A LIQUID DROP

ROTATIONAL MOTION OF SURFACE NUCLEONS INDUCES ASPHERICITY IN CENTRAL CORE

→ AFFECTS SHELL QUATUM STATES

SURFACE MOTION LEADS TO PERTURBATION OF SINGLE-PARTICLE SHELL STATES

→ GOOD PREDICTIONS FOR ELECTRIC & MAGNETIC MULTIPOLE MOMENTS

PHYSICALLY – COLLECTIVE MODEL IS SHELL + NON CENTRAL POTENTIAL

- SPHERICALLY SYMMETRIC NUCLEI ARE INSENSITIVE TO ROTATIONS
  - ↳ NON SPHERICAL NUCLEI CAN HAVE VIBRATIONAL & ROTATIONAL ENERGY LEVELS

ELLIPSOIDAL POTENTIAL

$$V(x,y,z) = \begin{cases} 0 & \text{FOR } ax^2 + by^2 + \frac{z^2}{ab} \leq R^2 \\ \infty & \text{OUTSIDE} \end{cases}$$

ROTATIONAL STATES HAVE HAMILTONIANS

$$\mathcal{H} = \frac{\vec{L}^2}{2I}$$

TOTAL ANGULAR MOMENTUM

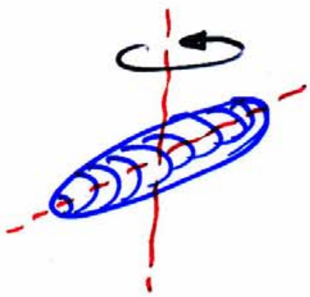
MOMENT OF INERTIA

EIGEN VALUES OF THIS HAMILTONIAN

$$\frac{\ell(\ell+1)\hbar^2}{2I}$$

$\Rightarrow$ $\Delta \ell = 2$ (QUADRUPOLE) TRANSITIONS OBSERVED

$\Rightarrow$ CORRESPOND TO



$\Delta \ell = 2$

NUCLEI HAVE EXCITED STATES

OBSERVE THAT LEVEL SPACING FROM GROUND STATE → 1ST EXCITED STATE OF EVEN-EVEN DECREASES WITH INCREASING A

→ LARGEST FOR NUCLEI WITH CLOSED SHELLS

ONE WOULD EXPECT THE ENERGY SPACING TO DECREASE WITH A

$$\mathcal{H} \sim \frac{\vec{L}^2}{2I}$$

GROWS WITH A

FOR CLOSED SHELLS ⇒ SPHERICAL NUCLEI

⇒ NO ROTATIONAL STATES

FOR CLOSED SHELLS FIRST EXCITED STATE — VIBRATIONAL

— CORE EXCITATION

— LARGE NUMBER OF NUCLEONS

— LARGE ENERGY SPACING

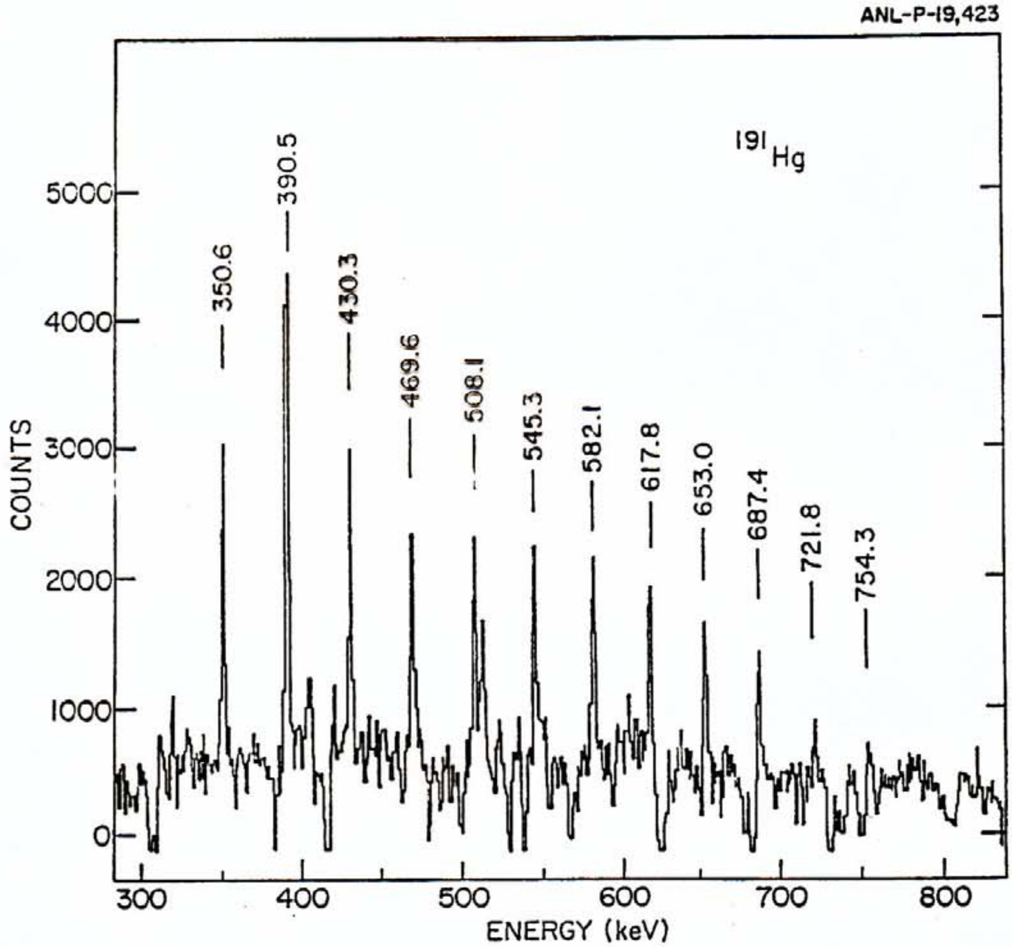


Figure 2 The γ-ray spectrum of the first superdeformed band observed in the $A = 190$ region (18). The 390-keV transition is an unresolved doublet consisting of a transition in the superdeformed band and the $17/2^{+}$-$13/2^{+}$ ground-state transition in ^{191}Hg.

CAN EXCITE NUCLEI
TO $\sim 60\hbar$