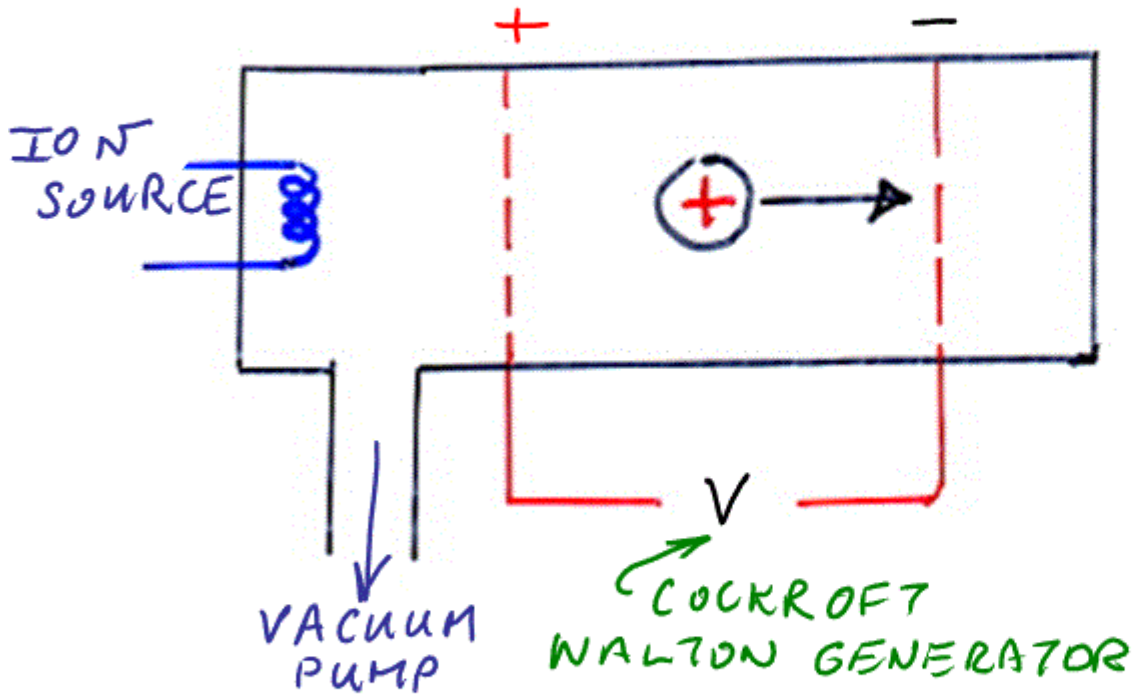
SIMPLE ELECTROSTATIC ACCELERATOR

USED BY COCKROFT & WALTON — ARTIFICIAL RADIOACTIVITY



ELECTRIC FIELD

$$\vec{F} = q\vec{E}$$

CHARGE ON PARTICLE

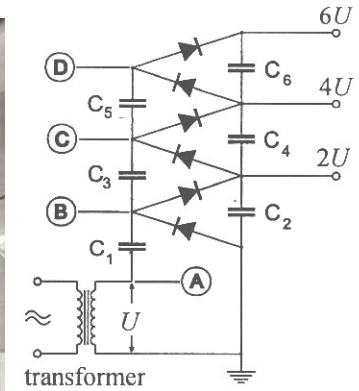
$$|\vec{E}| = \frac{V}{d}$$

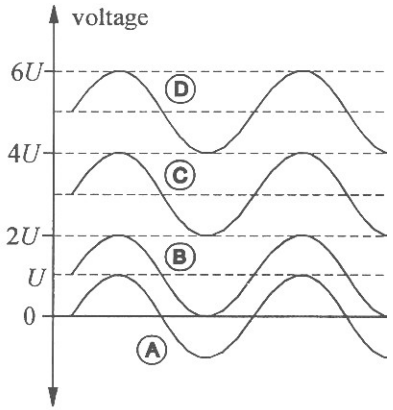
ENERGY GAINED BY CHARGED PARTICLE

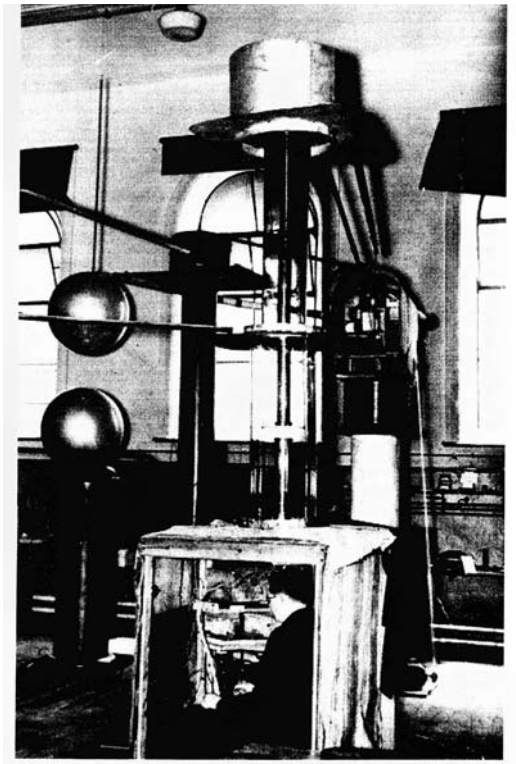
$$E_{Acc} = Fd = qV$$

- TWO SHORTCOMINGS:
  - GENERATING HIGH VOLTAGE
  - INSULATING BEYOND ~ 100 kV (100 keV)

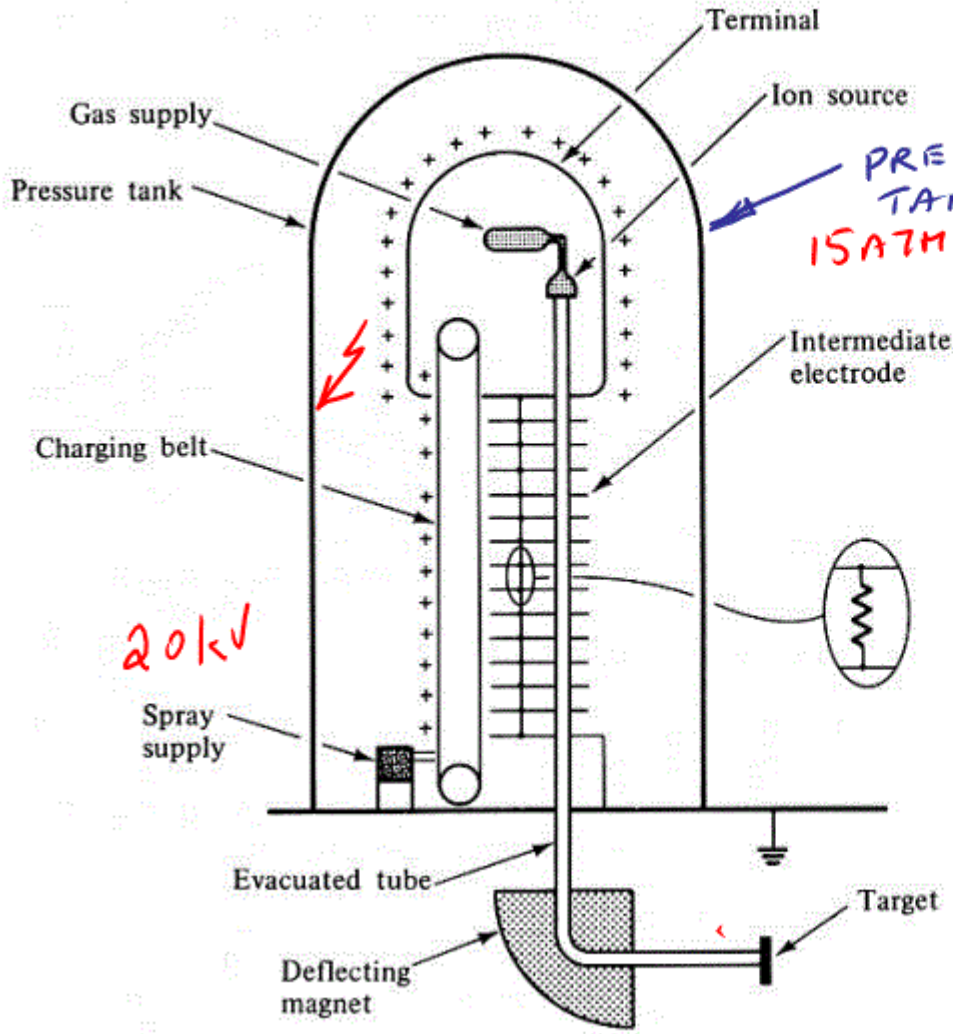
Cockcroft-Walton Generator







VAN DE GRAAFF



- TRANSPORT CHARGE

Q

TO TERMINAL OF CAPACITANE

C

$$V = \frac{Q}{C}$$

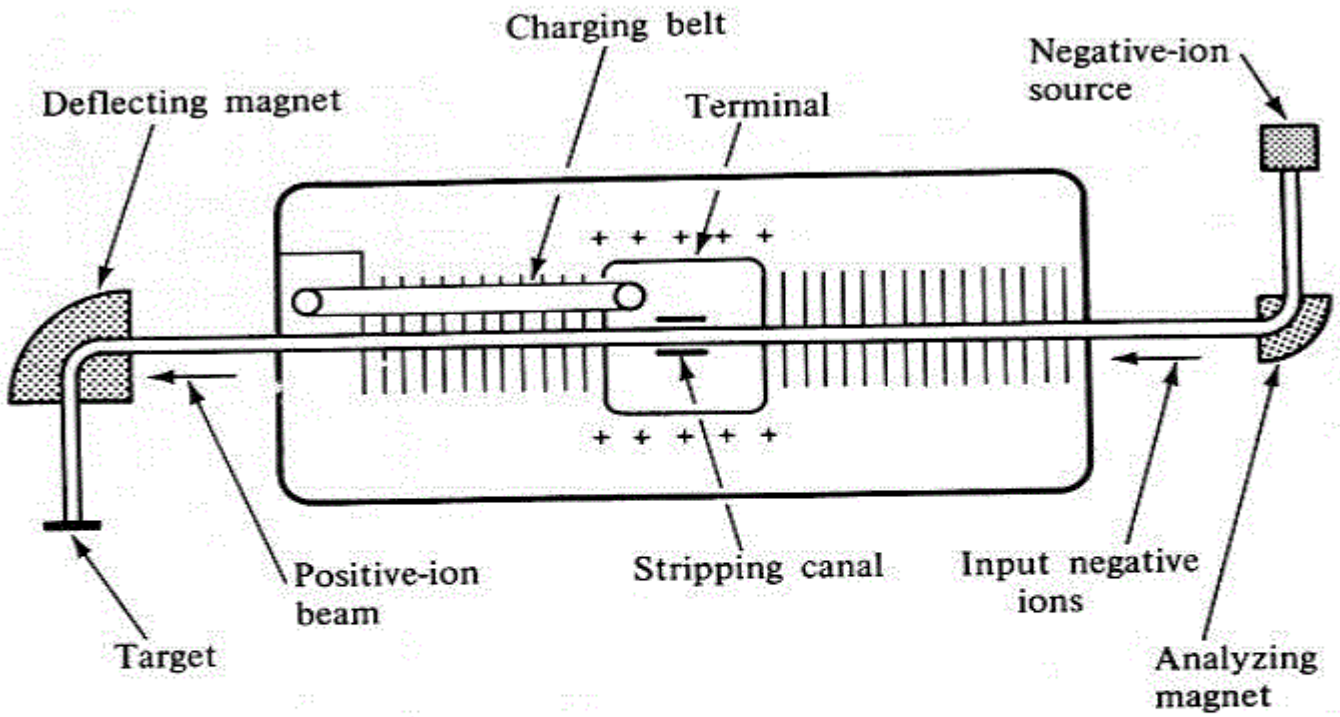
- LIMITATION ~ 12MV

→ VOLTAGE BREAKDOWN

→ NOT ENOUGH TO RESOLVE PROTONS IN THE NUCLEUS

~ 12 MeV

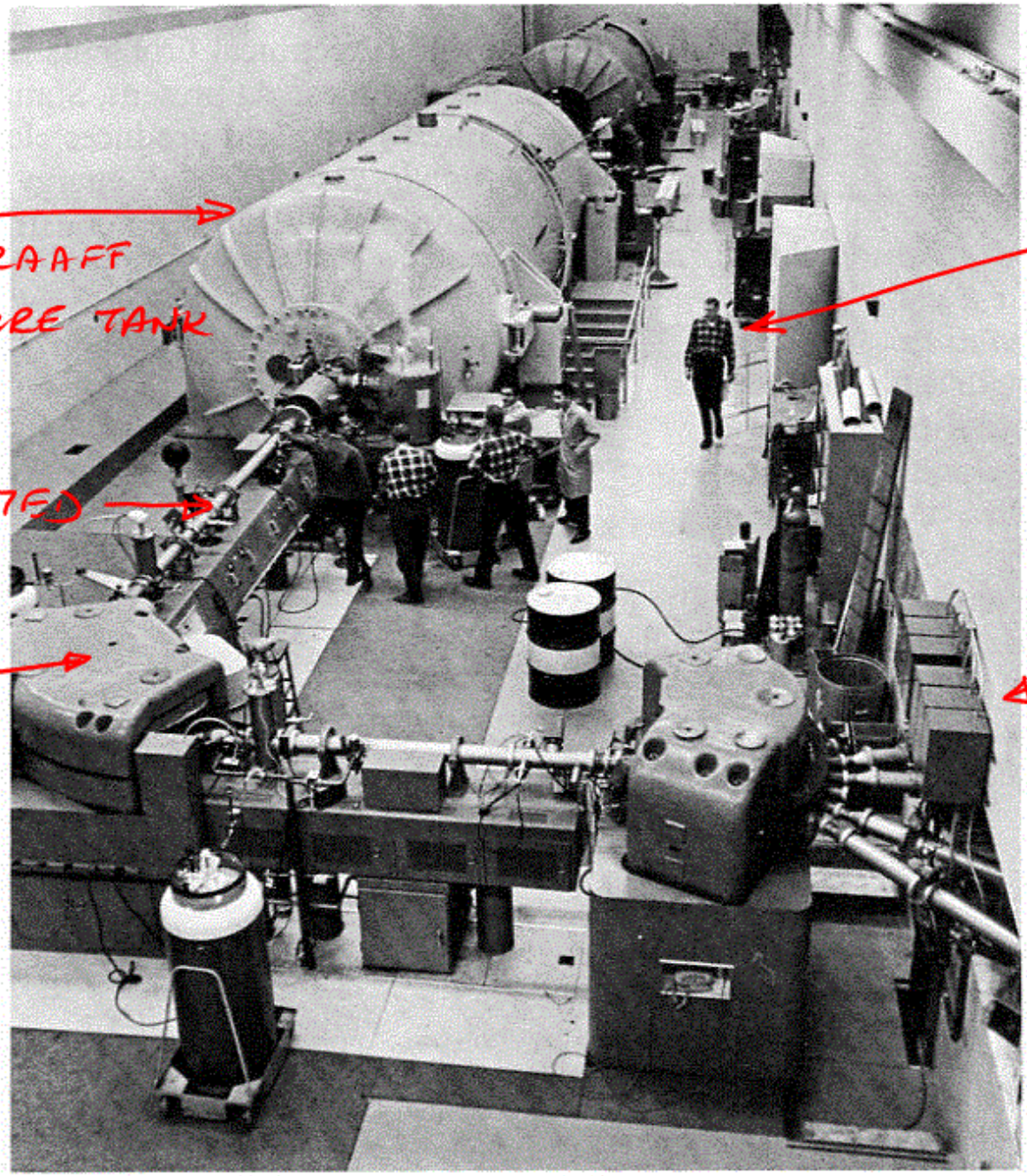
TANDEM VAN DE GRAAFF



- USE VOLTAGE ON TERMIAL TWICE
- ACCELERATE -VE IONS UP TO TERMINAL
- STRIP OFF TWO ELECTRONS INSIDE TERMINAL

— ACCELERATE AWAY

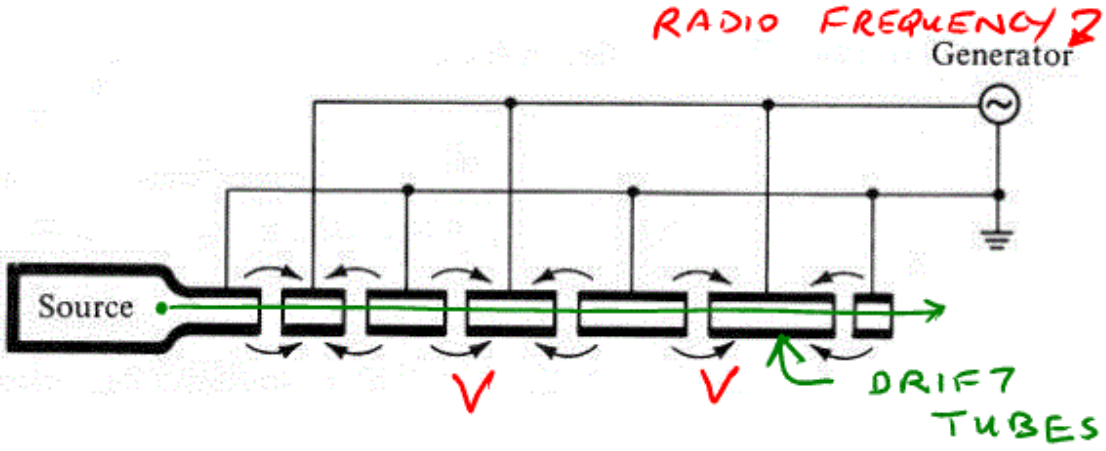
- 40 MeV CHALK RIVER HAD LARGE TANDEM


VAN DE GRAAFF IN PRESSURE TANK
1960's PHYSICIST
ACCELERATED BEAM
BENDING MAGNET
BEAMS TO EXPERIMENTS

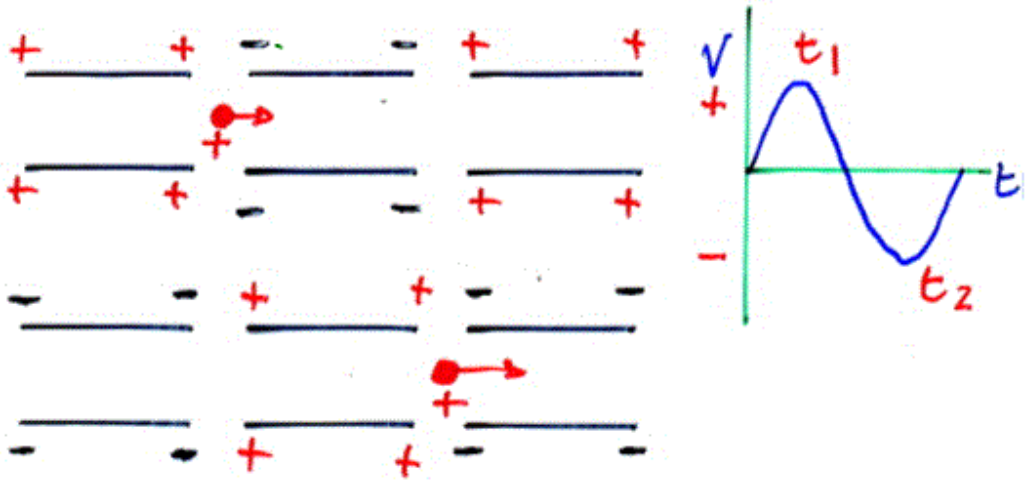
LINEAR ACCELERATOR (LINAC)

TORONTO USED TO HAVE 40 MeV LINAC

- INVENTED BY WIDEROE



- USE SAME RELATIVELY SMALL VOLTAGE IN MANY STEPS — REACH EQUIVALENT HIGH VOLTAGE



- FIELD ZERO INSIDE DRIFT TUBES
- PARTICLE MOVES ONE GAP → NEXT, IN TIME E-FIELD REVERSES
- PARTICLES ACCELERATING → LENGTH OF DRIF TUBES INCREASES → NON RELATINISTIC

- PARTICLE ENTERING DRIFT TUBE n, ENERGY $n \cdot eV$
  - n: # GAPS TRAVERSED
  - eV: VOLTAGE ACROSS GAP
- NON-RELATIVISTIC KINETIC ENERGY $T = \frac{1}{2} m v^2$

$$v = \left(\frac{2 \cdot n e V}{m} \right)^{\frac{1}{2}} \qquad \left(\frac{2T}{m} \right)^{1/2}$$

- THIS VELOCITY TAKES PARTICLE THRU DRIFT TUBE OF LEGTH L_n IN TIME FIELD TAKES TO REVERSE

$$t_n = L_n / v$$

- FREQUENCY OF RADIO FREQUECY OSCILLATOR

f (H_2) HAS REVERSAL TIME $\frac{1}{2f}$

$$L_n = \frac{1}{2f} \left(\frac{2 n e V}{m} \right)^{\frac{1}{2}} \longrightarrow L_n \propto \sqrt{n}$$

NUMERICAL VALUES

$$L_n = \frac{1}{2f} \cdot v_n$$

TYPICALLY $v_n = 0.5c$; $f = 7\,MHz \rightarrow L_n = 10.7\,m$

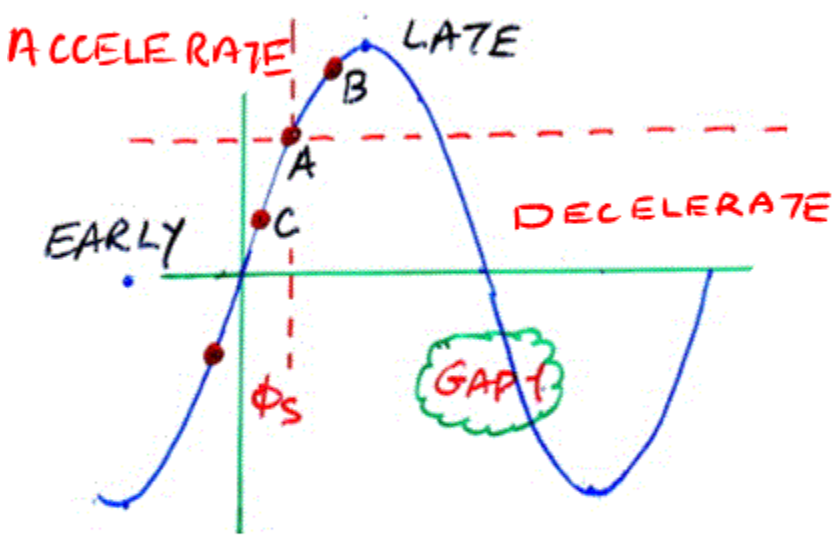
- LOW RADIO FREQUENCY LEADS TO VERY LONG STRUCTURES
- PRACTICALLY NEED HIGH RADIO FREQUENCIES

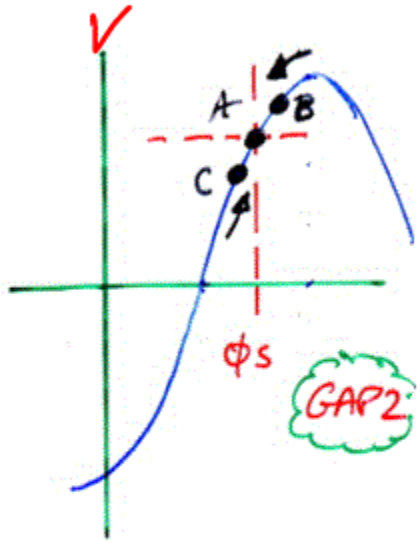
  KLYSTRONS $\rightarrow$ 100 MHz $\rightarrow$ 10 GHz

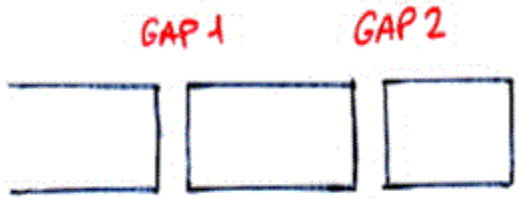
- THIS WIDEROE STRUCTURE IS OBSOLETE
  - $\rightarrow$ VERY INEFFICIENT
  - $\rightarrow$ RADIATION LOSS

PHASE STABILITY IN LINAC

- TO MAINTAIN PRECISE SYNCHRONISM BETWEEN PARTICLE MOTION & RF OSCILLATOR SEEMS DIFFICULT → NOT SO

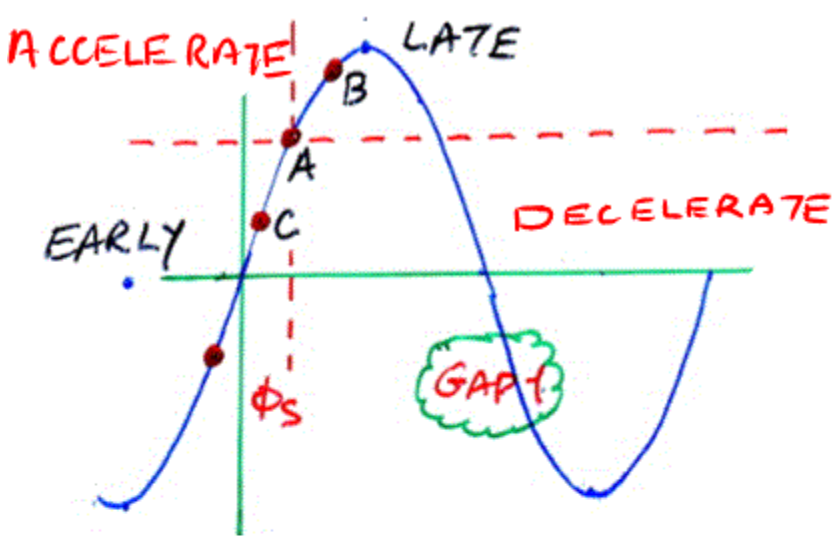


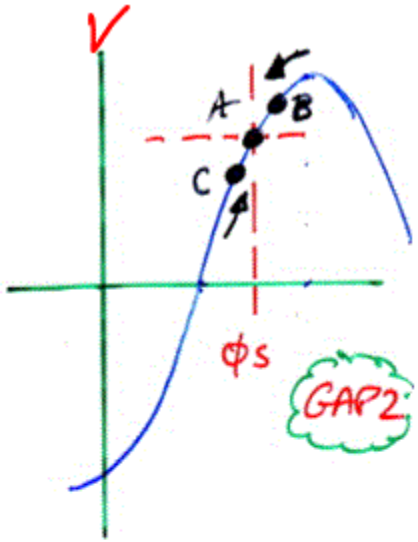


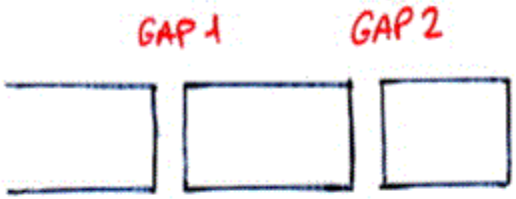


- PARTICLE A CROSSES GAP 1 PHASE ϕ IN STEP WITH VOLTAGE
- GAP 2 — SAME VOLTAGE PHASE — AGAIN ACCELERATED
- PARTICLE B ARRIVE LATE, VOLTAGE HIGHER

  ACCELERATED MORE ARRIVES AT GAP 2 EARLIER

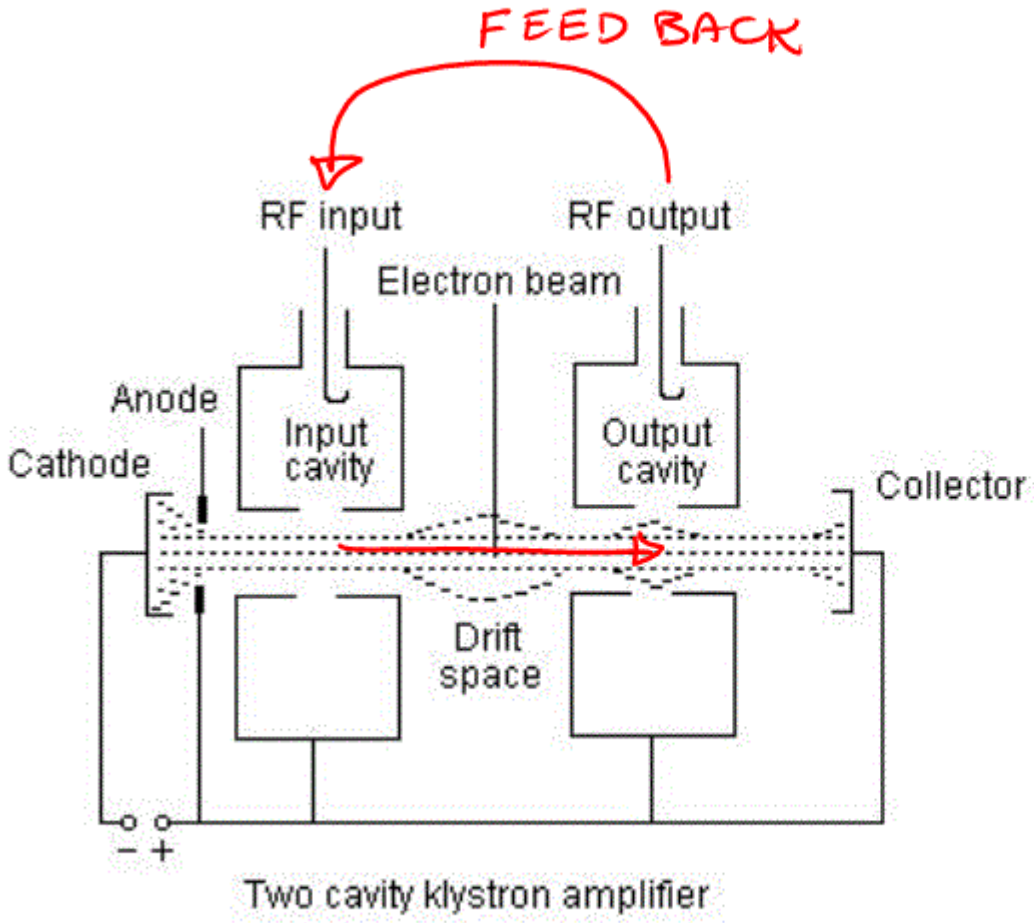






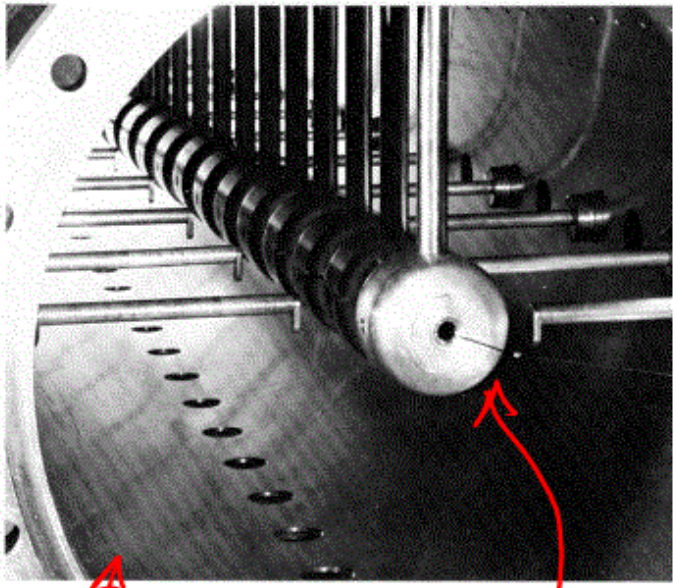
- PARTICLE C ARRIVES EARLIER AT GAP 1
  - VOLTAGE LOWER, ACCELERATED LESS
  - ARRIVES LATER IN PHASE AT GAP 2
- B AND C CONVERGE IN PHASE WITH A
- NO NEED TO START WITH PARTICLES ALL IN PHASE WITH RADIO FREQUENCY OSCILLATOR

RADIO FREQUENCY POWER GENERATION



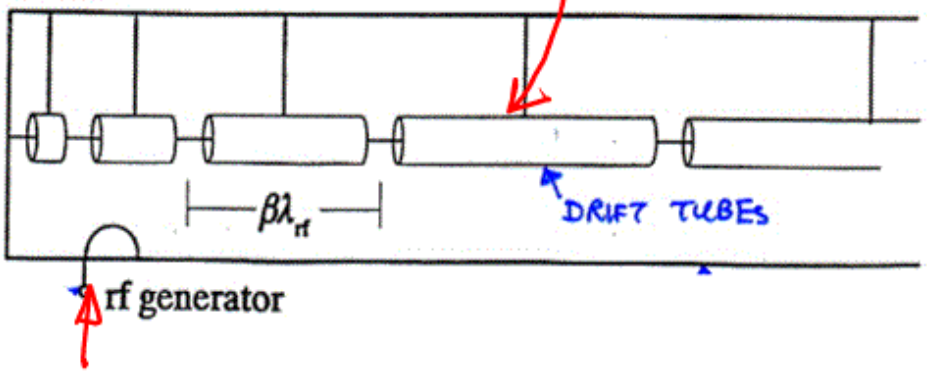
2 CAVITY KLYSTRON OSCILLATOR

ALVAREZ LINAC STRUCTURE



CONDUCTING ENCLOSURE

DRIFT TUBES

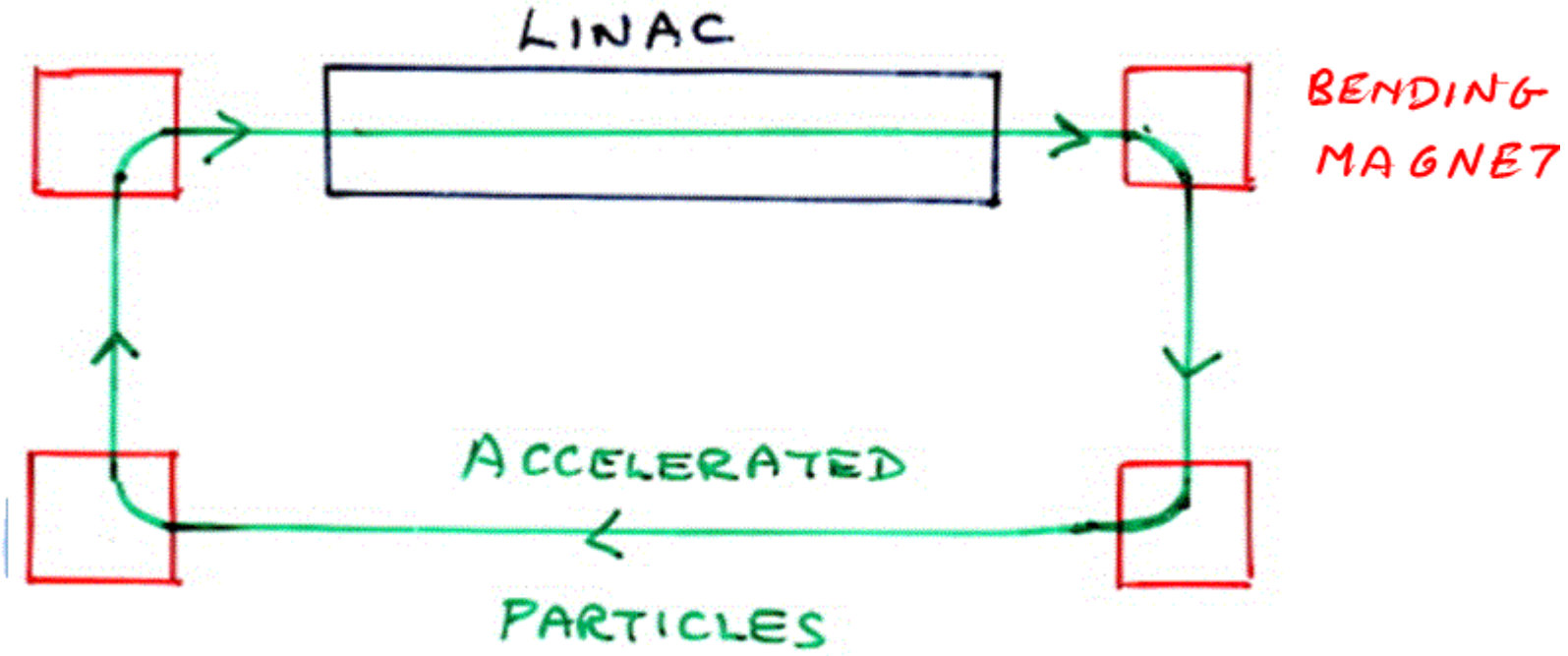


RADIO FREQUENCY INPUT

- WIDEROE STRUCTURE VERY INEFFICIENT — RADIO FREQUENCY RADIATION LOSS
- ALVAREZ STRUCTURE
  - RESONANT CAVITY LIKE KLYSTRON
  - USED FOR PROTON SYNCHROTRON INJECTOR
    100 MeV → 100MHz
  - HIGH ENERGY ELECTRON ACCELERATORS
    4 GeV - 500 GeV GHz RF

SLAC – 50 GeV Electron LINAC

CIRCULAR ACCELERATOR

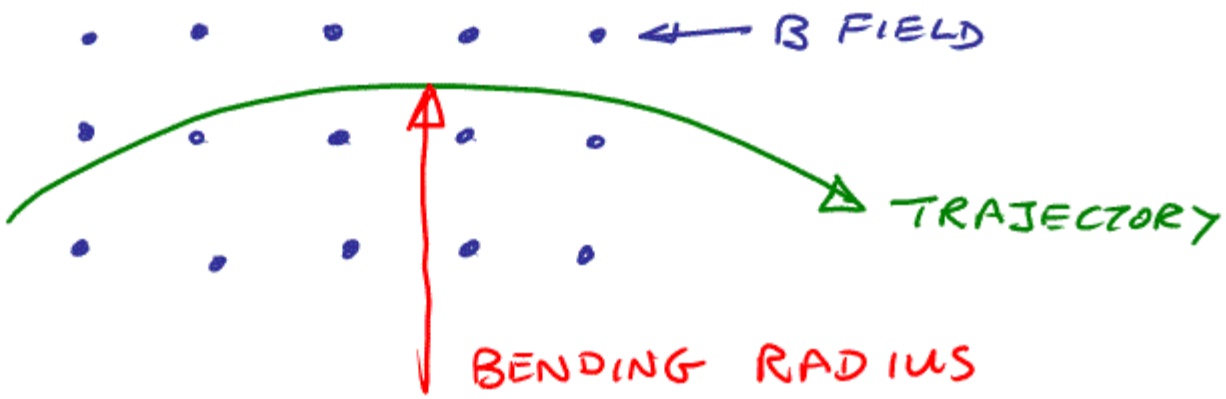


- REUSE ACCELERATING VOLTAGE UNTIL REACH VERY HIGH ENERGY

PARTICLE BENDING IN MAGNETIC FIELD

$$\bar{F} = q\left(\bar{E} + \frac{1}{c}\bar{v} \times \bar{B}\right)$$ LORENTZ

- FORCE FROM MAGNETIC FIELD NORMAL TO PARTICLE TRAJECTORY



- FOR NO ELECTRIC FIELD & B FIELD NORMAL TO PAGE

$$F = q\frac{v}{c}B\sin\theta \leftarrow 90° = 1 \rightarrow F = q\frac{vB}{c}$$

- FOR A PARTICLE MOVING IN A CIRCLE OF RADIUS ρ

CENTRIPETAL FORCE = LORENTZ FORCE

CIRCULAR ACCELERATORS

- AT PRESENT PARTICLE PHYSICS DOMINATED BY CIRCULAR ACCELERATORS

ELECTRONS	PROTONS
CESR	SPS @ CERN
PEP II	TEVATRON @ FERMILAB
KEK	AGS
LEP	LARGE HADRON COLLIDER

- MOST EFFICIENT & COMPACT WAY OF GETTING TO HIGH ENERGY – UNTIL SYNCHROTRON RADIATION BECOMES IMPORTANT

CENTRIPETAL FORCE = LORENTZ FORCE

$$\frac{\gamma m v^2}{\rho} = \frac{vBq}{c} \Rightarrow \rho = \frac{p \cdot c}{Bq}$$

BENDING RADIUS IN GAUSSIAN UNITS

• ACCELERATOR BUILDERS USE m, VOLT, TESLA

$$pc = \rho \cdot B \cdot q$$

eV/c ← pc; cm ← ρ; Gauss ← B

VOLT = STATVOLT / 300

TESLA = 10^4 GAUSS

m = 10^2 cm

$$pc\left[\frac{V}{c} \times 300\right] = \rho\left[m \times 10^2\right] B\left[T \times 10^4\right] e$$

$$pc\,[GeV/c] = 0.3\,\rho[m]\,B[T]$$

$$\rho[m] = \frac{p\,[GeV]}{0.3\,B[T]}$$

$$p[sv]\,c = \rho(cm)\,B(g)$$

$$p\left[\frac{V}{300}\right]\cdot c = \rho\,(m\times 10^2)\;B(T\times 10^4)$$

$$pc = \rho[m]\,B[T]\times 10^6\times 3\times 10^2$$

$$p[eV] = P[m]\,B[T]\times 3\times 10^8$$

$$p[GeV\times 10^9] = \rho\,[m]\,B[T]\times 3\times 10^8$$

$$pc[GeV] = \rho\;\;[m]\,B[T]\times 0.3$$

200m 8T

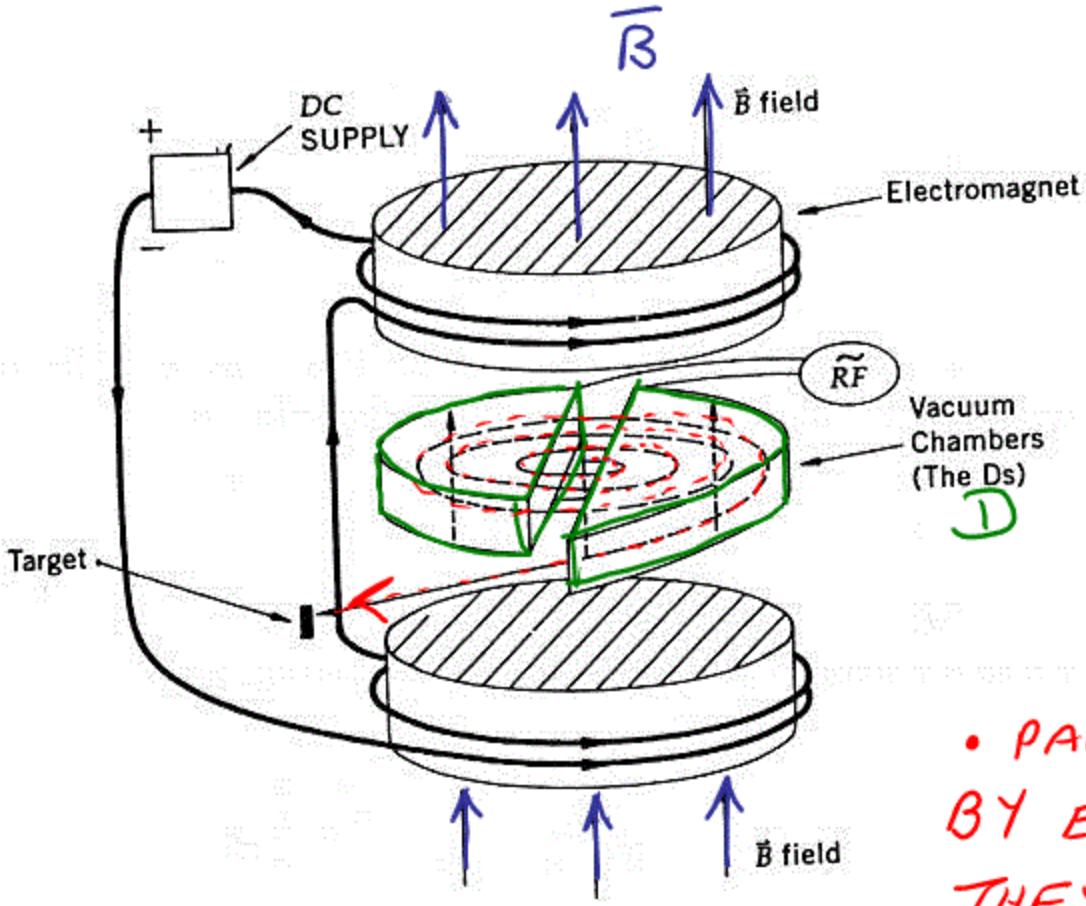
$$pc = 480\;\; GeV = 480\times 10^9\; eV$$

$$pc[eV] = \rho(cm)\times B(g)$$

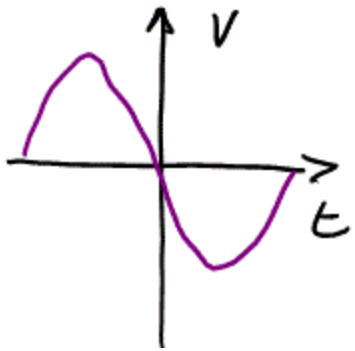
$$= 200\times 100\times 8\times 10000 = 1.6\times 10^9\; eSV$$

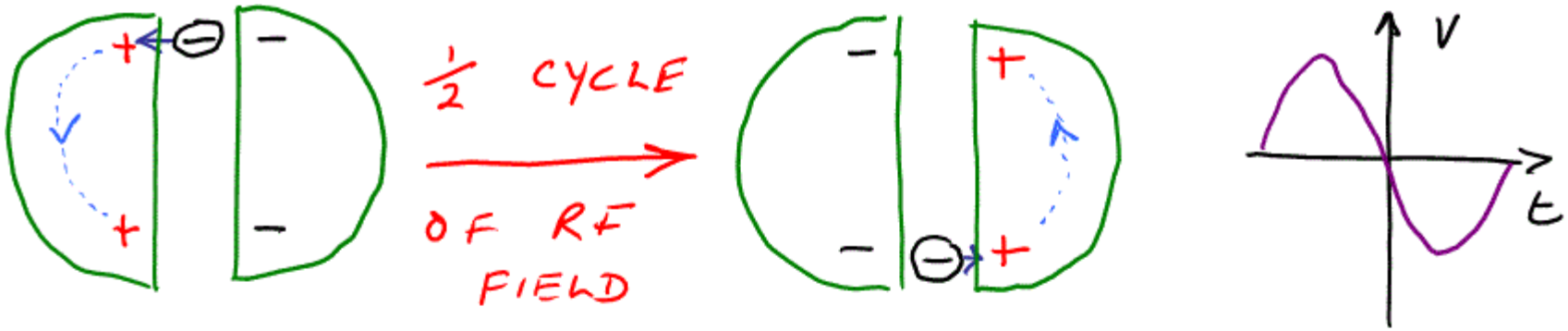
$$= 480\times 10^9\; eV$$

THE CYCLOTRON



- TWO HOLLOW D-SHAPED ELECTRODES, EXCITED BY HV RF OSCILLATOR
- NO ELECTRIC FIELD INSIDE "D_s"
- MAGNET FIELD INSIDE D_s
- PARTICLES ACCELERATED BY ELECTRIC FIELD AS THEY CROSS GAP BETWEEN D_s





CENTRIPETAL FORCE = LORENTZ FORCE
FOR AN ORBIT OF RADIUS r

$$\frac{m v^2}{r} = q \frac{v \cdot B}{c}$$

$$\frac{v}{r} = \frac{q B}{mc} = \text{CONSTANT}$$

TIME FOR ORBIT = $2\pi r / v$

ORBITAL FREQUENCY = $v / 2\pi r$

IF RADIO FREQUENCY f = ORBITAL FREQUENCY

CONTINUOUS ACCELERATION

CONTINUOUS ACCELERATION

RADIO FREQUENCY = ORBITAL FREQUENCY

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi}\frac{q}{m}\frac{B}{C} = \text{CONSTANT}$$

f → CYCLOTRON FREQUENCY

→ DOES NOT DEPEND ON RADIUS OF ORBIT

- PARTICLE STARTS AT SOURCE CLOSE TO CENTRE OF MACHINE
- SPIRALS OUT CONTINUOUSLY GAINING ENERGY FROM RESONANT RF.

THINK AGAIN ABOUT WHY A CYCLOTRON WORKS

$$F_C = F_L$$

$$\frac{mv}{r} = \frac{q \cdot B}{mc} = k$$

$$\frac{v}{r} = \text{CONSTANT} = \text{FREQUENCY}$$

AS r INCREASES, v INCREASES $\rightarrow \frac{v}{r} = \text{CONSTANT}$

FOR A RELATIVISTIC PARTICLE $v = c = \text{CONSTANT}$

$$\therefore \frac{v}{r} = \frac{c}{r} \neq \text{CONSTANT}$$

ELECTRON CYCLOTRON

"MICROTRON"

ELECTRON IS RELATIVISTIC FOR $E \sim 500\,keV$

ORBITS INCREASE IN RADIUS DURING ACCELERATION

ANOTHER RELATIVISTIC EFFECT

$$f = \frac{1}{2\pi}\frac{q}{m}\frac{B}{c}$$ = RF FREQUENCY = ORBITAL FREQUENCY

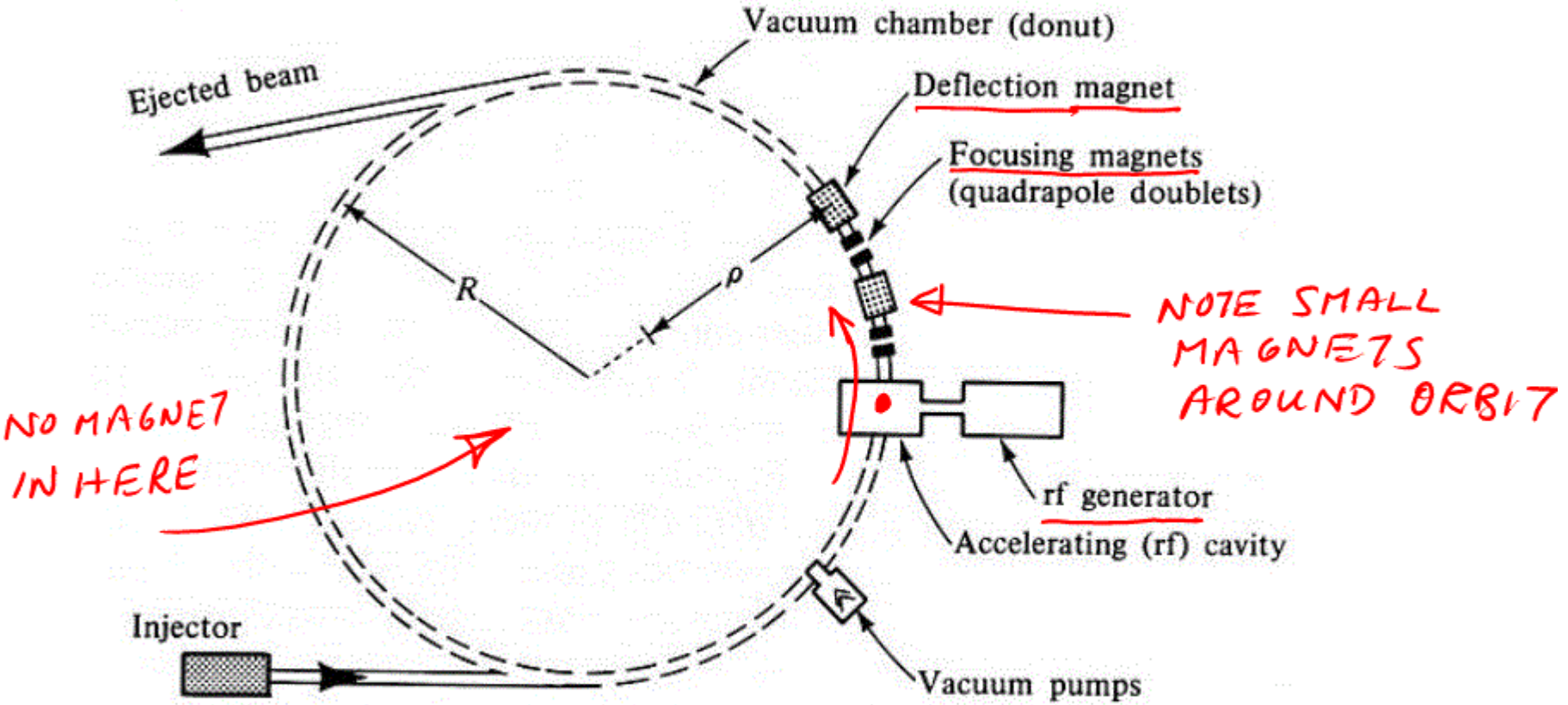
- AS PARTICLES ACCELERATE, TOTAL RELATIVISTIC ENERGY BECOMES $\approx$ MASS ENERGY
- IN THIS SITUATION $m \rightarrow m\gamma$ ← LORENTZ BOOST

$$f = \frac{1}{2\pi}\frac{q}{\gamma m}\frac{B}{c}$$

DURING ACCELERATION γ INCREASES & RESONANCE CONDITION FAILS

- INCREASE B — SYNCHROTRON
- DECREASE RF FREQUENCY — SYNCHROCYCLOTRON

SYNCHROTRON — CONSTANT RADIUS ORBIT



DUE TO MAGNETS ONLY AROUND ORBIT CAN BE MADE VERY LARGE → HIGHEST ENERGIES

SYNCHROTRON

AS USUAL $$f = \frac{1}{2\pi}\frac{q}{m}\frac{1}{\gamma}\frac{B}{C}$$

EQUAL FOR RESONANCE

IN RELATIVISTIC SITUATION ORBITAL PERIOD $\frac{2\pi R}{C}$

SO ORBITAL FREQUENCY $C/2\pi R$

CONDITION FOR CONSTANT ACCELERATION

RF FREQUENCY = INTEGER × ORBITAL FREQUENCY

$$\frac{1}{2\pi}\frac{q}{m}\frac{1}{\gamma}\frac{B}{C} = \frac{C}{2\pi r}\cdot n$$ ← HARMONIC NUMBER

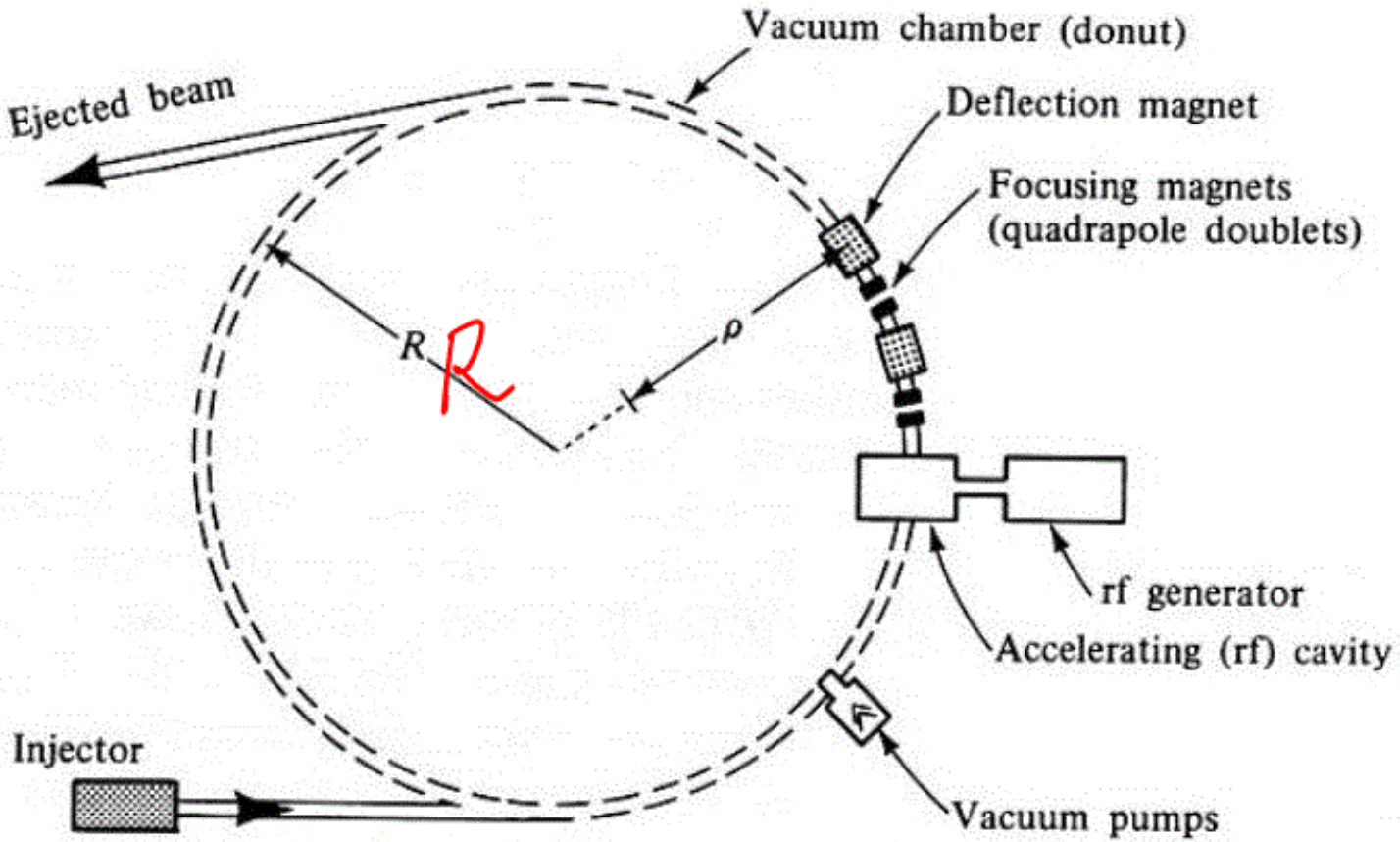
SINCE $v \approx C$ & $p = m\gamma C$ $\quad p \sim m\gamma C$

$$\frac{qB}{p} = \frac{nC}{R}$$

$$R = \frac{ncp}{qB}$$

AS ACCELERATION PROCEEDS p INCREASES

$\therefore$ B INCREASES FOR CONSTANT R



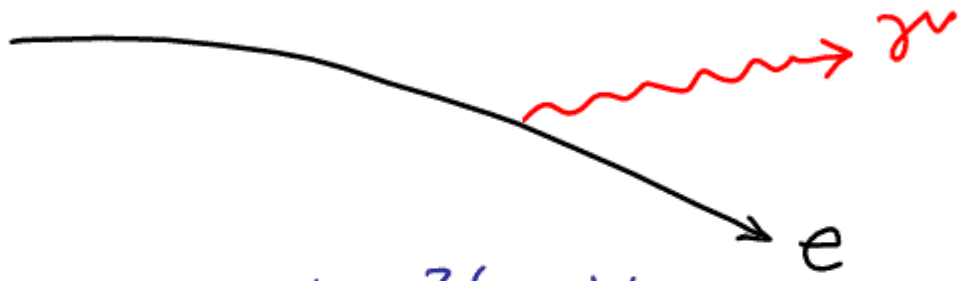
FOR A FIXED B_{MAX}

$$R = \frac{cp}{qB} \Rightarrow R_{MACHINE} \propto p_{MAX}$$

HIGHER ENERGIES $\Rightarrow$ LARGER MACHINES

ELECTRON VERSUS PROTON SYNCHROTRON

- ELECTRONS ACCELERATED AROUND CIRCULAR ORBIT → RADIATE



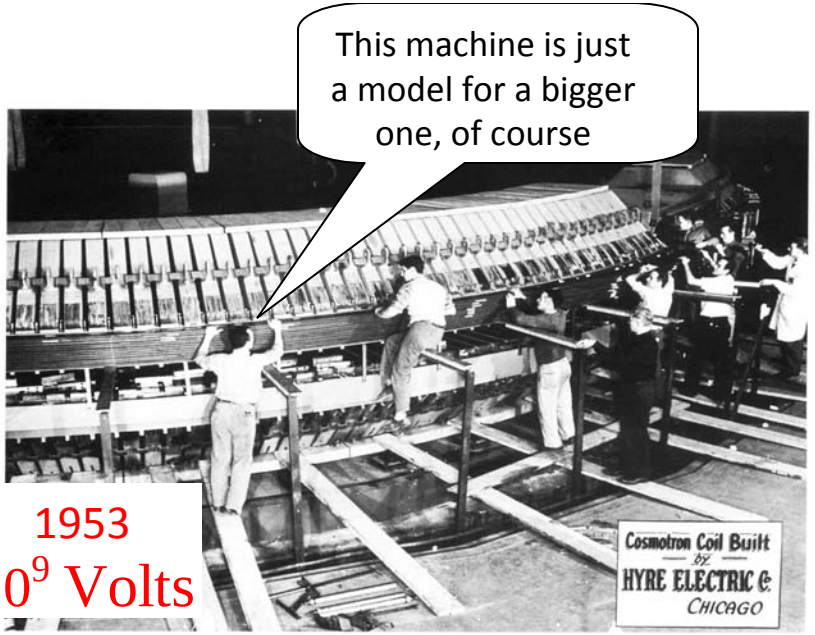
$$\text{ENERGY LOSS} \propto \frac{4\pi e^2}{R}\left(\frac{E}{mc^2}\right)^4$$

$$\frac{\Delta E\,(\text{PROTON})}{\Delta E\,(\text{ELECTRON})} = \left(\frac{m_e}{m_p}\right)^4 \approx 10^{-13}$$

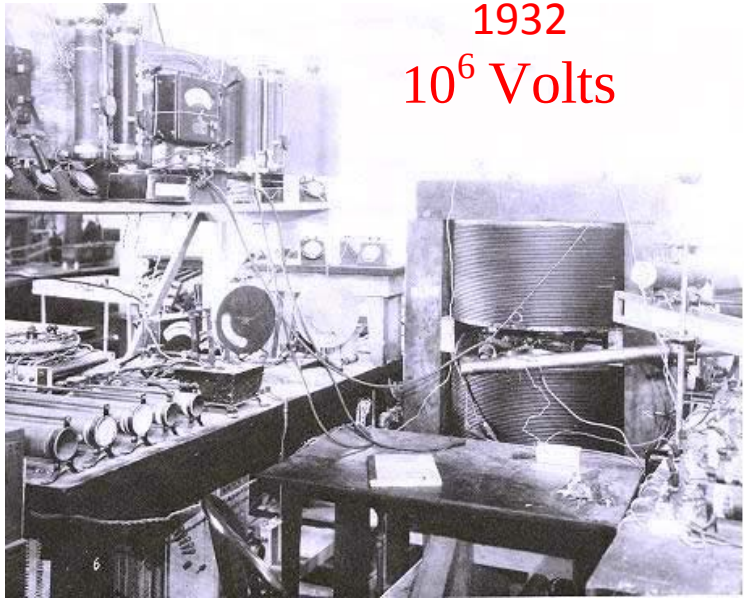
THIS IS WHY ELECTRONS IN CERN TUNNEL GO TO 50 GeV WHILE PROTONS TO 7000 GeV

LIMITED BY BENDING MAGNET

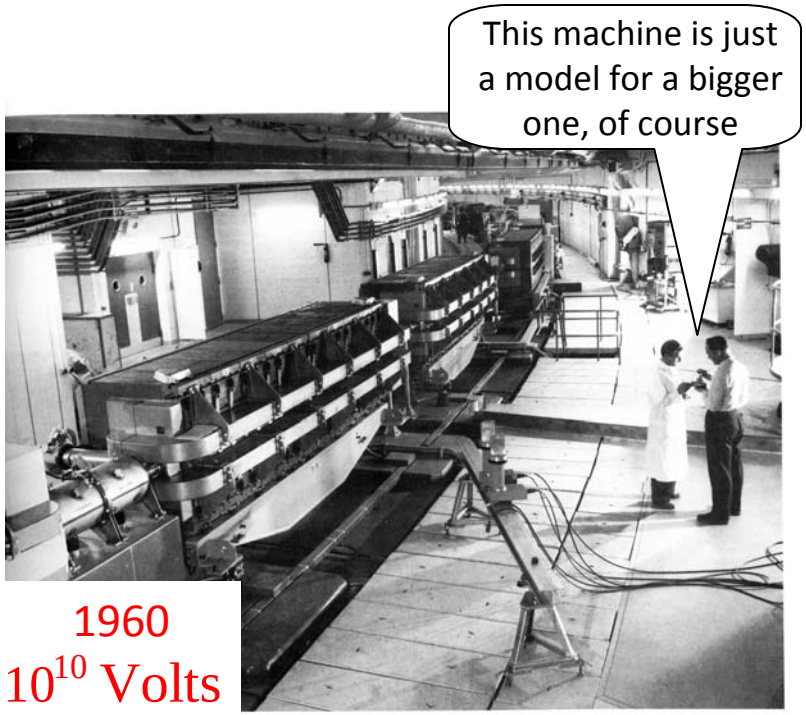
1931
10^4 Volts



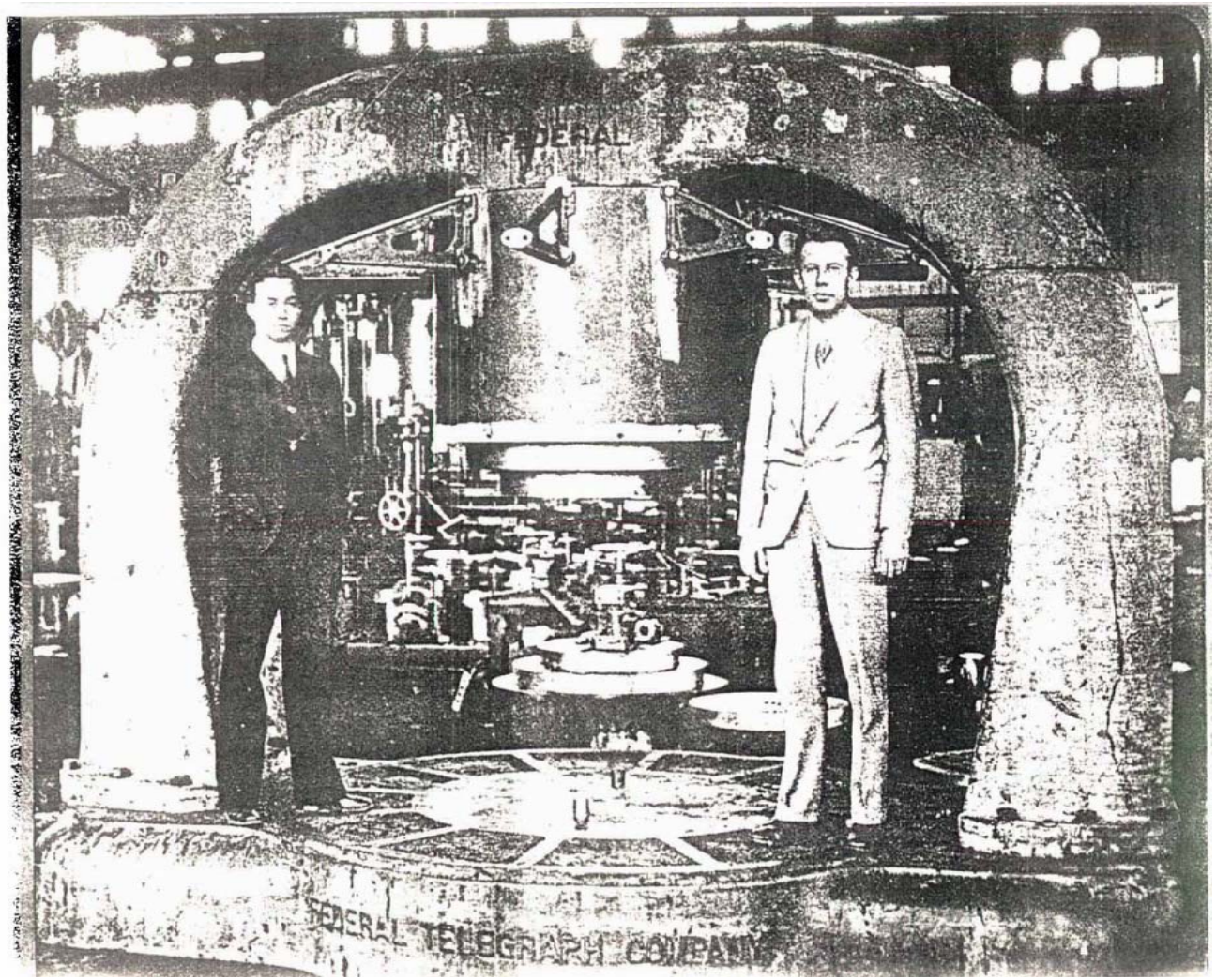
1953
10^9 Volts



1932
10^6 Volts



1960
10^{10} Volts

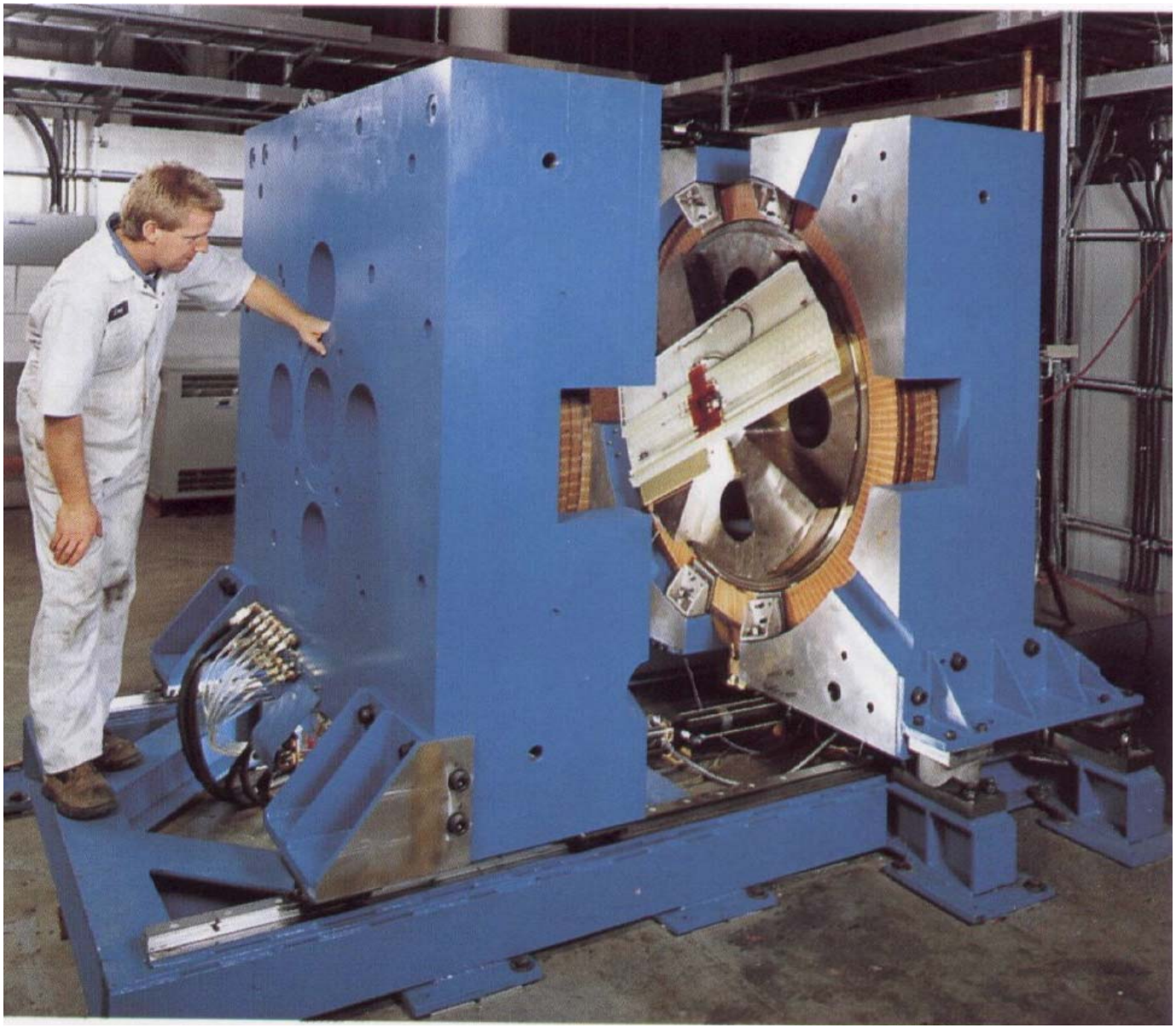


Livingston and Lawrence with the magnet of the “27-inch” (later “37-inch”) cyclotron on which most of Berkeley’s 1930s nuclear physics was performed. Lab wear was different then!

THE 184-INCH SYNCHROCYCLOTRON

The Berkeley 184” was begun in 1939 as a classical cyclotron, to be operated with Vrf = 1 MV, but WWII interrupted rf installation and it was used to test mass spectrographic separation of uranium isotopes. FM rf was installed in 1946, yielding 190 MeV d+ (700 MeV p in 1959).

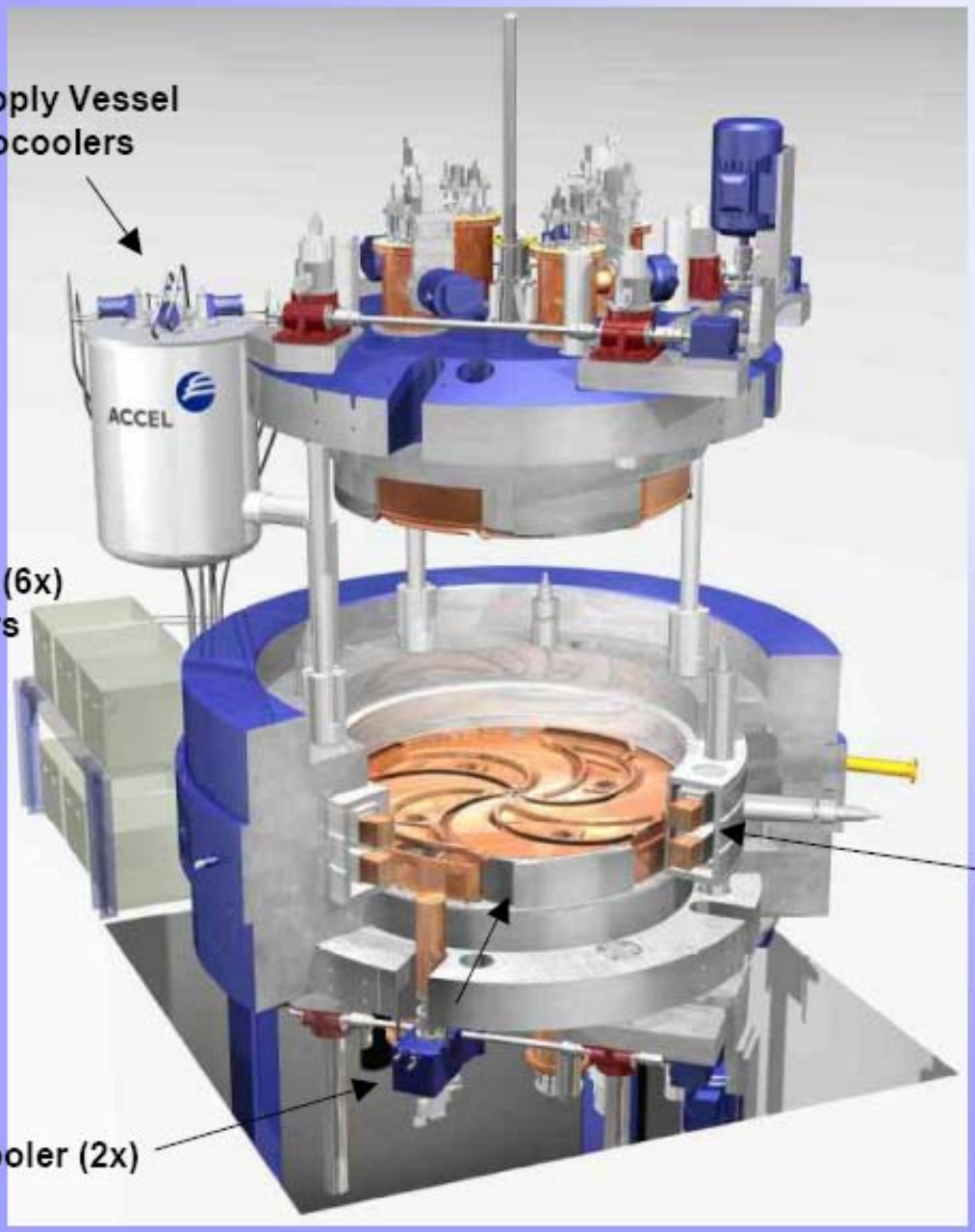
PET Medical Cyclotron

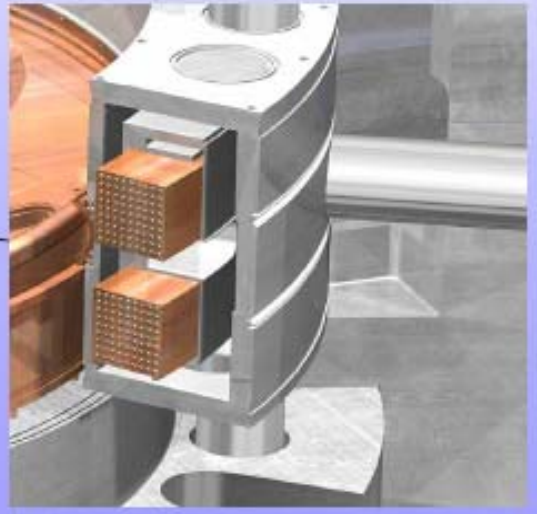


TRIUMF (Vancouver) 500 MeV Cyclotron

ACCEL

250 MeV Superconducting Proton Cyclotron



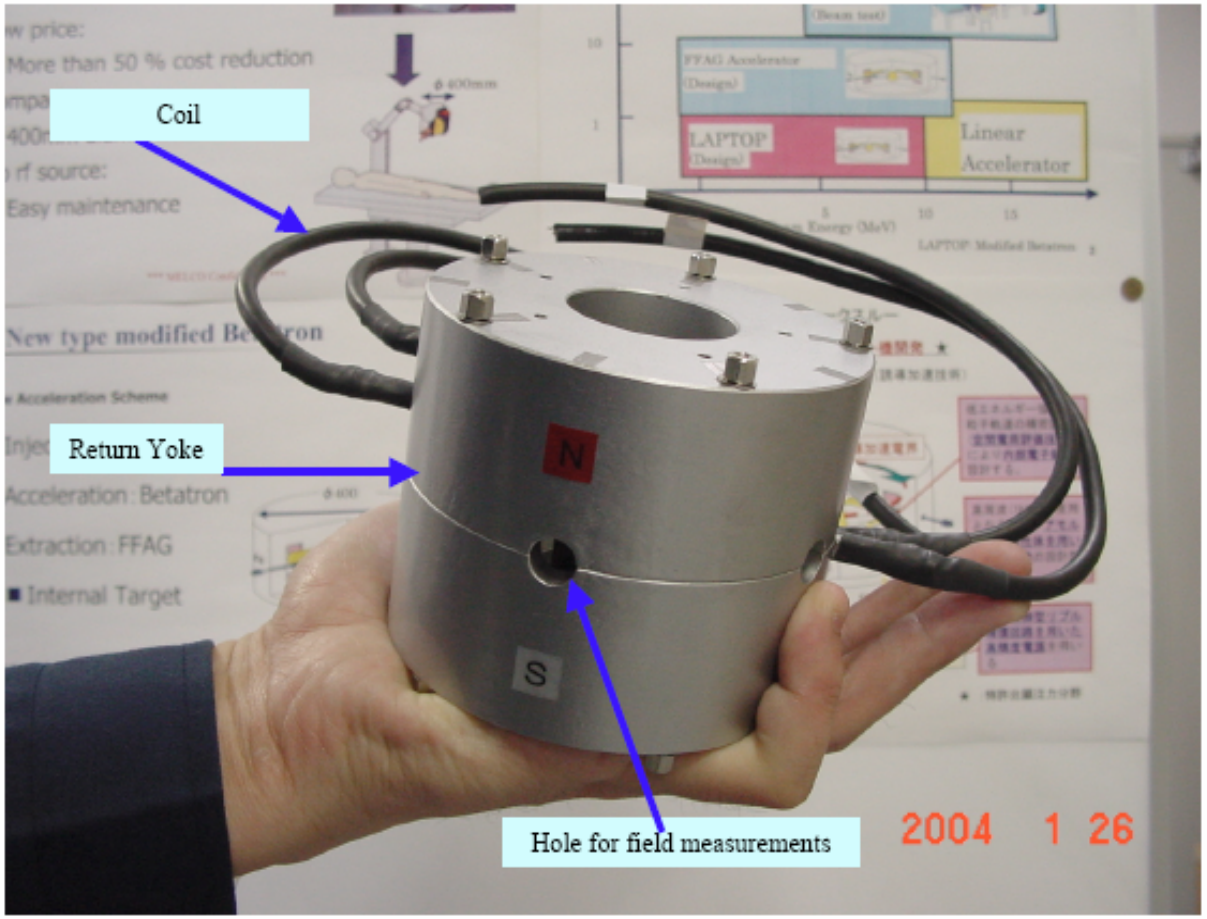


Superconducting Coil

MITSUBISHI ELECTRIC

Spiral Magnet

Changes for the Better



The present study is partially supported by the REIMEI Research Resources of Japan Atomic Energy Research Institute.

You can have your own cyclotron – from Mitsubishi

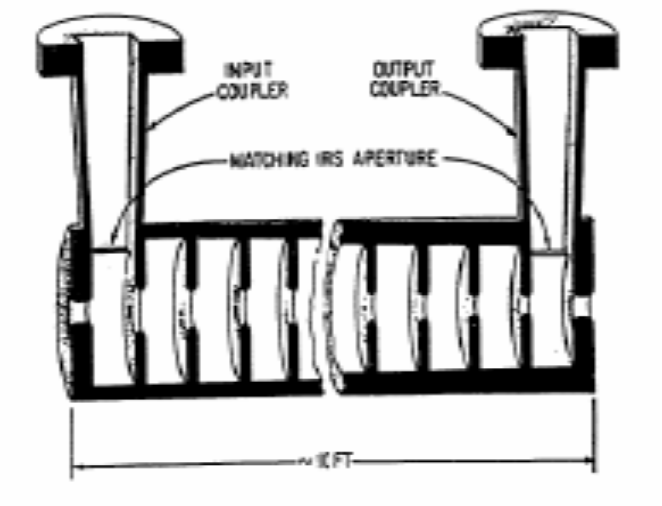

Alors, c’est fini!
Et maintenant?

DC HIGH-VOLTAGE ACCELERATORS – TANDEM VAN DE GRAAFFS

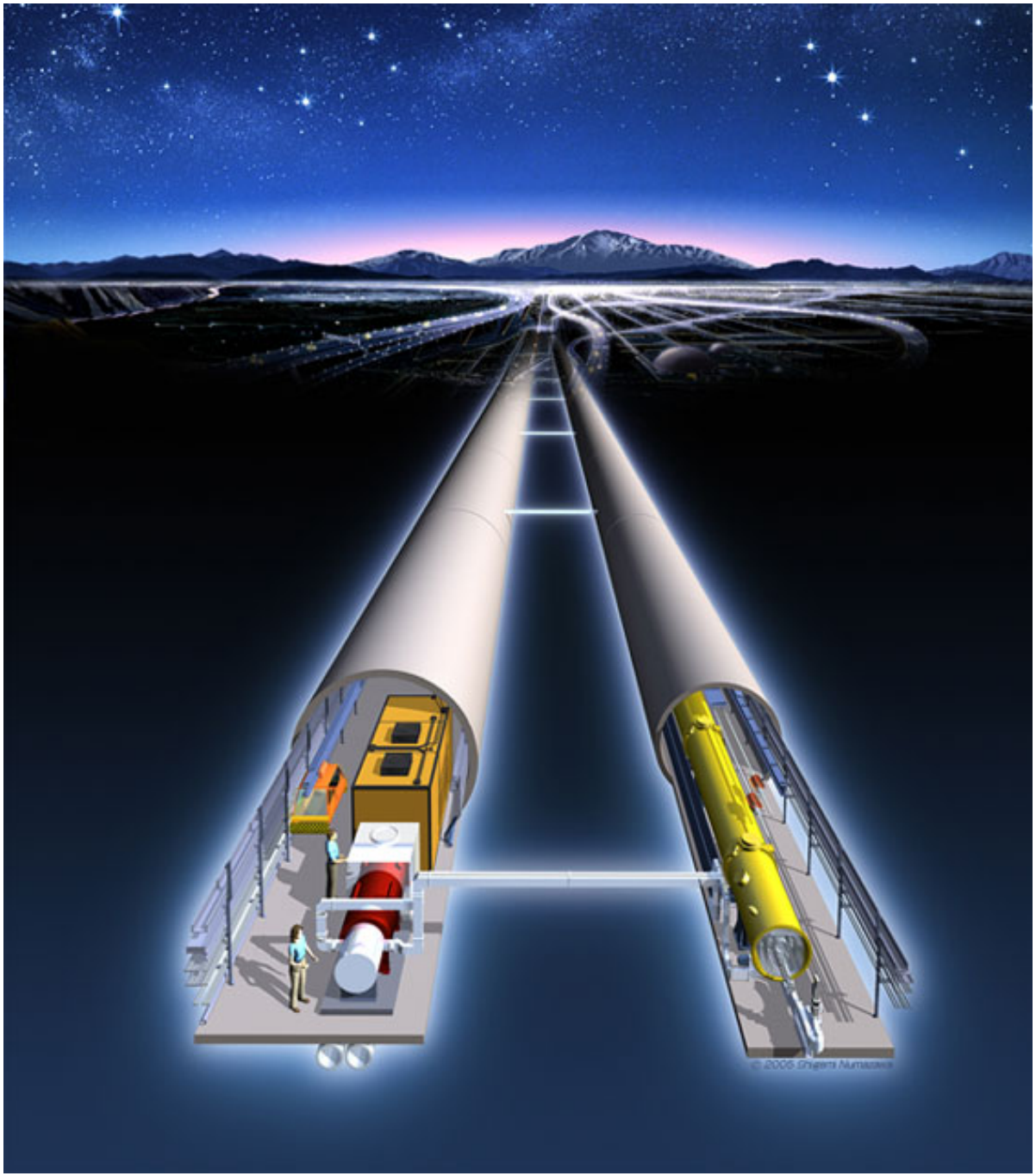
Yale 22-MV tandem.

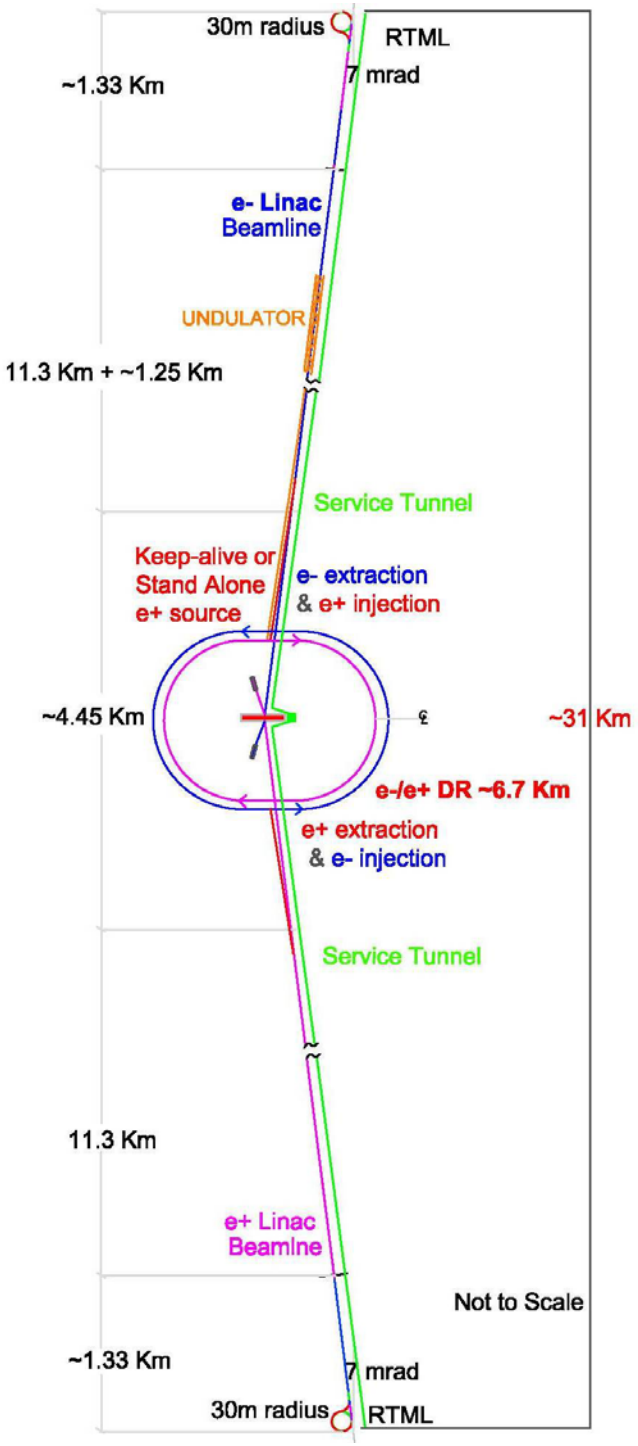
Daresbury folded tandem
(20 MV in a 230-ft tower).

The ISAC 150-keV/u RFQ linac



500 keV electron LINAC for Cancer Therapy




30m radius
RTML
7 mrad
~1.33 Km
e- Linac
Beamline
UNDULATOR
11.3 Km + ~1.25 Km
Service Tunnel
Keep-alive or
Stand Alone
e+ source
e- extraction
& e+ injection
~4.45 Km
~31 Km
e-/e+ DR ~6.7 Km
e+ extraction
& e- injection
Service Tunnel
11.3 Km
e+ Linac
Beamline
Not to Scale
~1.33 Km
7 mrad
30m radius
RTML