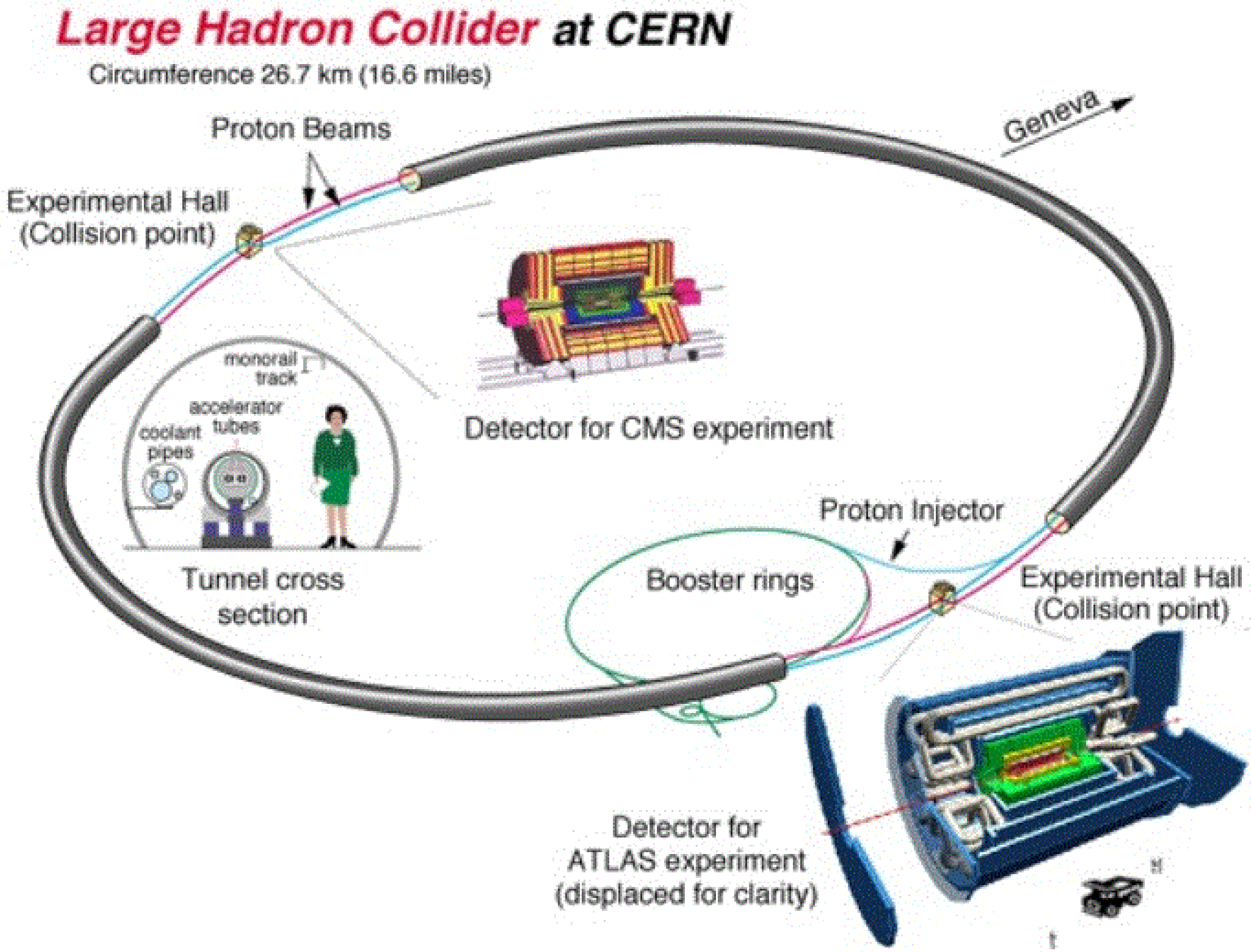
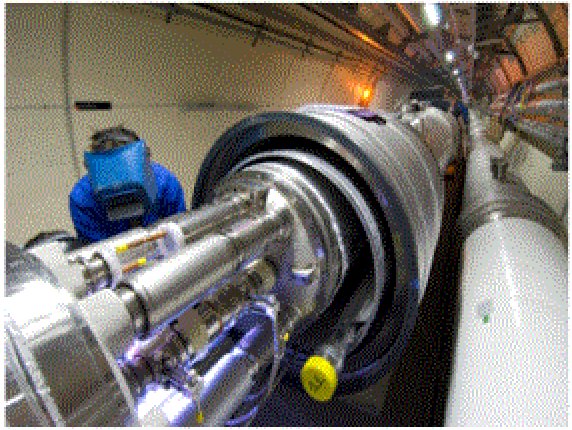
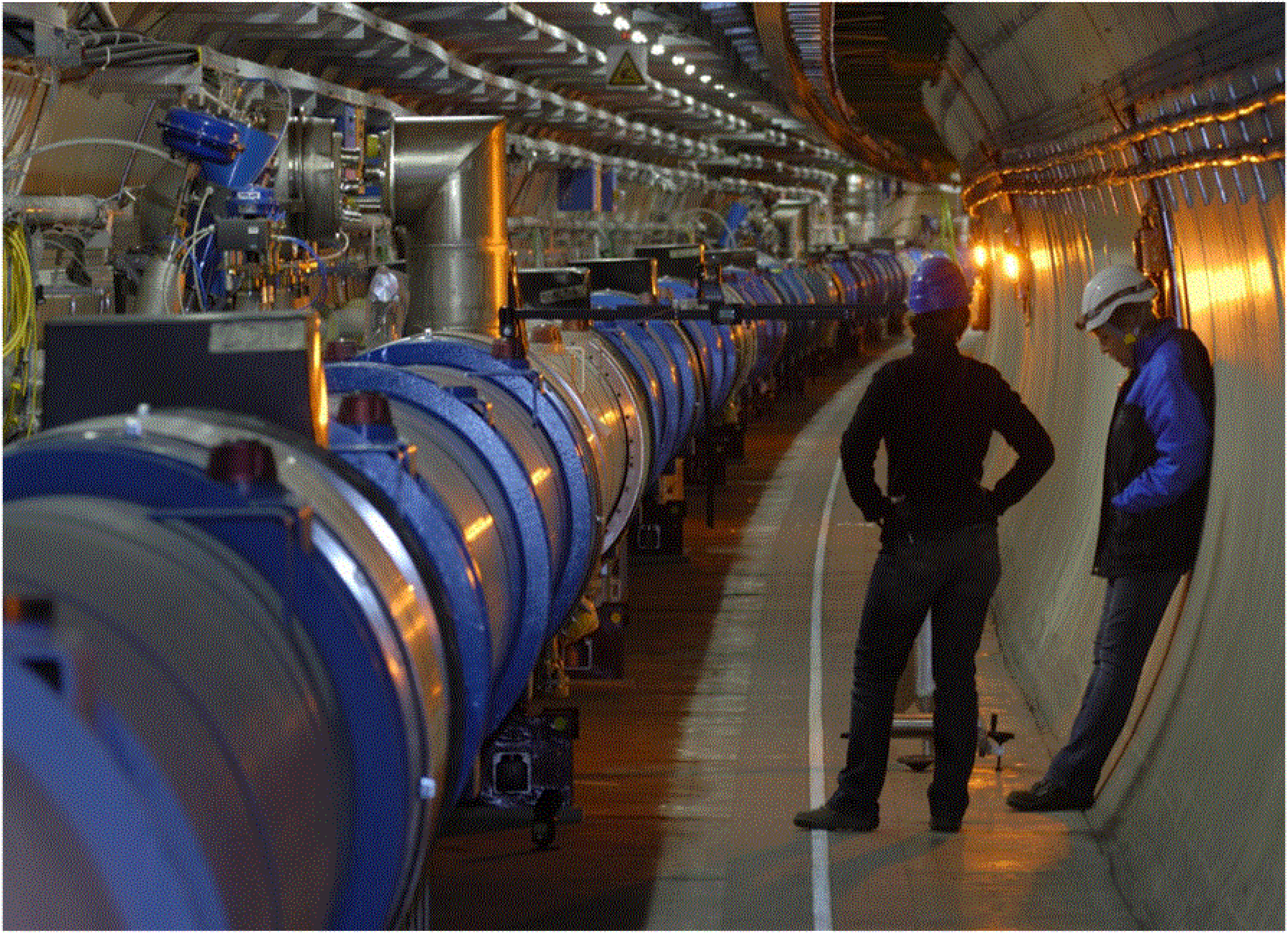
CARTOON OF MODERN ACCELERATOR COMPLEX



BUILDING THE LHC





ACCELERATORS

- GEIGER & MARSDEN – RADIOACTIVE SOURCE
  7 MeV α

- SIZE & SHAPE OF NUCLEUS 40 MeV p
- SIZE & SHAPE OF PROTON 500 MeV e^-
- PATTERN OF π, p, Δ^{++}, K^+ MASSES 10 GeV p
- DISCOVERY OF QUARKS 50 GeV e^-
- DISCOVERY OF TOP QUARK 1000 GeV p
- DISCOVERY OF HIGGS BOSON 14000 GeV p

- AS WE WANT TO RESOLVE SMALLER STRUCTURES
  PRODUCE HIGER MASS

- INCREASE ACCELERATOR ENERGY

TWO PURPOSES

- PRODUCE MASSIVE UNSTABLE PARTICLES

  CONVERT KINETIC ENERGY OF BEAMS → MASS

$$E = mc^2 \rightarrow m = E/c^2$$

- PROBE SMALLER DISTANCES → SCATTERING

BEAM TARGET

LOOK AT ANGULAR DISTRIBUTION IN FINAL STATE

MUST BE SMALLER

$$\Psi = A e^{-i\bar{p}\cdot\bar{r}/\hbar}$$

DEFINITE MOMENT

— PLANE WAVE

$$\Delta p \Delta x \sim \hbar \rightarrow \Delta x \sim \frac{\hbar}{\Delta p}$$

WAVELENGTH OF BEAM PARTICLES

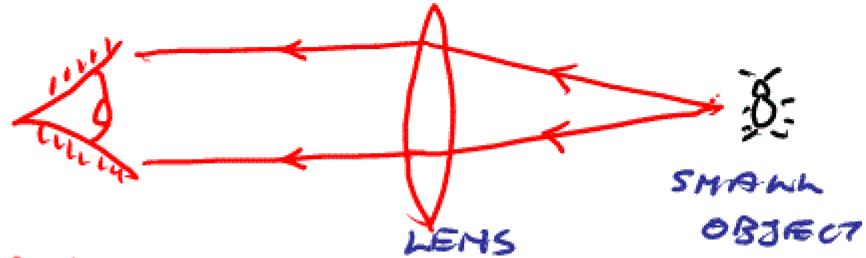
de BROGLIE $\lambda = h/p$

RESOLVING LENGTH

BEAM MOMENTUM

SPATIAL RESOLUTION & MOMENTUM TRANSFER

RESOLUTION IN OPTICS



RESOLUTION DEFINED BY

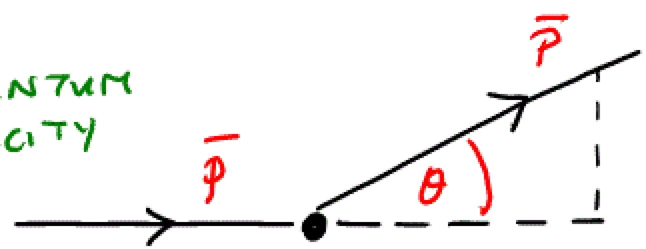
$$\Delta r \sim \frac{\lambda}{\sin\theta}$$

λ – WAVELENGTH OF LIGHT

$\sin\theta$ – APERTURE

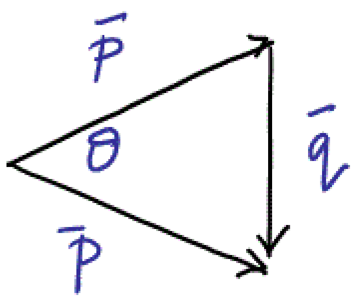
PARTICLE SCATTERING → RESOLVE SMALL OBJECTS USING MATTER WAVES

3-MOMENTUM – SIMPLICITY



$$\Delta p = \bar{q}$$

$$\Delta r \approx \frac{\lambda}{\sin\theta} = \frac{h}{p\sin\theta}$$



$$|\bar{q}| = 2|\bar{p}|\sin\frac{\theta}{2} \sim p\sin\theta$$

$$\Delta r \sim h/q$$

HIGH SPATIAL RESOLUTION ← HIGH MOMENTUM TRANSFER

- PROTON DIAMETER $\sim 1\,fm = 10^{-15}\,m$
- WHAT MOMENTUM TRANSFER DO WE NEED TO RESOLVE PROTONS INSIDE NUCLEUS ?

NEED $1\,fm = \dfrac{h}{q} = \dfrac{hc}{qc} \rightarrow = 1.23\ GeV\cdot fm$

$qc = 1.23\,GeV\cdot fm\,/\,1\,fm$

$q \sim 1\ GeV/c$ MOMENTUM TRANSFER

MOMENTUM OF BEAM MUST BE $>$ MOMENTUM TRANSFER

- PRESENT LIMIT ON QUARK DIAMETER $< 10^{-18}\,m$

MOMENTUM TRANSFER

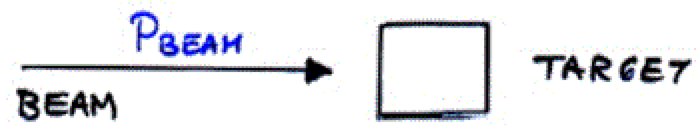
$\sim 1000\ GeV/c = 1\ TeV/c$

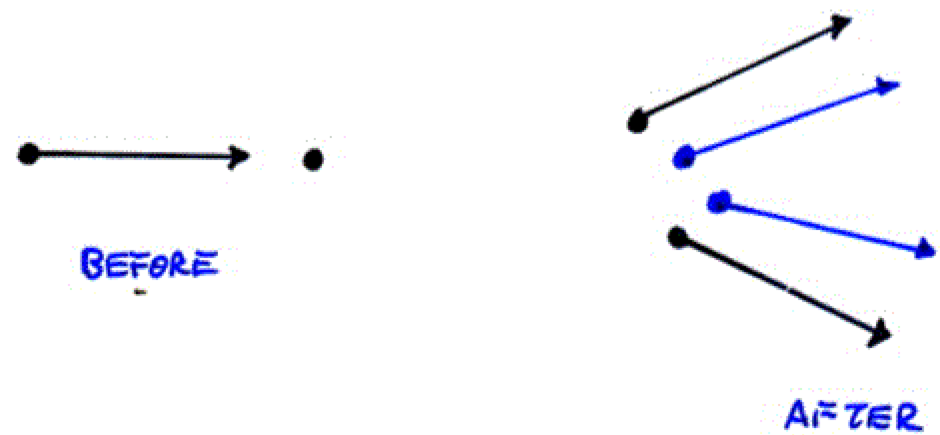
THIS NEEDS AN ACCELERATING VOLTAGE $\sim 10^{12}$ VOLTS

HOW?

LABORATORY & CENTRE OF MASS (MOMENTUM) FRAMES

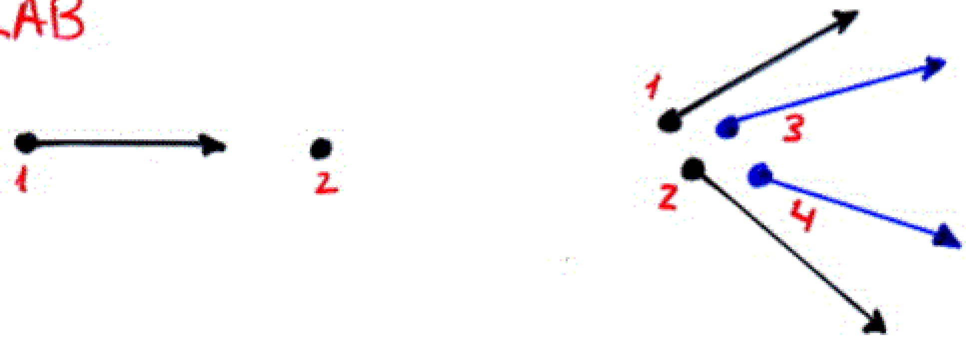
IN ORDER TO UNDERSTAND HOW TO PRODUCE HIGH MASS OBJECTS → SPECIAL RELATIVITY





AS P_{BEAM} INCREASES MORE & MORE ENERGY GOES INTO INCREASING VELOCITY OF CENTRE-OF-MASS AS SEEN IN THE LAB FRAME

LAB



OBVIOUS FROM SPECIAL RELATIVITY

- MOTION OF CENTRE-OF-MASS IN LAB IS IRRELEVANT TO PHYSICS
- ONLY ENERGY IN C of M IS USEFUL FOR NEW PARTICLE PRODUCTION

DO A LORENTZ BOOST INTO CM FRAME

"CENTRE-OF-MOMENTUM"

BEFORE

TOTAL MOMENTUM = 0

AFTER

TOTAL MOMENTUM = 0

TWO BODY COLLISION $a + b \rightarrow c + d$

BEAM (→ a) TARGET (→ b)

IN THE LAB TARGET IS AT REST $\bar{p} = 0$, $E = m$

BEAM PARTICLE STRIKES TARGET WITH ENERGY & MOMENTUM E^{LAB}, $\bar{p}^{LAB}$

$\bar{P}_b^{LAB} = 0$, $E_b^{LAB} = m_b c^2$, $\bar{p}_a^{LAB} = \bar{p}_{beam}$, $E_a^{LAB} = E_{beam}$

- AFTER COLLISION c + d BOTH MOVING IN LAB
- MOMENTA DEPEND ON MASSES GENERALLY DIFFERENT FRO m_a + m_b

IN CM $a + b \rightarrow c + d$

BEFORE COLLISION

$\bar{p}_a^{CM} + \bar{p}_b^{CM} = 0$ BY DEFINITION

AFTER COLLISION

$\bar{p}_c^{CM} + \bar{p}_d^{CM} = 0$

ONLY RELATIVE MOTION OF a & b IN CMS IS RELEVANT FOR CHANGING MASSES of c, d

→ OR PRODUCING NEW PARTICLES.

CONTINUE COMPARISON OF LAB & CMS

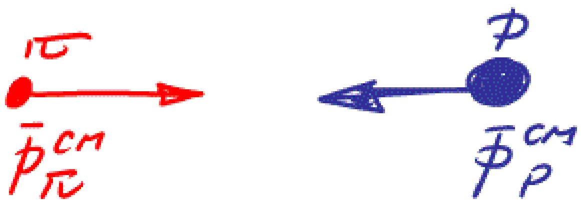
LAB: CONSIDER $\pi p \to \pi N^*$ ← EXCITED PROTON

$m_{N^*} > m_p > m_\pi$

IN CMS, BY DEFINITION, π + p COLLIDE WITH EQUAL & OPPOSITE MOMENTA

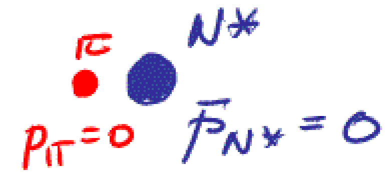
CMS:

BEFORE COLLISION



$$\bar{p}_\pi^{\,CM} = -\bar{p}_p^{\,CM}$$

AFTER COLLISION



π + N^* AT REST

ALL ENERGY OF RELATIVE MOTION OF INITIAL π, p HAS GONE INTO PRODUCING MASS OF N^*

$$\bar{p}_\pi^{\,CM} = -\bar{p}_{N^*}^{\,CM} = 0$$

IN FINAL STATE WHERE π & P BOTH AT REST

TOTAL CMS ENERGY

AFTER COLLISION $\left\{ W^{CM} = (m_\pi + m_{N*})c^2 \approx m_{N*}c^2 \right.$

$m_{N*} \gg m_\pi$

NOTICE $(W^{CM})^2 = [m]^2$

RELATIVISTIC INVARIANT

TOTAL RELATIVISTIC ENERGY IS CONSERVED IN COLLISION SO

BEFORE COLLISION $\left\{ W^{CM} = E_\pi^{CM} + E_p^{CM} = (m_\pi + m_{N*})c^2 \right.$

OR $(W^{CM})^2 = (E_\pi^{CM} + E_p^{CM})^2 \approx m_{N*}^2 c^2$

RELATIVISTIC INVARIANT

MUST HAVE SAME VALUE IN LABORATORY FRAME.

- REMEMBER ABOUT LORENTZ INVARIANT
  (SOME TIMES CALL IT RELATIVISTIC INVARIANT

- CONSIDER REST MASS m

$$E^2 = p^2c^2 + m^2c^4 \longrightarrow m^2c^4 = E^2 - p^2c^2$$

SAME IN ALL FRAMES

IF WE HAVE A GROUP OF i PARTICLES
4-VECTOR OF EACH IS $(E_i, \bar{p}_i)$
TOTAL 4-VECTOR:

LORENTZ INVARIANT

$$\left(\sum_i E_i\right)^2 - \left(\sum \bar{p}_i\right)^2 c^2 = M^2c^2$$

ENERGIES MOMENTUM INVARIANT MASS

INVARIANT MASS OF A GROUP PARTICLES IS A LORENTZ INVARIAT $\rightarrow$ JUST LIKE A SINGLE PARTICLE

$$\left(\sum E_i\right)^2 - \left(\sum \bar{p}_i\right)^2 c^2 = M^2 c^4$$

- INVARIANT MASS OF AN i PARTICLE SYSTEM IS A LORENTZ INVARIANT YOU CAN CALCULATE IT IN ANY FRAME YOU LIKE
- INVARIANT MASS OF AN i PARTICLE SYSTEM IS ONLY EQUAL TO SUM OF MASSES IF RELATIVE MOTION IS ZERO
- THIS MAKES PERFECT SENSE IN SPECIAL RELATIVITY

THE KINETIC ENERGY DUE TO RELATIVE MOTION } CONTRIBUTES TO INVARIANT MASS (EFFECTIVE MASS)

CONNECTING CM + LAB FRAMES

- USE LORENTZ TRANSFORM? UMM — OK — COMPLEX
- EXPLOIT LORENTZ INVARIANCE OF INVARIANT MASS

$$M^2_{LAB}\, c^4 = M^2_{CM}\, c^4$$

CENTRE OF MASS — LAB

$$\left(E_a^{CM} + E_b^{CM}\right)^2 - \big(\underbrace{\bar{p}_a^{CM} + p_b^{CM}}_{\bar{p}_a^{CM} = -p_b^{CM}}\big)^2 c^2 = \big(\underbrace{E_a^{LAB}}_{E_{beam}} + \underbrace{E_b^{LAB}}_{m_{TARG}c^2}\big)^2 - \big(\underbrace{\bar{p}_a^{LAB}}_{p_{beam}} + \underbrace{\bar{p}_b^{LAB}}_{\to 0}\big)^2 c^2$$

USE $m^2 = E^2 - p^2$

$$W^2 = \left(E_a^{CM} + E_b^{CM}\right)^2 = 2E_{beam}\, m_{targ}\, c^2 + \left(m^2_{beam} + m^2_{target}\right)c^4$$

COMMON NOTATION $\quad W^2 = \left(E^{CM}_{TOT}\right)^2 = \left(\sqrt{s}\right)^2$

$$W^2 = 2 E_{bm} \cdot m_{tg} c^4 + \left(m_{bm}^2 + m_{tg}^2 \right) c^4$$

TYPICAL

1000 GeV

1 GeV/c^2

PROTON – PROTON
FIXED TARGET COLLISIONS

FOR SUCH HIGH ENERGY BEAM – APPROXIMATE

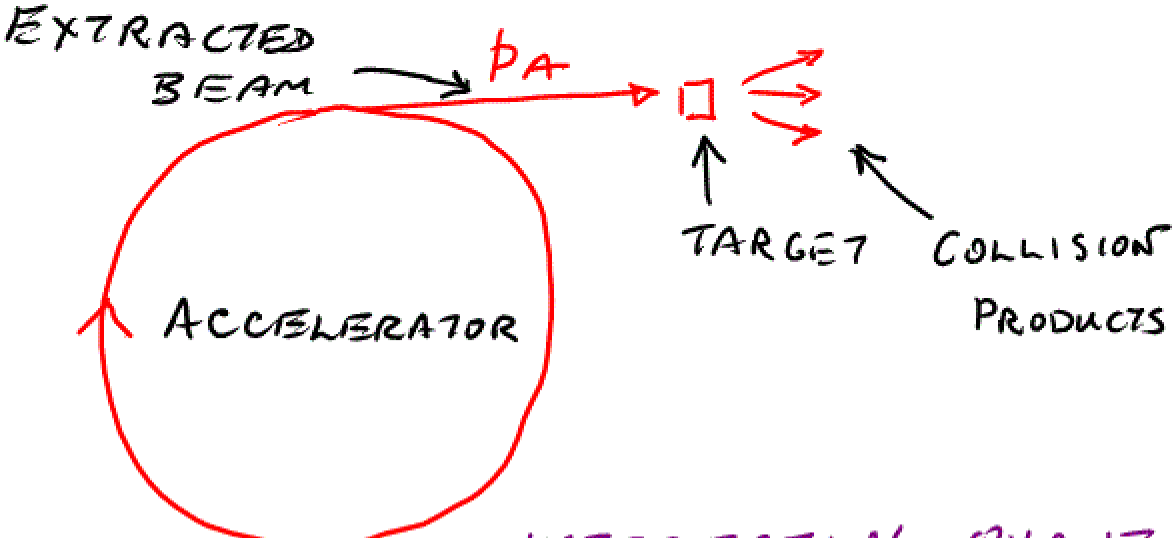
$$W \sim \left(2 E_{bm} m_{tg} c^2 \right)^{1/2}$$

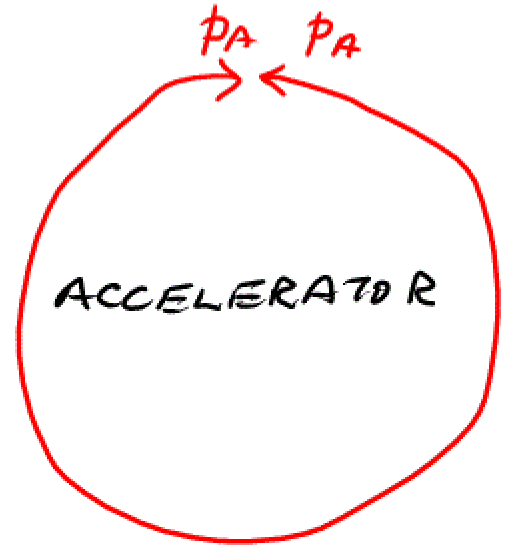
SO

$$W \propto \sqrt{E_{beam}}$$

Fixed Target v Colliding Beams

Will discuss details of accelerators later





Interesting quantity

E_{CM}

- Most of p_A goes into motion of CM in lab frame
- Accelerator is in lab frame

- Colliding beams accelerator is in CM frame
- All of p_a goes into E_{CM}.

USE 4-MOMENTUM & C = 1

- 2 PARTICLE COLLISION IN AN ARBITRARY FRAME

$$A + B \rightarrow C + D$$

$m_A \quad (E_A, \bar{p}_A) \qquad m_B \quad (E_B, \bar{p}_B)$

- TOTAL (4-MOMENTUM)^2 p^2 OF SYSTEM

$$p^2 = (p_A + p_B)^2 \leftarrow \text{4-VECTORS}$$

$$p^2 = p_A^2 + p_B^2 + 2 p_A p_B = m_A^2 + m_B^2 + 2E_A E_B - 2\vec{p}_A \cdot \vec{p}_B$$

$$p_A^2 = E_A^2 - \bar{p}_A^2 = m_A^2$$

3 VECTOR

$$p^2 = m_A^2 + m_B^2 + 2E_A E_B - 2|\bar{p}_A||\bar{p}_B|\cos\theta$$

SCATTERING ANGLE

IN CMS FRAME $\sum \bar{p} = 0$

$$p^{*2} = E^{*2} - |\bar{p}^*|^2 = E^{*2} \leftarrow \text{(4-VECTOR)}^2$$

$\rightarrow$ INVARIANT

$* = $ CMS

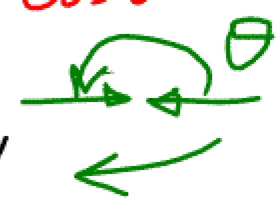
$$p^{*2} = E^{*2} = m_A^2 + m_B^2 + 2E_A E_b - 2|\bar{p}_A||\bar{p}_B| \cos\theta$$

THIS IS TRUE IN ANY FRAME ∴ CAN CHOOSE WHICH FRAME WE EVALUATE IT IN

CHOOSE LAB FRAME $\underbrace{|\bar{p}_B| = 0; \quad E_B = m_b}_{\text{STATIONARY TARGET}}$

$$E^{*2} = m_A^2 + m_B^2 + 2m_B E_A \rightarrow E^* \propto \sqrt{E_A} \leftarrow \text{ACCELERATOR ENERGY}$$

• COLLIDING BEAMS : $|p_A| = |p_B|$ $\cos\theta = -1$



$- \times - = +$

$$E^{*2} = m_A^2 + m_B^2 + 2E_A E_B + 2\underbrace{|p_A||p_B|}_{2E_A^2}$$

FOR $E >> m$

$$E^{*2} = 4E_A^2$$

$$E^* = 2E_A \leftarrow \text{ACCELERATOR ENERGY} \longrightarrow$$

$E_A \propto \$$

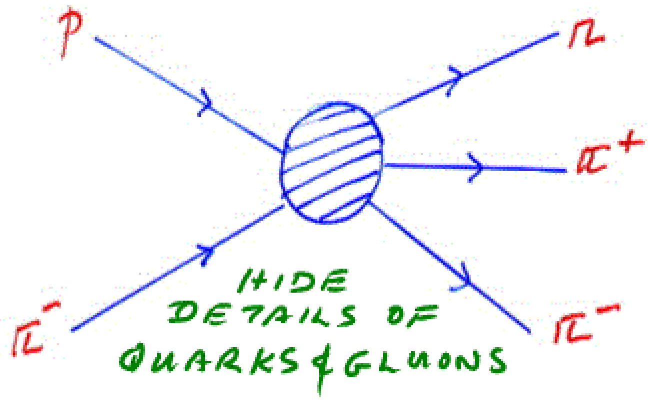
$E^*_{FIXED} \propto \sqrt{\$}$

$E^*_{COLLIDER} \propto \$$

ANOTHER USE OF INVARIANT MASS

PARTICLES DECAYING THRU STRONG FORCE $\tau \sim 10^{-23}\,s$

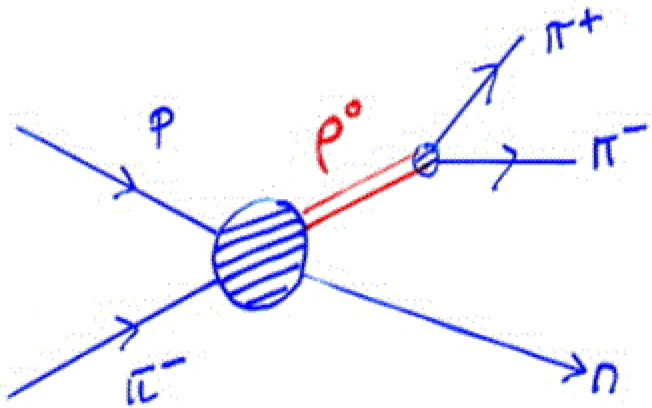
HOW CAN ONE MEASURE SUCH A SHORT LIFETIME?



COLLIDE $p\pi^-$

$\pi^- p \to n\pi^+\pi^-$

STRONG INTERACTION

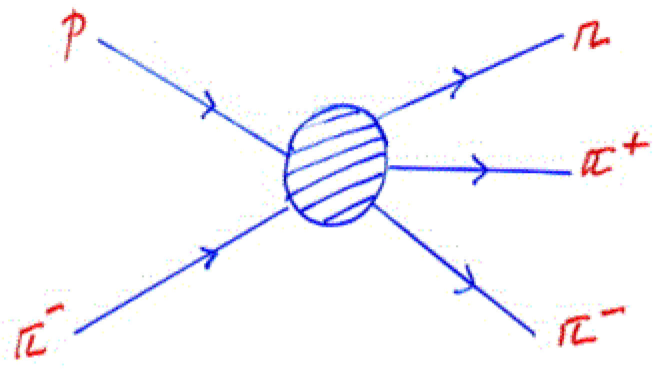


CAN PRODUCE VERY SHORT LIVED INTERMEDIATE STATE

"RESONANCE"

$\tau_{\rho_0} \sim 10^{-23}\,s$

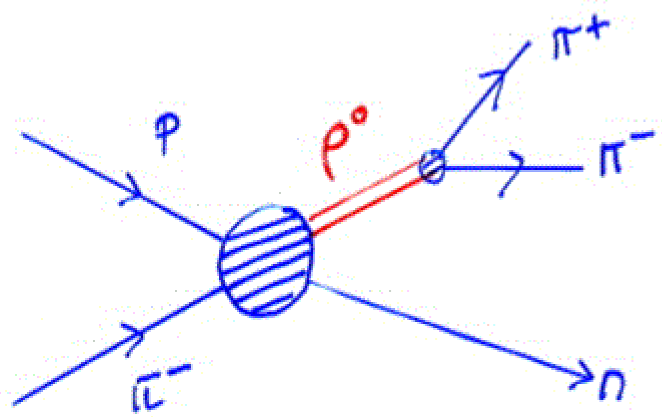
TRAVELS $\sim 1f = 10^{-15}\,m$



NO INTERMEDIATE STATE

ENERGY + MOMENTUM SHARED BETWEEN $\pi^+ \pi^- n$ IN RANDOM STATISTICAL FASHION

"PHASE SPACE"



IF INTERMEDIATE STATE

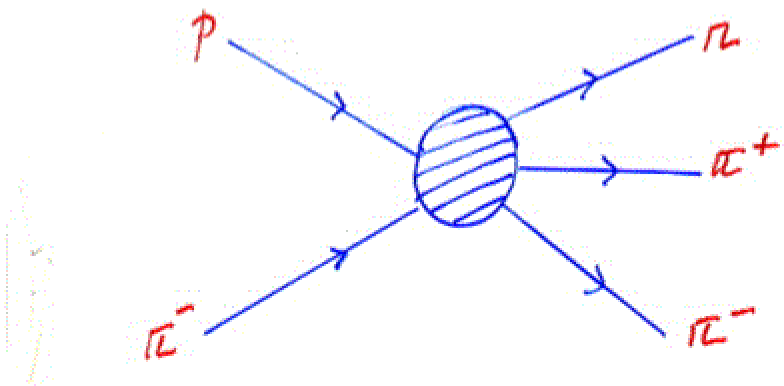
$$E_\rho = E_{\pi^+} + E_{\pi^-}$$

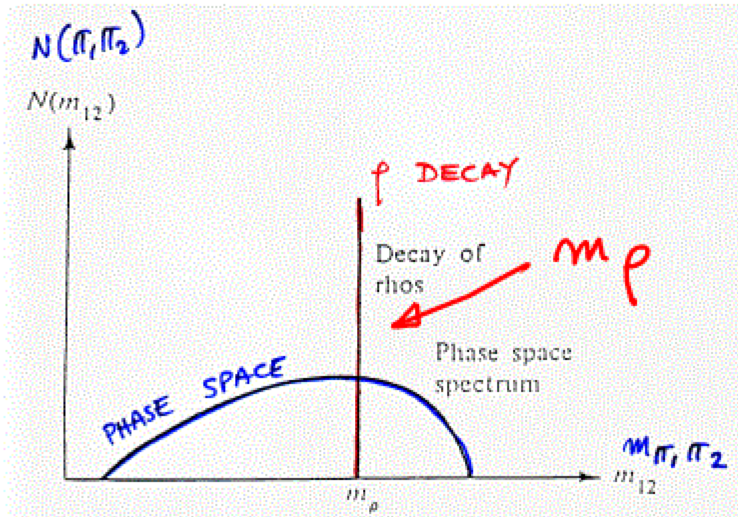
$$\bar{p}_\rho = \bar{p}_{\pi^+} + \bar{p}_{\pi^-}$$

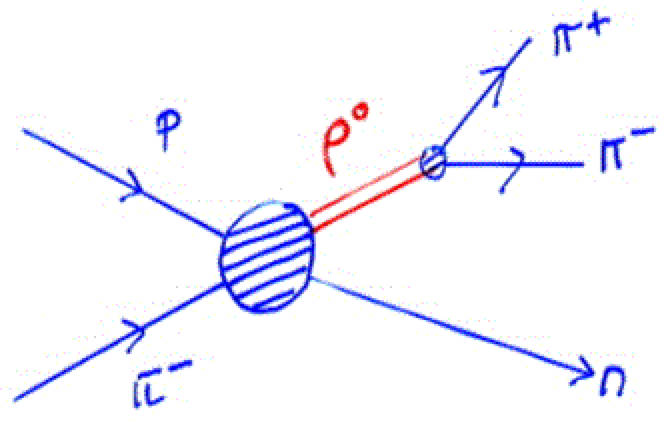
$$m_\rho^2 = \left[E_\rho^2 - \bar{p}_\rho^2\right]$$

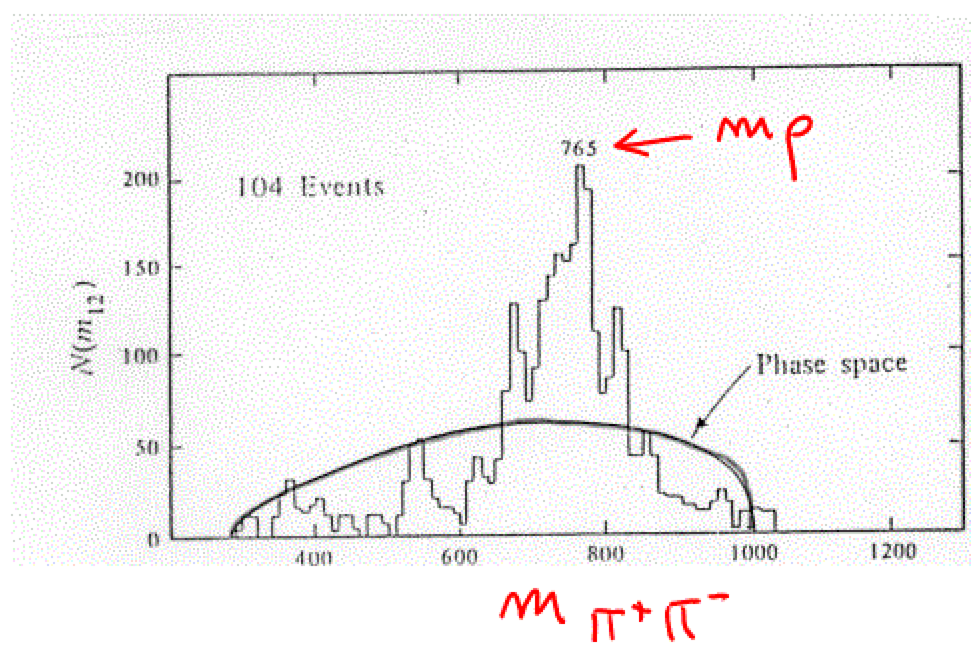
$$= \left[(E_{\pi^+} + E_{\pi^-})^2 - (\bar{p}_{\pi^+} + \bar{p}_{\pi^-})^2\right]$$

MASS OF ρ° = INVARIANT MASS OF DAUGHTER $\pi\pi$ PAIR


p
π
π+
π−
π−


N(π1π2)
N(m12)
ρ DECAY
Decay of rhos
mρ
Phase space spectrum
PHASE SPACE
mπ1π2
m12
mρ


p
ρ°
π+
π−
π−
n


765
mρ
104 Events
N(m12)
200
150
100
50
0
400
600
800
1000
1200
Phase space
m π+π−

EVIDENCE FOR A $T=0$ THREE-PION RESONANCE*

B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson
Lawrence Radiation Laboratory and Department of Physics, University of California, Berkeley, California

(Received August 14, 1961)

The existence of a heavy neutral meson with $T=0$ and $J=1^-$ was predicted by Nambu[1] in an attempt to explain the electromagnetic form factors

$$|Q|=0:\ \pi^+\pi^-\pi^0\ (800\times 4 \text{ combinations}), \qquad (4)$$

$$|Q|=1:\ \pi^\pm\pi^\pm\pi^\pm\ (800\times 4 \text{ combinations}), \qquad (4')$$

VOLUME 7, NUMBER 5 PHYSICAL REVIEW LETTERS SEPTEMBER 1, 1961

PHASE SPACE

ω MESON

$\omega \to \pi^+\pi^-\pi^0$

$M(\pi^+\pi^-\pi^0)$

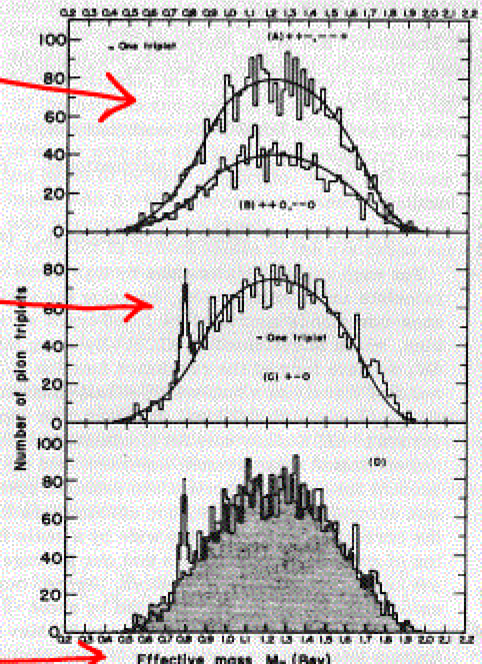
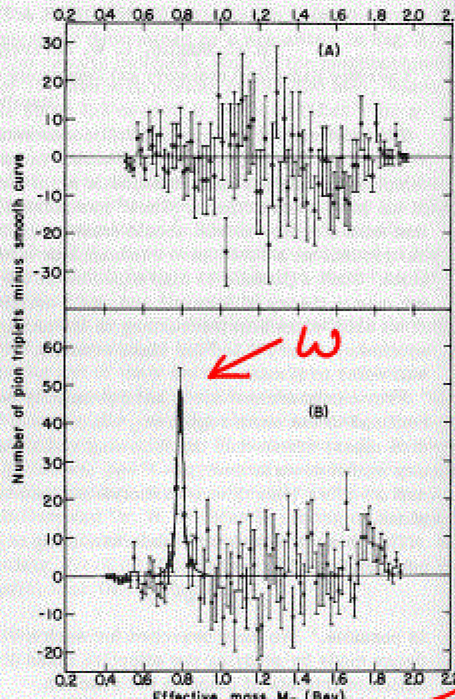


FIG. 1. Number of pion triplets versus effective mass (M_3) of the triplets for reaction $\bar{p}+p\to 2\pi^+ + 2\pi^- + \pi^0$. (A) is the distribution for the combination (4'), $|Q|=1$; (B) is for the combination (4"), $|Q|=2$; and (C) for (4), $Q=0$, with 3200, 1600, and 3200 triplets, respectively. Full width of one interval is 20 Mev. In (D), the combined distributions (A) and (B) (shaded area) are contrasted with distribution (C) (heavy line).

ω

PHASE SPACE SUBTRACTED



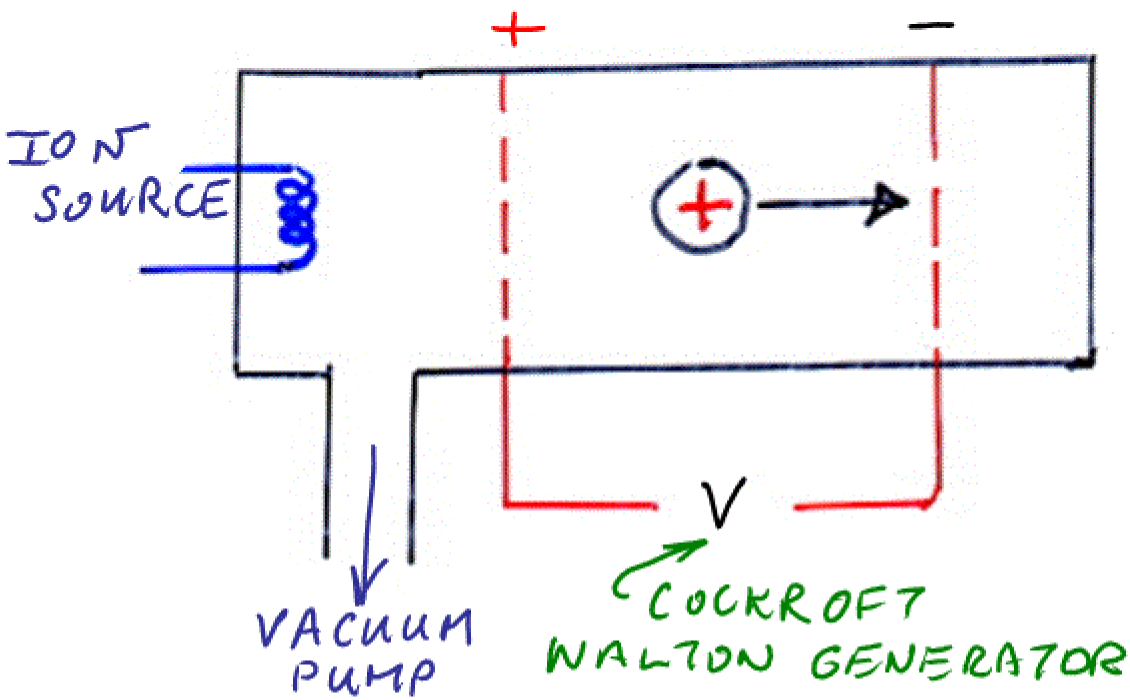
INVARIANT MASS $\pi^+\pi^-\pi^0$

FIG. 2. (A) M_3 spectrum of the pion triplets in the combined distributions 1(A) and 1(B), with the smooth curve subtracted. (B) M_3 spectrum of the neutral pion triplets in distribution 1(C), again with the smooth background subtracted; a resonance curve is drawn through the peak at 787 Mev with $\Gamma/2=15$ Mev. The error flags are $\sqrt{N}$, where N is the total number of triplets per 20-Mev interval before subtraction of the smooth background curve.

(2). The missing-mass distributions in the two

SIMPLE ELECTROSTATIC ACCELERATOR

USED BY COCKROFT & WALTON — ARTIFICIAL RADIOACTIVITY



ELECTRIC FIELD

$$\bar{F} = q\,\bar{E}$$

CHARGE ON PARTICLE

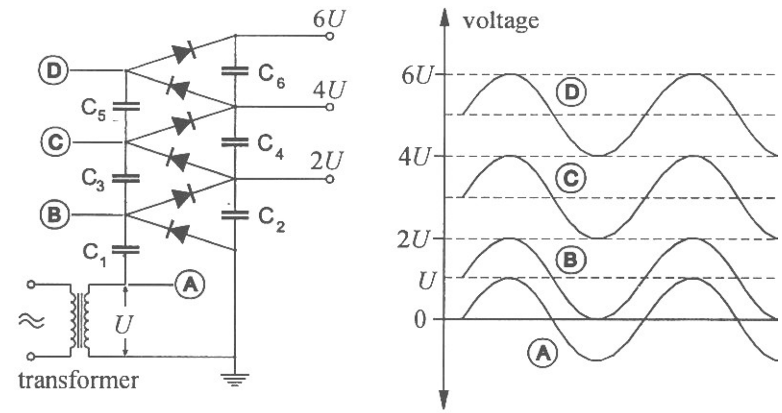
$$|\bar{E}| = \frac{V}{d}$$

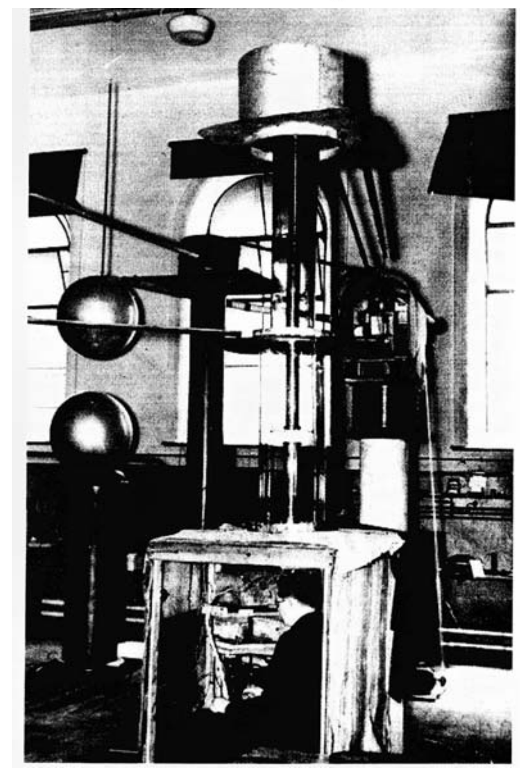
ENERGY GAINED BY CHARGED PARTICLE

$$E_{Acc} = Fd = qV$$

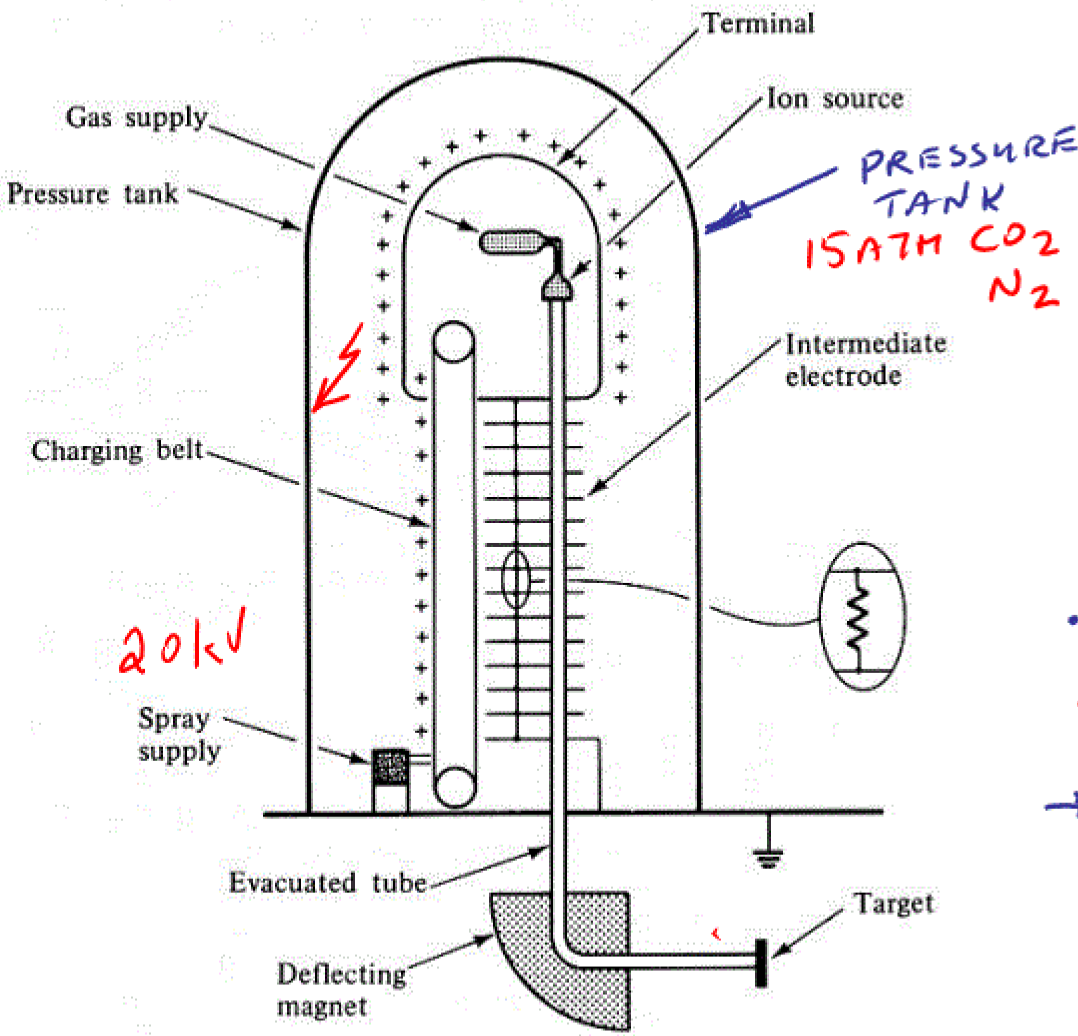
- TWO SHORTCOMINGS:
  - GENERATING HIGH VOLTAGE
  - INSULATING BEYOND ~ 100 kV (100 keV)

Cockcroft-Walton Generator





VAN DE GRAAFF



- TRANSPORT CHARGE

Q

TO TERMINAL OF CAPACITANCE

C

$$V = \frac{Q}{C}$$

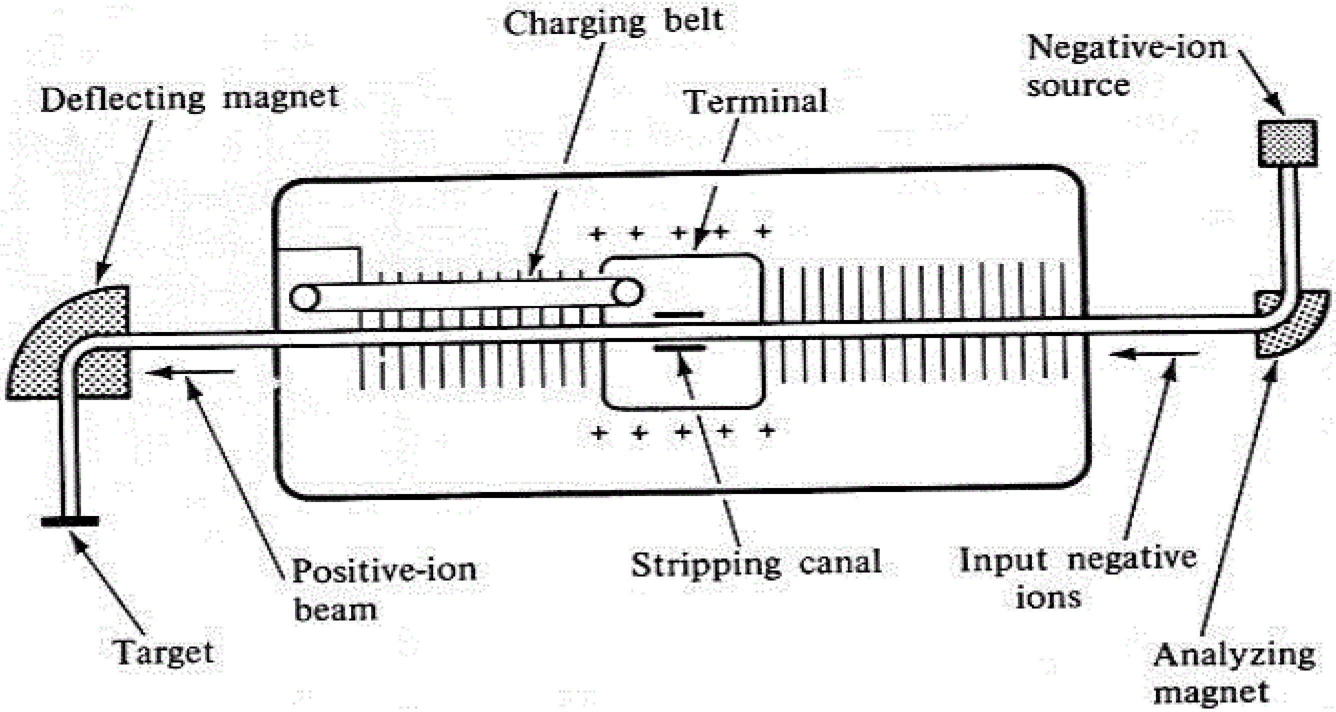
- LIMITATION ~ 12MV

→ VOLTAGE BREAKDOWN

→ NOT ENOUGH TO RESOLVE PROTONS IN THE NUCLEUS

~ 12 MeV

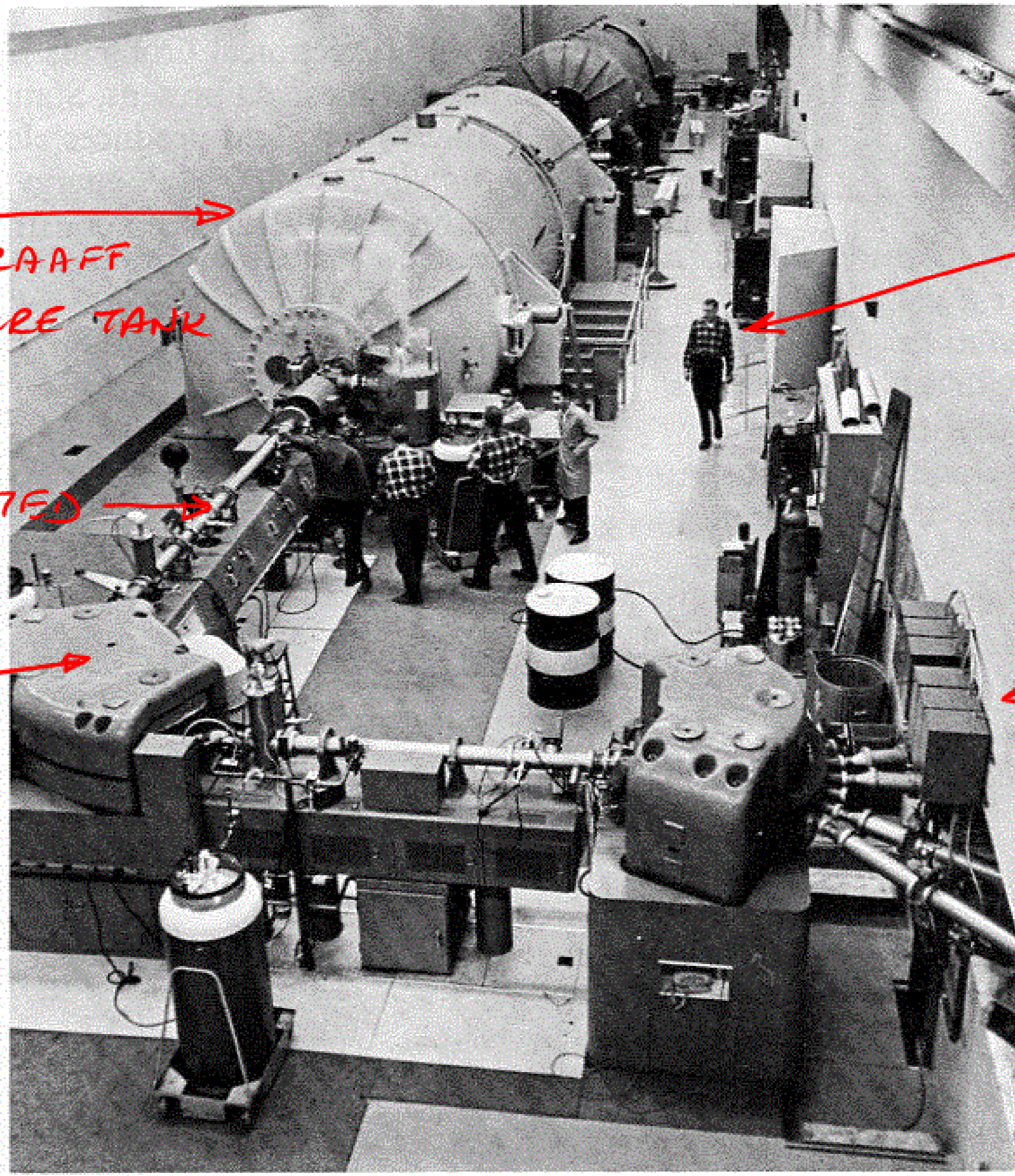
TANDEM VAN DE GRAAFF



- USE VOLTAGE ON TERMIAL TWICE
- ACCELERATE −VE IONS UP TO TERMINAL
- STRIP OFF TWO ELECTRONS INSIDE TERMINAL

  — ACCELERATE AWAY

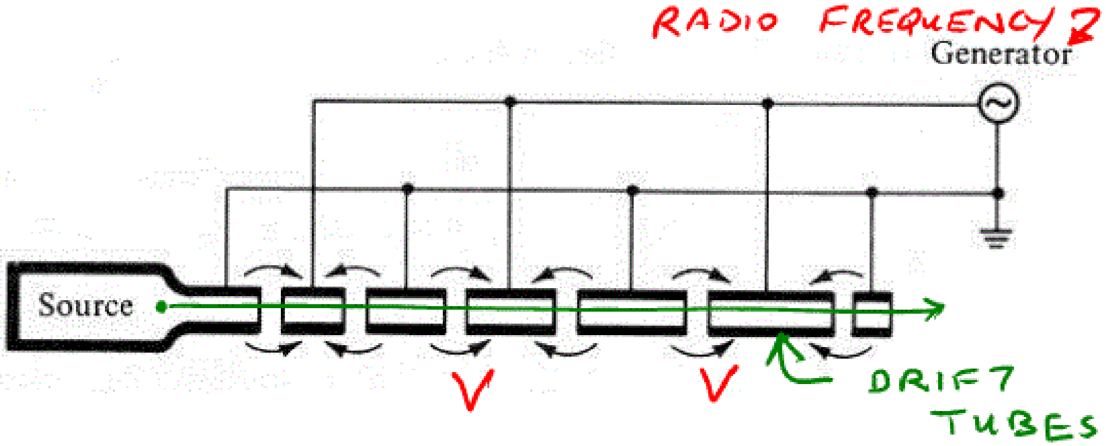
- 40 MeV   CHALK RIVER HAD LARGE TANDEM


VAN DE GRAAFF
IN PRESSURE TANK
1960'S
PHYSICIST
ACCELERATED
BEAM
BENDING
MAGNET
BEAMS TO
EXPERIMENTS

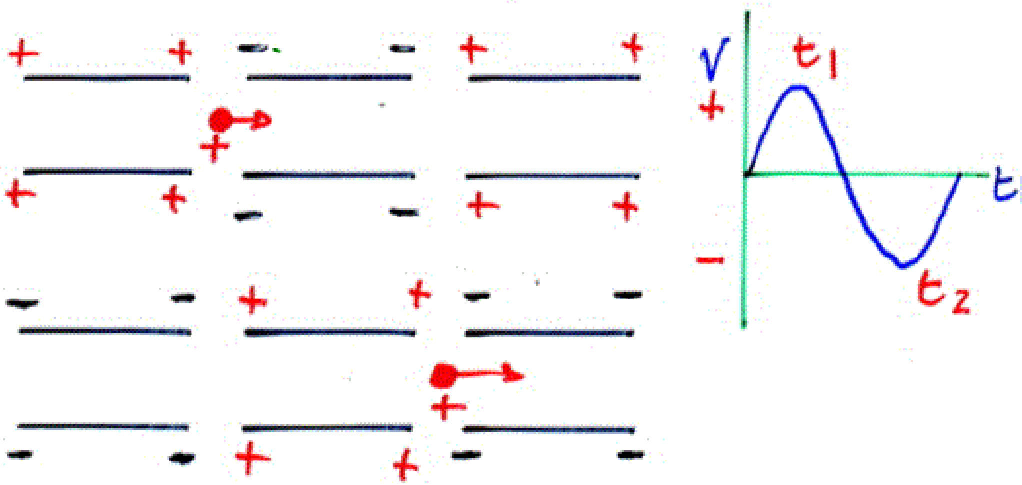
LINEAR ACCELERATOR (LINAC)

TORONTO USED TO HAVE 40 MeV LINAC

• INVENTED BY WIDEROE



• USE SAME RELATIVELY SMALL VOLTAGE IN MANY STEPS — REACH EQUIVALENT HIGH VOLTAGE



• FIELD ZERO INSIDE DRIFT TUBES

• PARTICLE MOVES ONE GAP → NEXT, IN TIME E-FIELD REVERSES

• PARTICLES ACCELERATING → LENGTH OF DRIF TUBES INCREASES

→ NON RELATIVISTIC

- PARTICLE ENTERING DRIFT TUBE n, ENERGY $n \cdot eV$

GAPS TRAVERSED

VOLTAGE ACROSS GAP

- NON-RELATIVISTIC

KINETIC ENERGY $T = \frac{1}{2} m v^2$

$$v = \left(\frac{2 \cdot neV}{m} \right)^{\frac{1}{2}} \qquad \left(\frac{2T}{m} \right)^{1/2}$$

- THIS VELOCITY TAKES PARTICLE THRU DRIFT TUBE OF LEGTH L_n IN TIME FIELD TAKES TO REVERSE

$$t_n = L_n / v$$

- FREQUENCY OF RADIO FREQUECY OSCILLATOR f (H_2) HAS REVERSAL TIME $\frac{1}{2f}$

$$L_n = \frac{1}{2f} \left(\frac{2neV}{m} \right)^{\frac{1}{2}} \longrightarrow L_n \propto \sqrt{n}$$

NUMERICAL VALUES

$$L_n = \frac{1}{2f} \cdot v_n$$

TYPICALLY $v_n = 0.5c$; $f = 7\,MHz \longrightarrow L_n = 10.7\,m$

- LOW RADIO FREQUENCY LEADS TO VERY LONG STRUCTURES

- PRACTICALLY NEED HIGH RADIO FREQUENCIES

  KLYSTRONS $\longrightarrow$ 100 MHz $\rightarrow$ 10 GHz

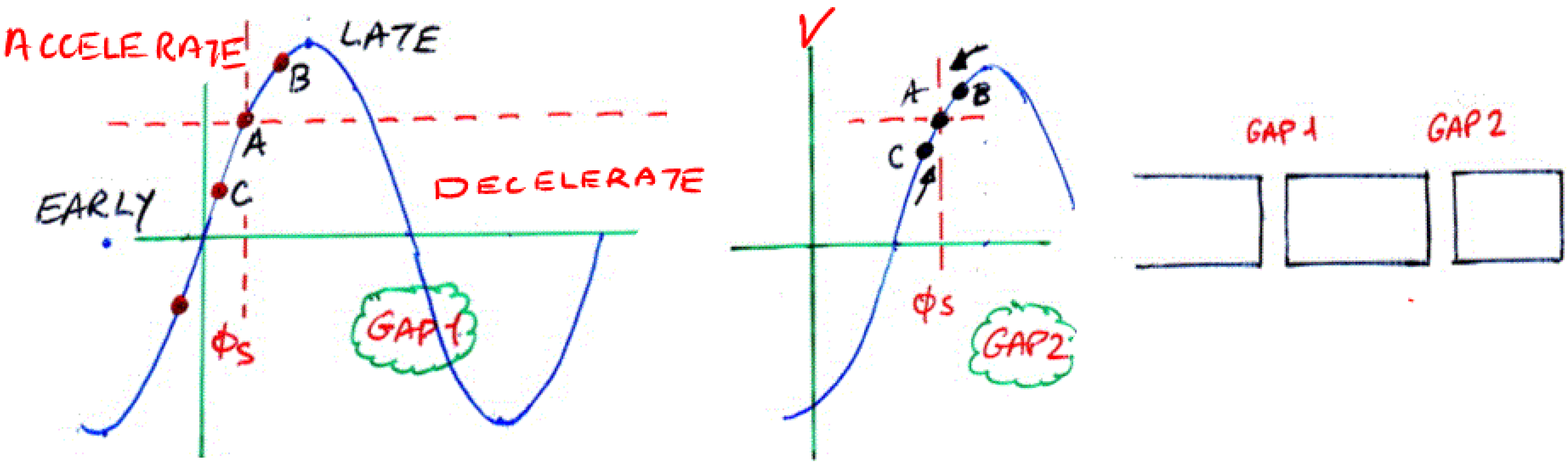
- THIS WIDEROE STRUCTURE IS OBSOLETE

  $\longrightarrow$ VERY INEFFICIENT

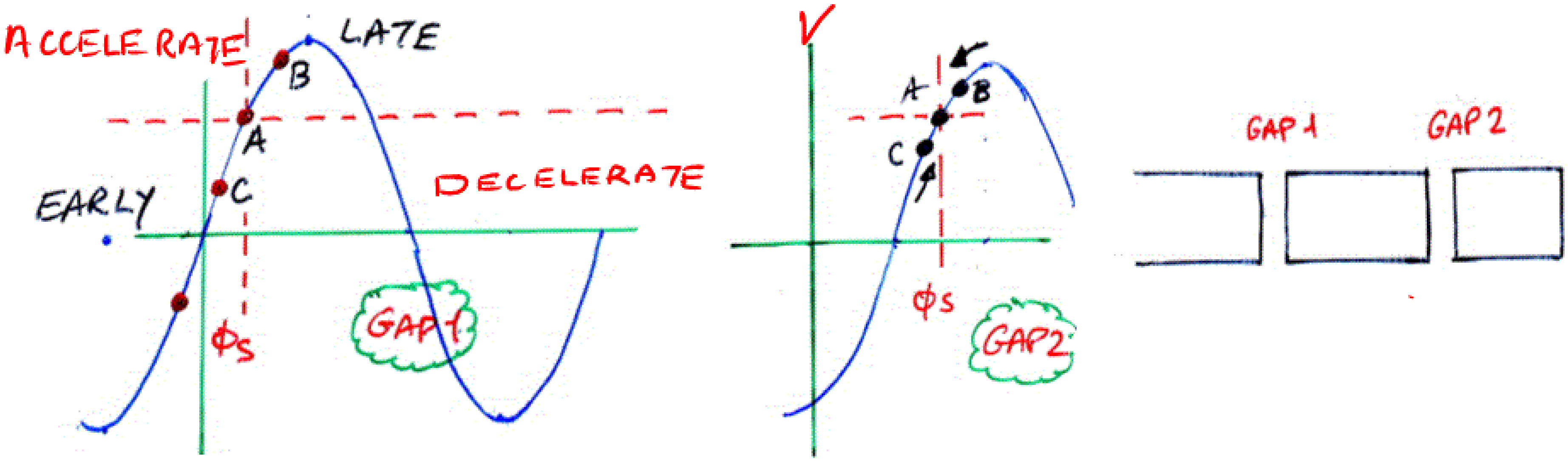
  $\longrightarrow$ RADIATION LOSS

PHASE STABILITY IN LINAC

- TO MAINTAIN PRECISE SYNCHRONISM BETWEEN PARTICLE MOTION & RF OSCILLATOR SEEMS DIFFICULT → NOT SO

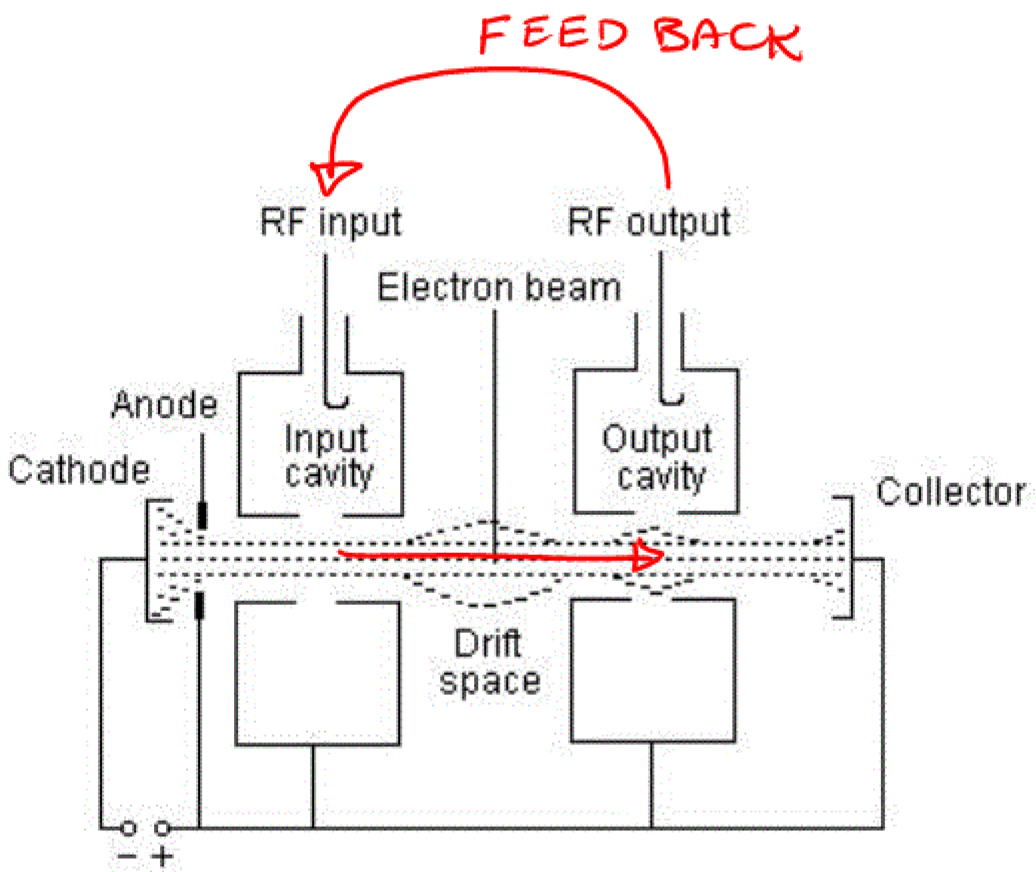


- PARTICLE A CROSSES GAP 1 PHASE ϕ IN STEP WITH VOLTAGE
- GAP 2 — SAME VOLTAGE PHASE — AGAIN ACCELERATED
- PARTICLE B ARRIVE LATE, VOLTAGE HIGHER ACCELERATED MORE ARRIVES AT GAP 2 EARLIER



- PARTICLE C ARRIVES EARLIER AT GAP 1
  - VOLTAGE LOWER, ACCELERATED LESS
  - ARRIVES LATER IN PHASE AT GAP 2
- B AND C CONVERGE IN PHASE WITH A
- NO NEED TO START WITH PARTICLES ALL IN PHASE WITH RADIO FREQUENCY OSCILLATOR

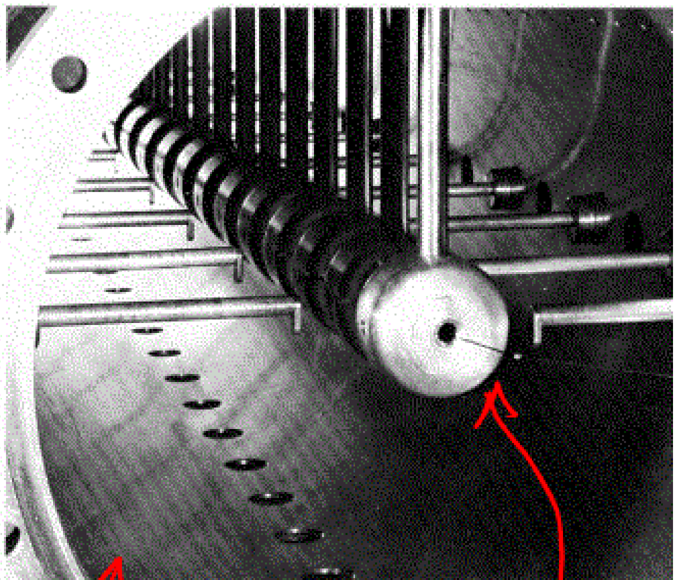
RADIO FREQUENCY POWER GENERATION



Two cavity klystron amplifier

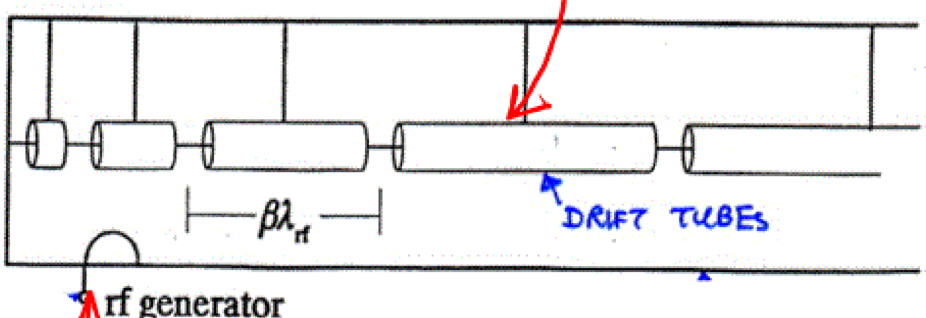
2 CAVITY KLYSTRON OSCILLATOR

ALVAREZ LINAC STRUCTURE



CONDUCTING ENCLOSURE

DRIFT TUBES



RADIO FREQUENCY INPUT

- WIDEROE STRUCTURE VERY INEFFICIENT — RADIO FREQUENCY RADIATION LOSS

- ALVAREZ STRUCTURE
  — RESONANT CAVITY LIKE KLYSTRON

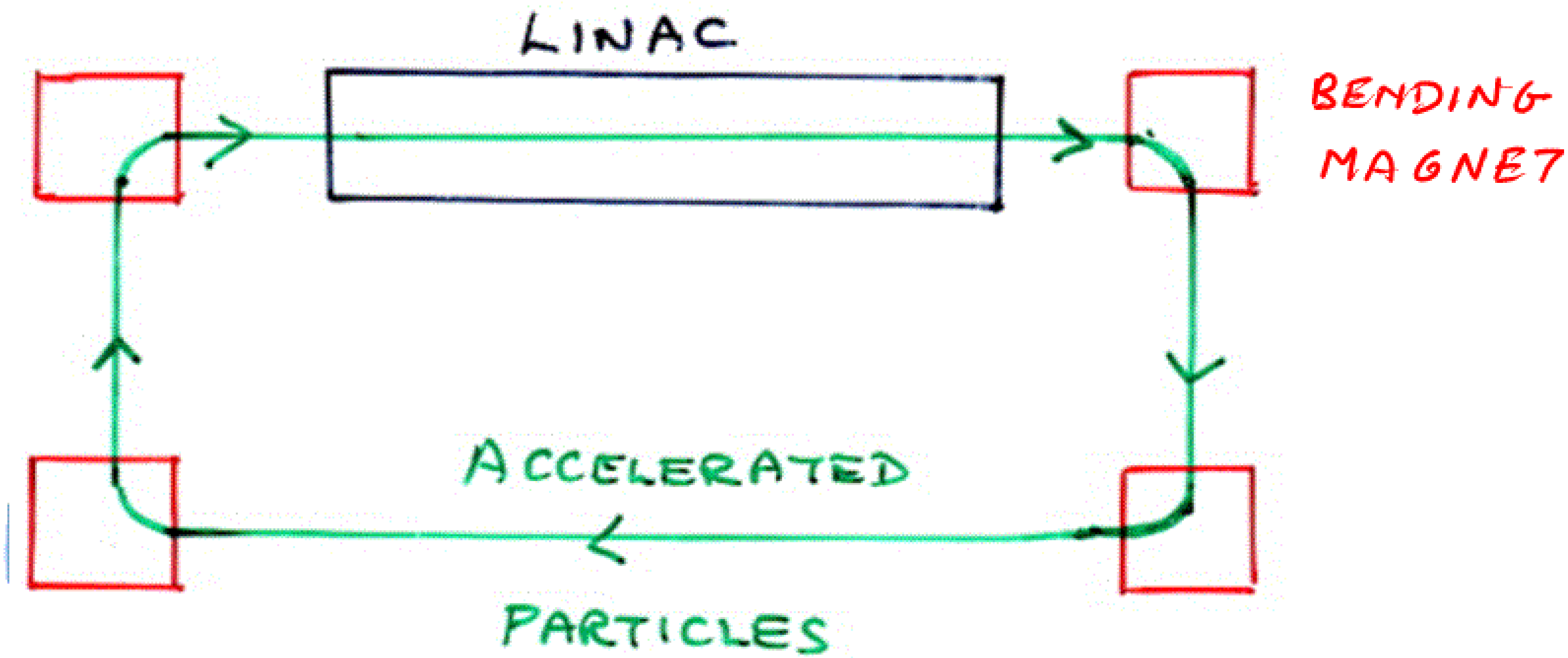
- USED FOR PROTON SYNCHROTRON INJECTOR
  100 MeV → 100 MHz

- HIGH ENERGY ELECTRON ACCELERATORS

40 GeV - 500 GeV GHz RF

SLAC – 50 GeV Electron LINAC

CIRCULAR ACCELERATOR

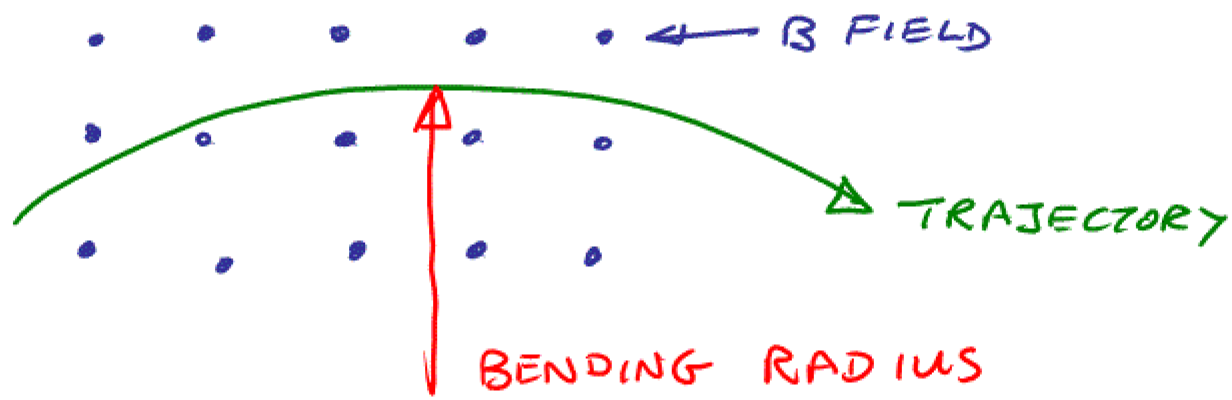


- REUSE ACCELERATING VOLTAGE UNTIL REACH VERY HIGH ENERGY

PARTICLE BENDING IN MAGNETIC FIELD

$$\bar{F} = q\left(\bar{E} + \frac{1}{c}\bar{v} \times \bar{B}\right)$$ LORENTZ

- FORCE FROM MAGNETIC FIELD NORMAL TO PARTICLE TRAJECTORY



- FOR NO ELECTRIC FIELD & B FIELD NORMAL TO PAGE

$$F = q\frac{v}{c}B\sin\theta \qquad 90^\circ = 1 \qquad F = q\frac{vB}{c}$$

- FOR A PARTICLE MOVING IN A CIRCLE OF RADIUS ρ

CENTRIPETAL FORCE = LORENTZ FORCE

CIRCULAR ACCELERATORS

- AT PRESENT PARTICLE PHYSICS DOMINATED BY

CIRCULAR ACCELERATORS

<u>ELECTRONS</u>

CESR

PEP II

KEK

LEP

<u>PROTONS</u>

SPS @ CERN

TEVATRON @ FERMILAB

AGS

LARGE HADRON COLLIDER

- MOST EFFICIENT & COMPACT WAY OF GETTING TO HIGH ENERGY – UNTIL SYNCHROTRON RADIATION BECOMES IMPORTANT

CENTRIPETAL FORCE = LORENTZ FORCE

$$\frac{\gamma m v^2}{\rho} = \frac{vBq}{c} \Rightarrow \rho = \frac{p \cdot c}{Bq}$$

BENDING RADIUS IN GAUSSIAN UNITS

• ACCELERATOR BUILDERS USE m, VOLT, TESLA

$$pc = \rho \cdot B \cdot q$$

esV/c   cm   Gauss

VOLT = STATVOLT / 300

TESLA = 10^4 GAUSS

m = 10^2 cm

$$pc \left[\frac{V}{c} \times 300\right] = \rho\left[m \times 10^2\right] B \left[T \times 10^4\right] e$$

$$pc\,[\mathrm{GeV}/c] = 0.3\,\rho\,[m]\,B\,[T]$$

$$\rho\,[m] = \frac{p\,[\mathrm{GeV}]}{0.3\,B\,[T]}$$

$$p[\mathrm{eV}]\,c = \rho(\mathrm{cm})\,B(\mathrm{g})$$

$$p\left[\frac{\mathrm{V}}{300}\right]\cdot c = \rho\,(\mathrm{m}\times 10^{2})\;B(\mathrm{T}\times 10^{4})$$

$$pc = \rho[\mathrm{m}]\,B[\mathrm{T}] \times 10^{6} \times 3\times 10^{2}$$

$$p[\mathrm{eV}] = \rho[\mathrm{m}]\,B[\mathrm{T}] \times 3\times 10^{8}$$

$$p[\mathrm{GeV}\times 10^{9}] = \rho\,[\mathrm{m}]\,B[\mathrm{T}] \times 3\times 10^{8}$$

$$pc\,[\mathrm{GeV}] = \rho\,[\mathrm{m}]\,B[\mathrm{T}]\times 0.3$$

zoom 87

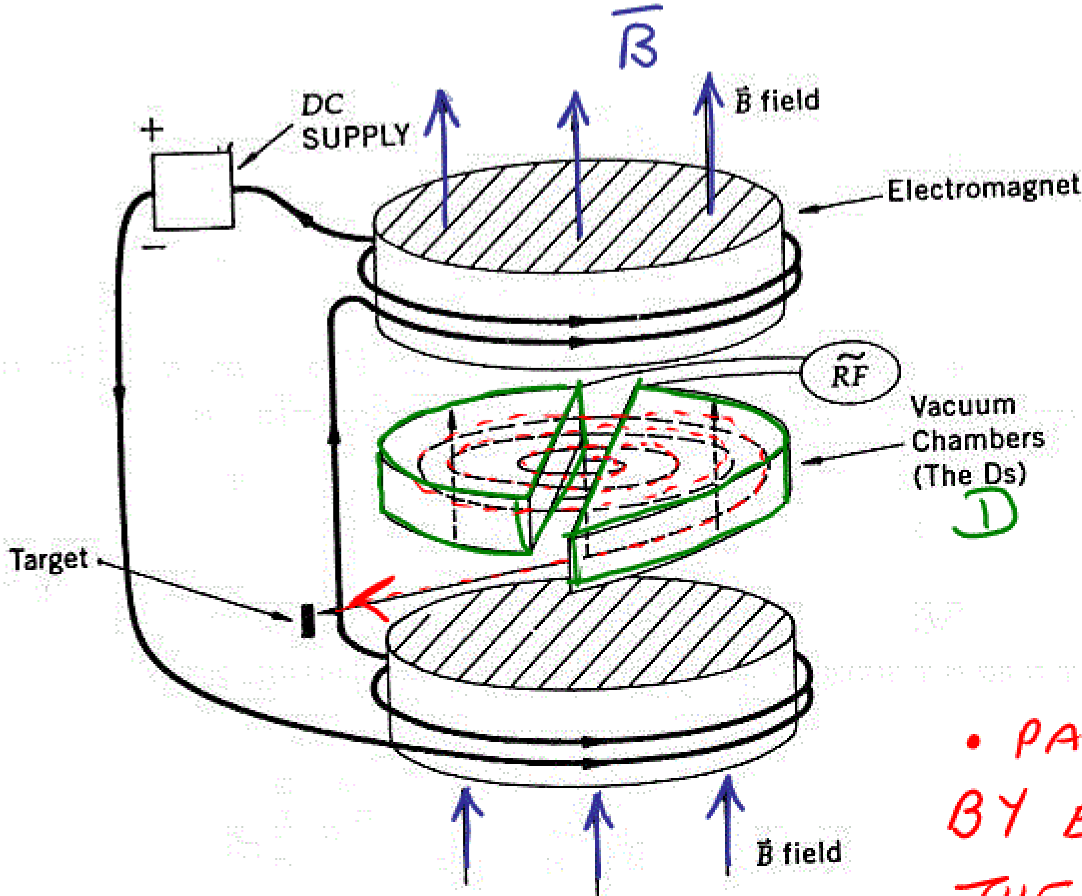
$$pc = 480 \;\; \mathrm{GeV} = 480\times 10^{9}\ \mathrm{eV}$$

$$pc\,[\mathrm{eV}] = \rho(\mathrm{cm}) \times B(\mathrm{g})$$

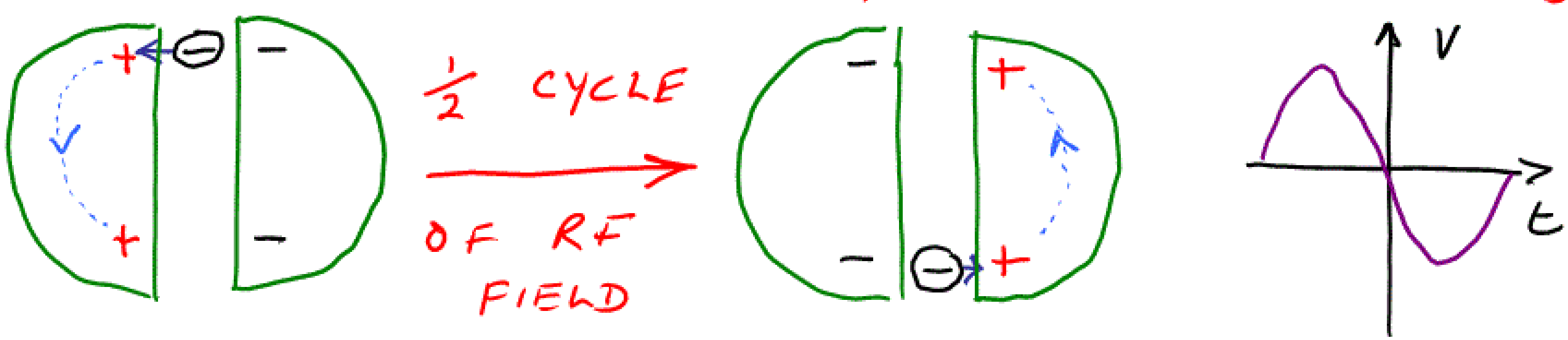
$$= 200\times 100 \times 8 \times 10000 = 1.6\times 10^{9}\ \mathrm{eSV}$$

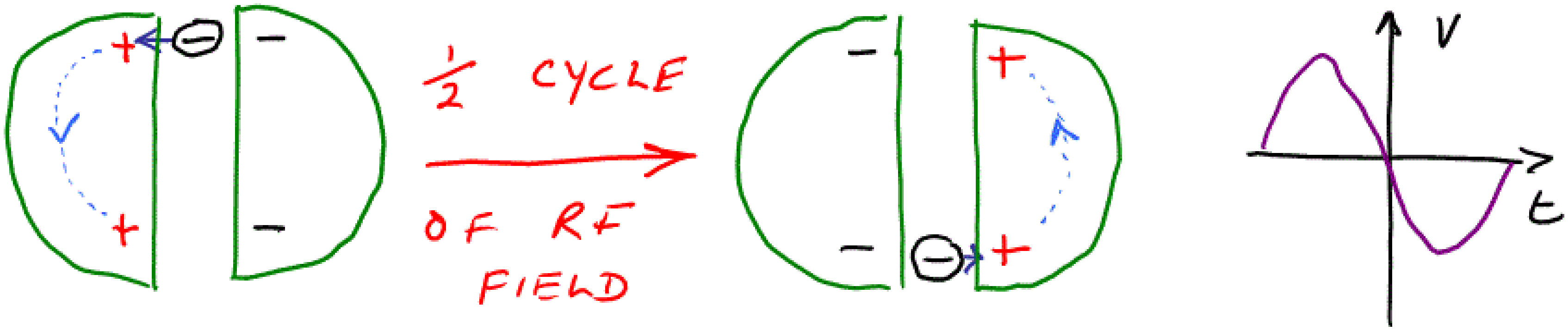
$$= 480\times 10^{9}\ \mathrm{eV}$$

THE CYCLOTRON



- TWO HOLLOW D-SHAPED ELECTRODES, EXCITED BY HV RF OSCILLATOR
- NO ELECTRIC FIELD INSIDE "Ds"
- MAGNET FIELD INSIDE Ds
- PARTICLES ACCELERATED BY ELECTRIC FIELD AS THEY CROSS GAP BETWEEN D_s





CENTRIPETAL FORCE = LORENTZ FORCE
FOR AN ORBIT OF RADIUS r

$$\frac{m v^2}{r} = q \frac{v \cdot B}{c}$$

$$\frac{v}{r} = \frac{q B}{m c} = \text{CONSTANT}$$

TIME FOR ORBIT = $2\pi r / v$

ORBITAL FREQUENCY = $v / 2\pi r$

IF RADIO FREQUENCY f = ORBITAL FREQUENCY

CONTINUOUS ACCELERATION

CONTINUOUS ACCELERATION

RADIO FREQUENCY = ORBITAL FREQUENCY

$$f = \frac{v}{2\pi r} = \frac{1}{2\pi}\frac{q}{m}\frac{B}{c} = \text{CONSTANT}$$

CYCLOTRON FREQUENCY

DOES NOT DEPEND ON RADIUS OF ORBIT

- PARTICLE STARTS AT SOURCE CLOSE TO CENTRE OF MACHINE
- SPIRALS OUT CONTINUOUSLY GAINING ENERGY FROM RESONANT RF.

THINK AGAIN ABOUT WHY A CYCLOTRON WORKS

$$F_c = F_L$$

$$\frac{mv}{r} = \frac{q \cdot B}{mc} = k$$

$$\frac{v}{r} = \text{CONSTANT} = \text{FREQUENCY}$$

AS r INCREASES, v INCREASES $\rightarrow$ $\frac{v}{r}$ = CONSTANT

FOR A RELATIVISTIC PARTICLE $v = c =$ CONSTANT

$$\therefore \frac{v}{r} = \frac{c}{r} \neq \text{CONSTANT}$$

ELECTRON CYCLOTRON

"MICROTRON"

ELECTRON IS RELATIVISTIC FOR $E \sim 500\,\text{keV}$

ORBITS INCREASE IN RADIUS DURING ACCELERATION

ANOTHER RELATIVISTIC EFFECT

$$f = \frac{1}{2\pi}\frac{q}{m}\frac{B}{c}$$ = RF FREQUENCY = ORBITAL FREQUENCY

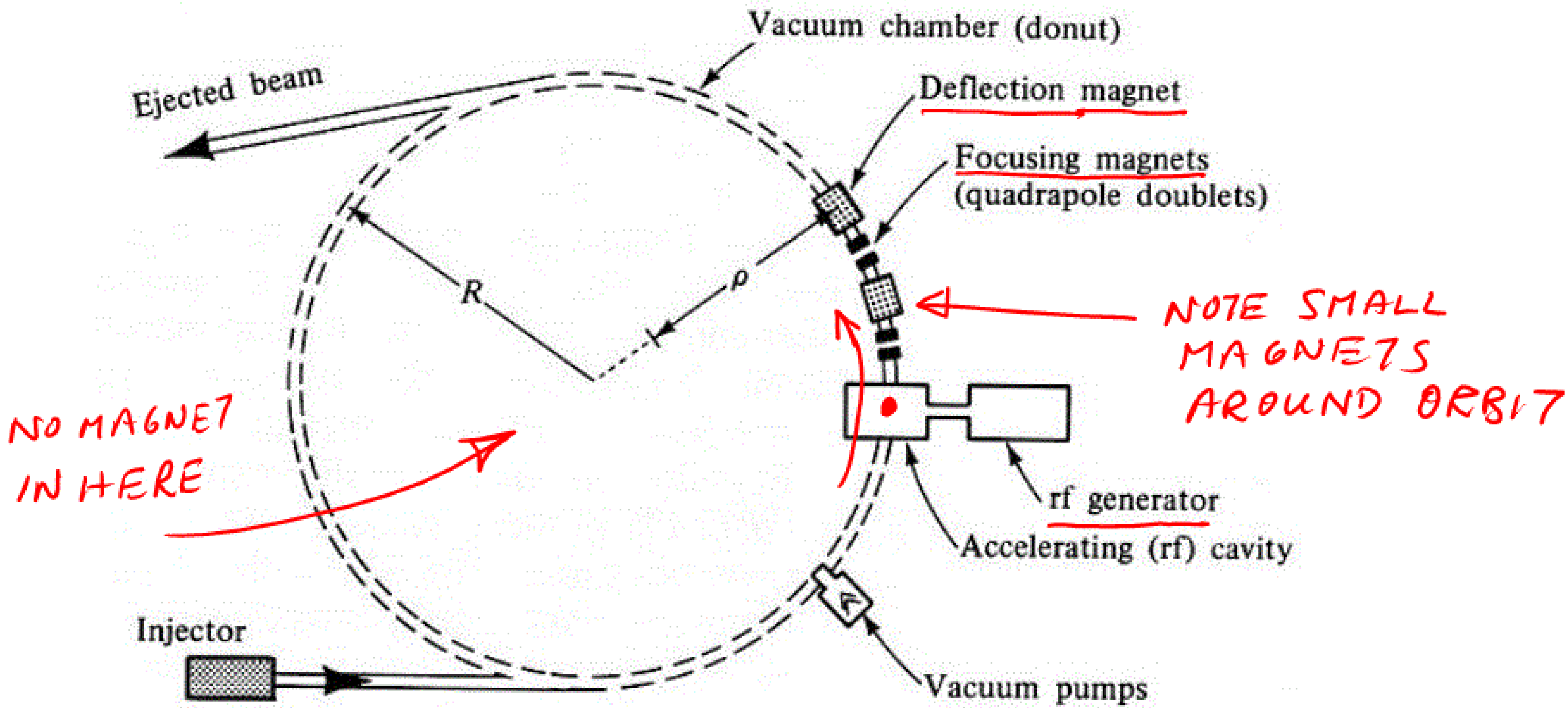
- AS PARTICLES ACCELERATE, TOTAL RELATIVISTIC ENERGY BECOMES ≈ MASS ENERGY
- IN THIS SITUATION $m \rightarrow m\gamma$ LORENTZ BOOST

$$f = \frac{1}{2\pi}\frac{q}{\gamma m}\frac{B}{c}$$

DURING ACCELERATION γ INCREASES & RESONANCE CONDITION FAILS

- INCREASE B — SYNCHROTRON
- DECREASE RF FREQUENCY — SYNCHROCYCLOTRON

SYNCHROTRON — CONSTANT RADIUS ORBIT



DUE TO MAGNETS ONLY AROUND ORBIT CAN BE MADE VERY LARGE → HIGHEST ENERGIES

SYNCHROTRON

AS USUAL

$$f = \frac{1}{2\pi}\frac{q}{m}\frac{1}{\gamma}\frac{B}{c}$$

EQUAL FOR RESONANCE

IN RELATIVISTIC SITUATION ORBITAL PERIOD $\frac{2\pi R}{c}$

SO ORBITAL FREQUENCY $c/2\pi R$

CONDITION FOR CONSTANT ACCELERATION

RF FREQUENCY = INTEGER x ORBITAL FREQUENCY

$$\frac{1}{2\pi}\underbrace{\frac{q}{m}\frac{1}{\gamma}\frac{B}{c}} = \frac{c}{2\pi r}\cdot n$$

HARMONIC NUMBER

SINCE $v \approx c$ & $p = m\gamma c$

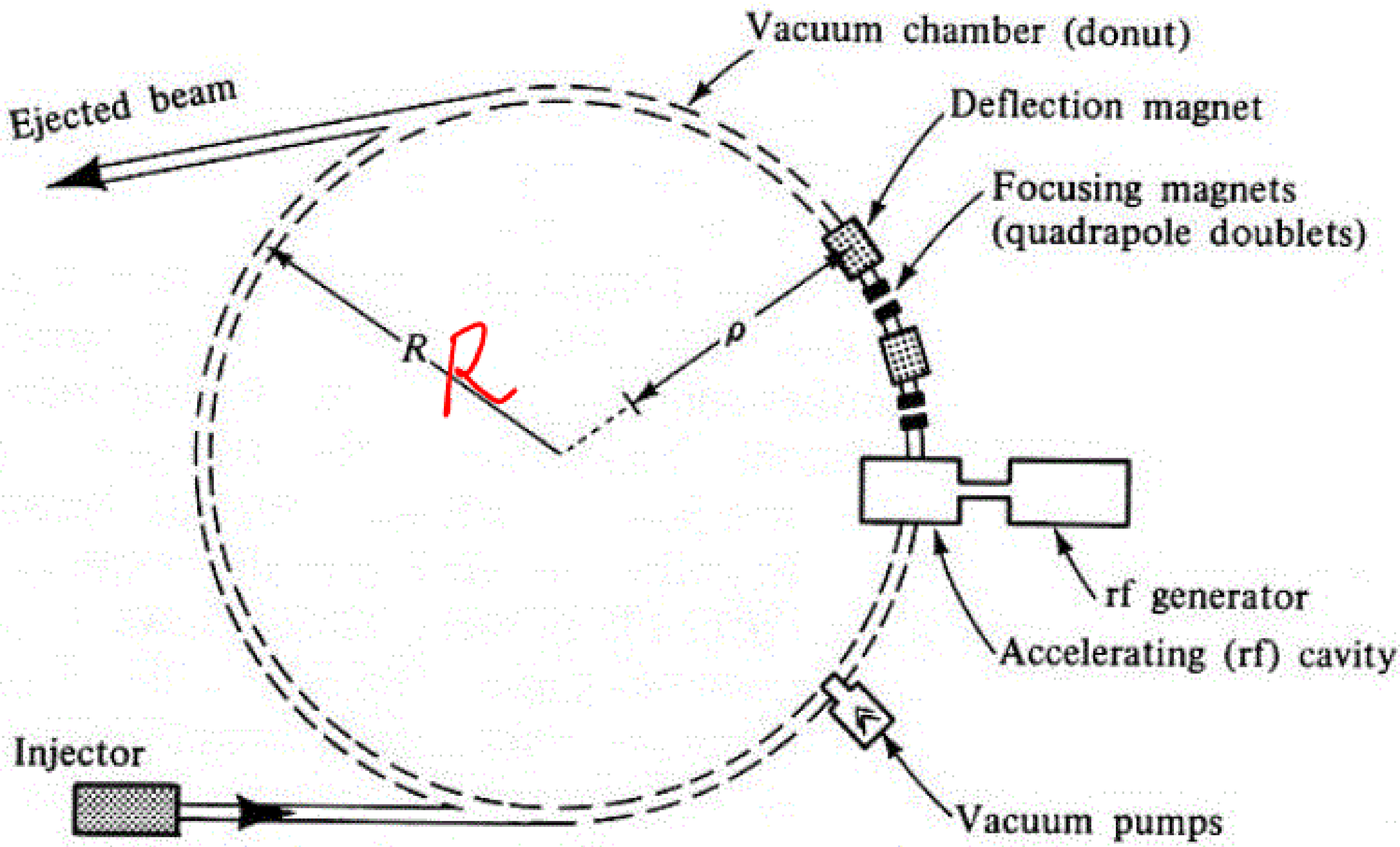
$p \sim m\gamma c$

$$\frac{qB}{p} = \frac{nc}{R}$$

$$R = \frac{ncp}{qB}$$

AS ACCELERATION PROCEEDS p INCREASES

$\therefore$ B INCREASES FOR CONSTANT R



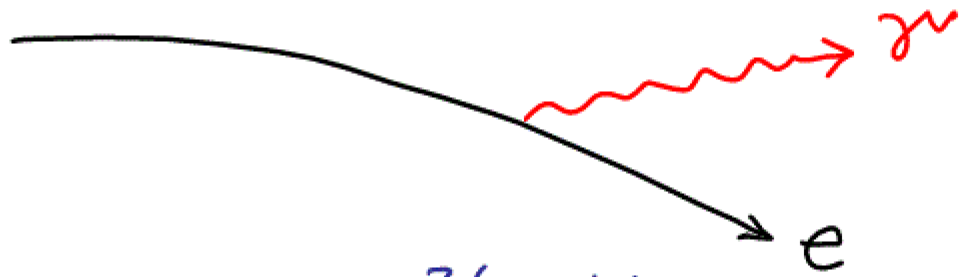
FOR A FIXED B_{MAX}

$$R = \frac{cP}{qB} \Rightarrow R_{MACHINE} \propto p_{MAX}$$

HIGHER ENERGIES $\Longrightarrow$ LARGER MACHINES

ELECTRON VERSUS PROTON SYNCHROTRON

- ELECTRONS ACCELERATED AROUND CIRCULAR ORBIT $\longrightarrow$ RADIATE



$$\text{ENERGY LOSS} \propto \frac{4\pi e^2}{R}\left(\frac{E}{mc^2}\right)^4$$

$$\frac{\Delta E(\text{PROTON})}{\Delta E(\text{ELECTRON})} = \left(\frac{m_e}{m_p}\right)^4 \approx 10^{-13}$$

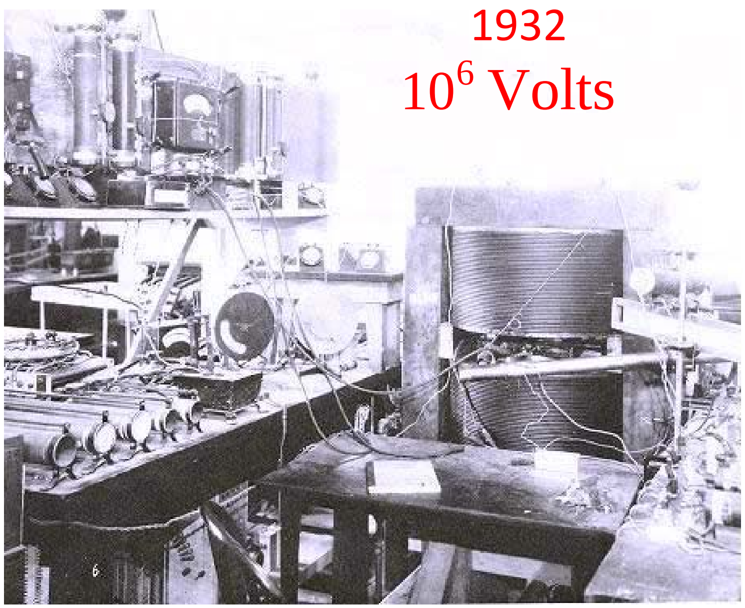
THIS IS WHY ELECTRONS IN CERN TUNNEL GO TO 50 GeV WHILE PROTONS TO 7000 GeV

LIMITED BY BENDING MAGNET

This machine is just
a model for a bigger
one, of course

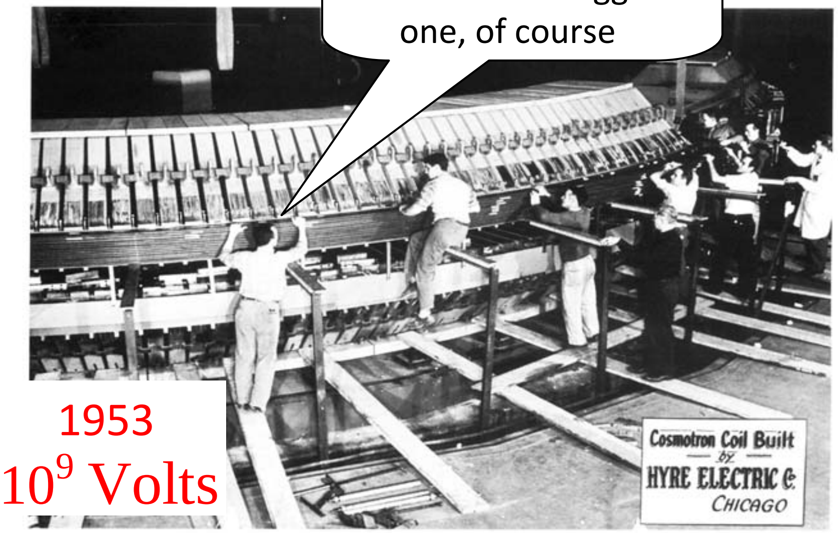
1931

10^4 Volts



1932

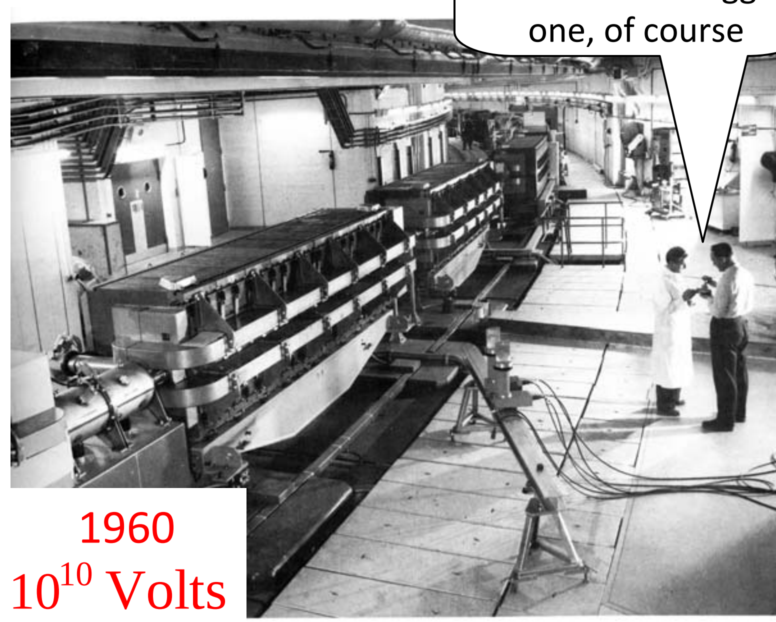
10^6 Volts



This machine is just
a model for a bigger
one, of course

1953

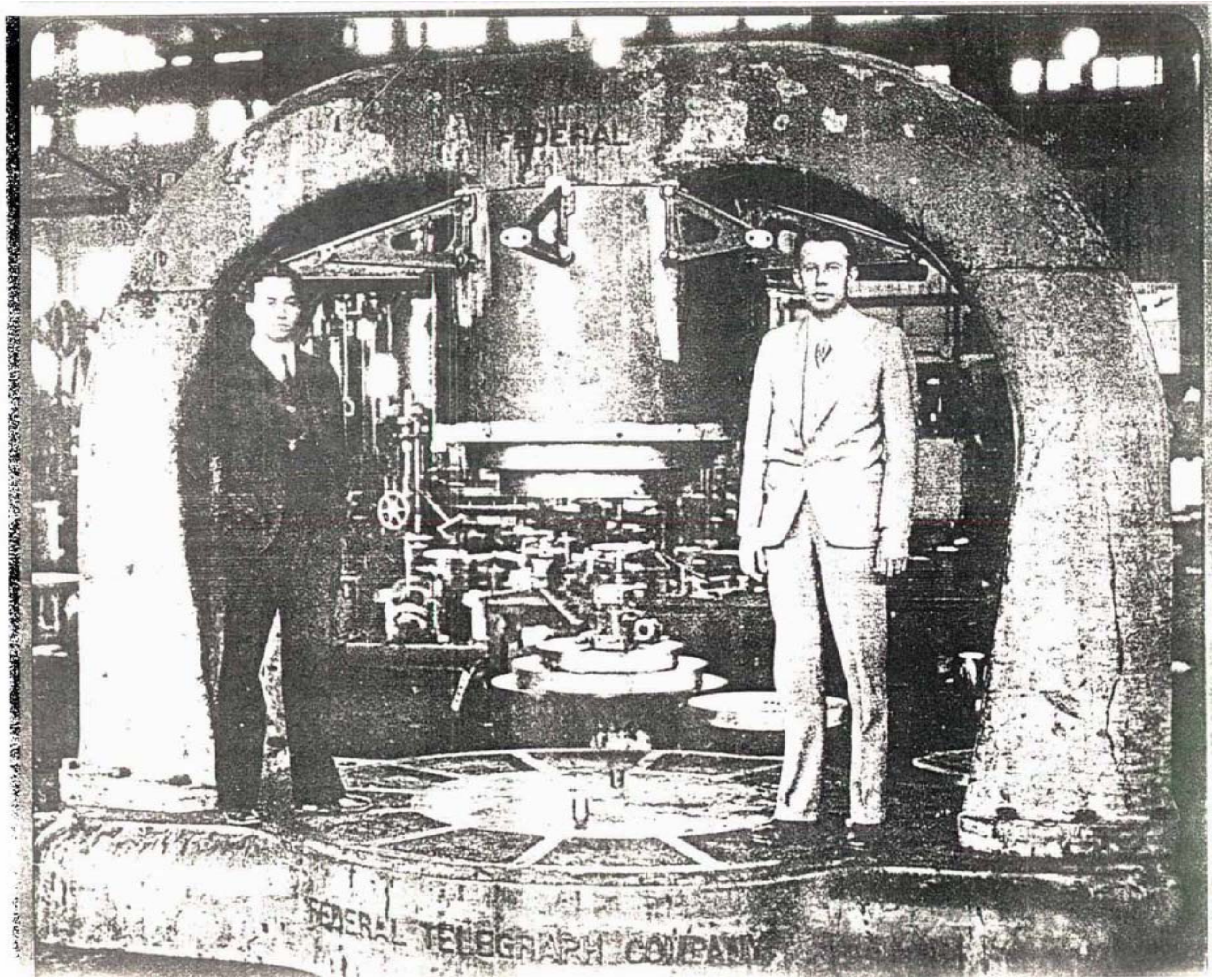
10^9 Volts



This machine is just
a model for a bigger
one, of course

1960

10^{10} Volts

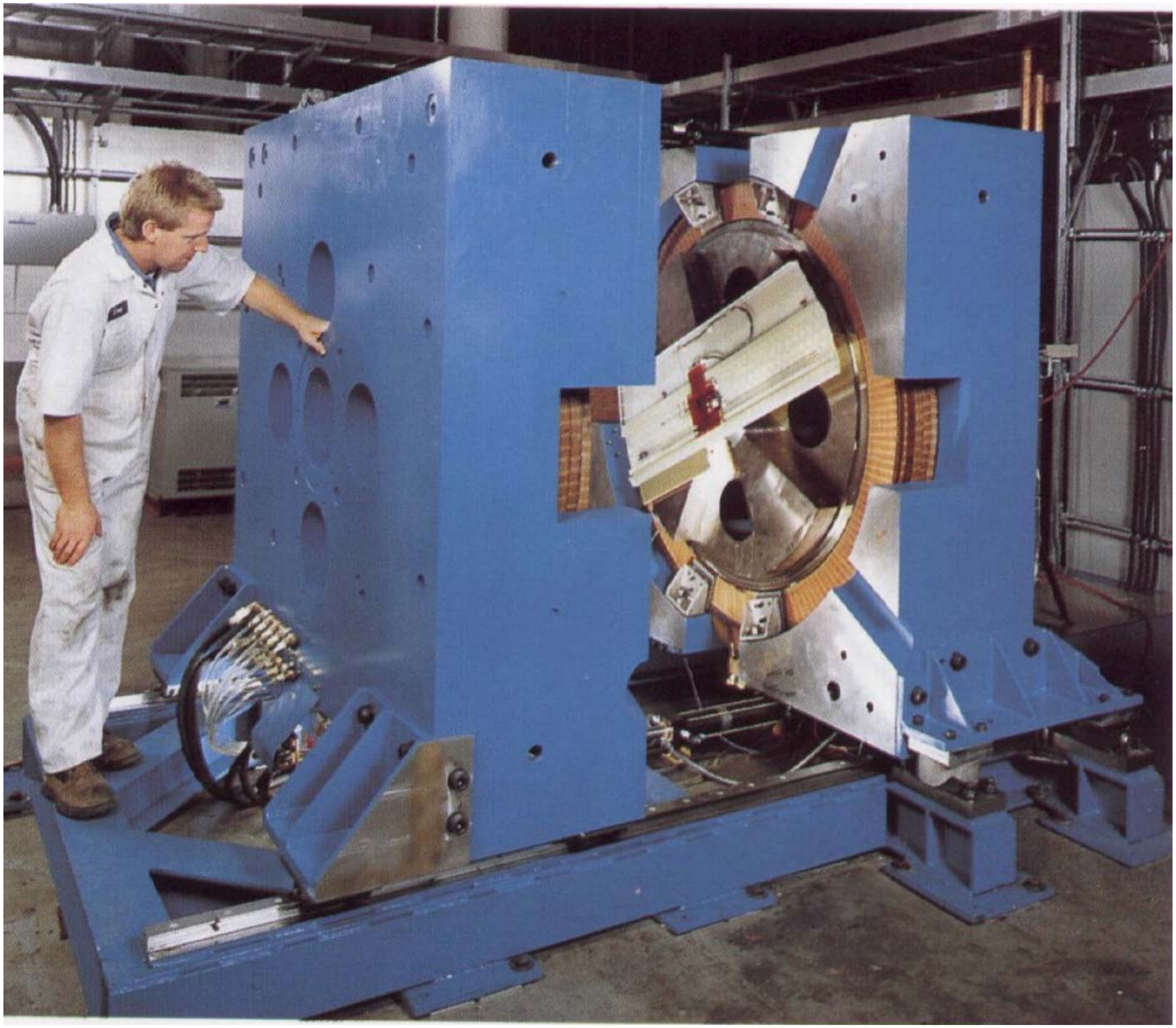


Livingston and Lawrence with the magnet of the “27-inch” (later “37-inch”) cyclotron on which most of Berkeley’s 1930s nuclear physics was performed. Lab wear was different then!

THE 184-INCH SYNCHROCYCLOTRON

The Berkeley 184” was begun in 1939 as a classical cyclotron, to be operated with Vrf = 1 MV, but WWII interrupted rf installation and it was used to test mass spectrographic separation of uranium isotopes. FM rf was installed in 1946, yielding 190 MeV d+ (700 MeV p in 1959).

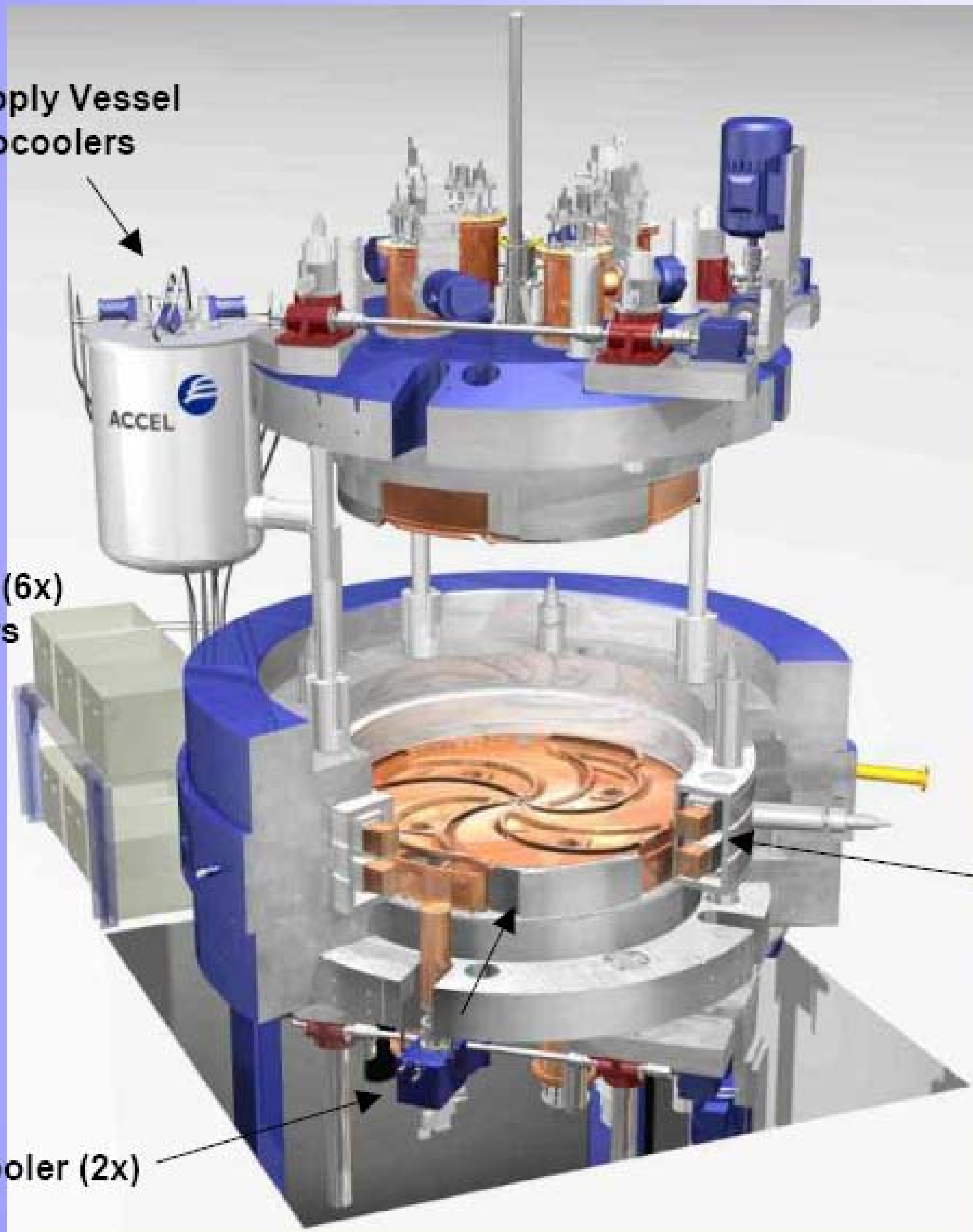
PET Medical Cyclotron

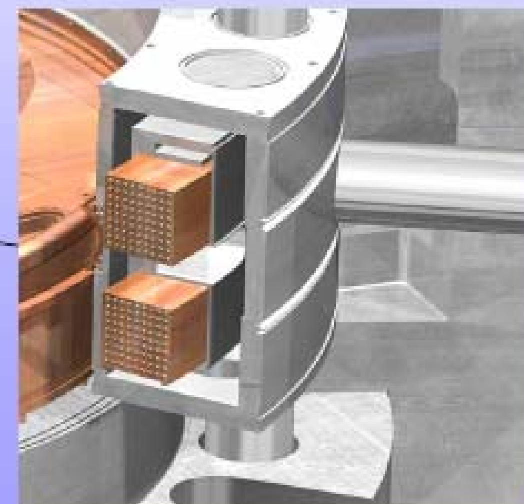


TRIUMF (Vancouver) 500 MeV Cyclotron

ACCEL

250 MeV Superconducting Proton Cyclotron

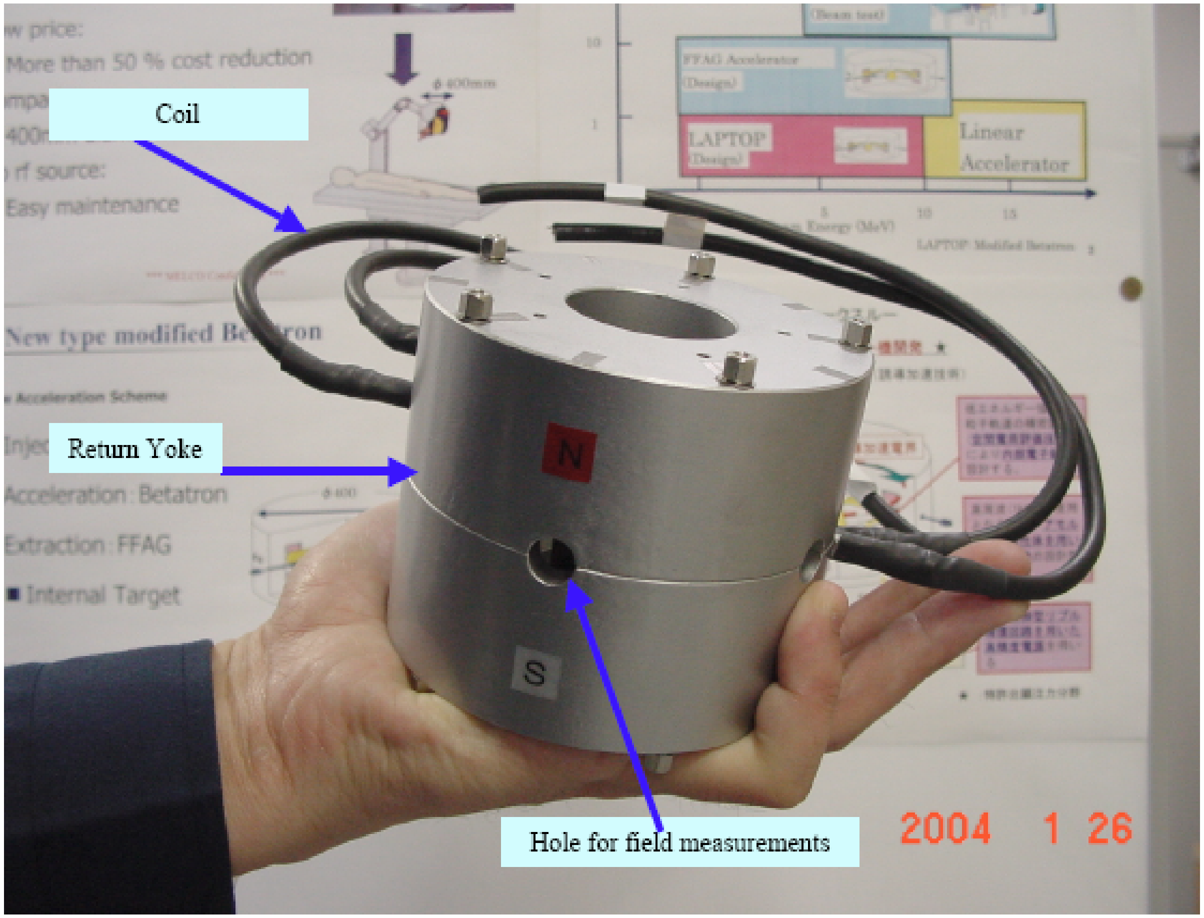




Superconducting Coil

Spiral Magnet

Changes for the Better



The present study is partially supported by the REIMEI Research Resources of Japan Atomic Energy Research Institute.

You can have your own cyclotron – from Mitsubishi

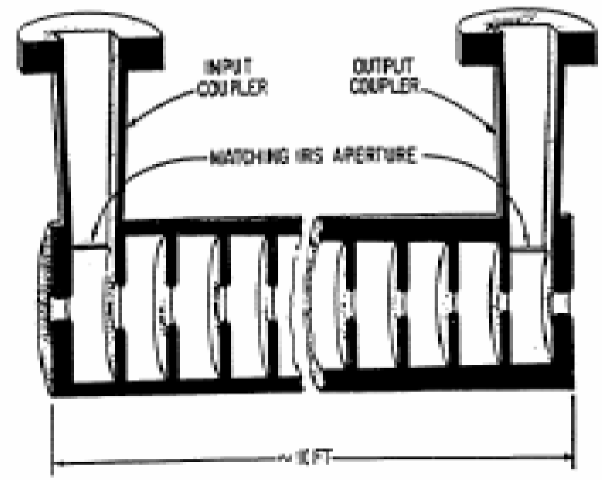

Alors, c’est fini!
Et maintenant?

DC HIGH-VOLTAGE ACCELERATORS – TANDEM VAN DE GRAAFFS

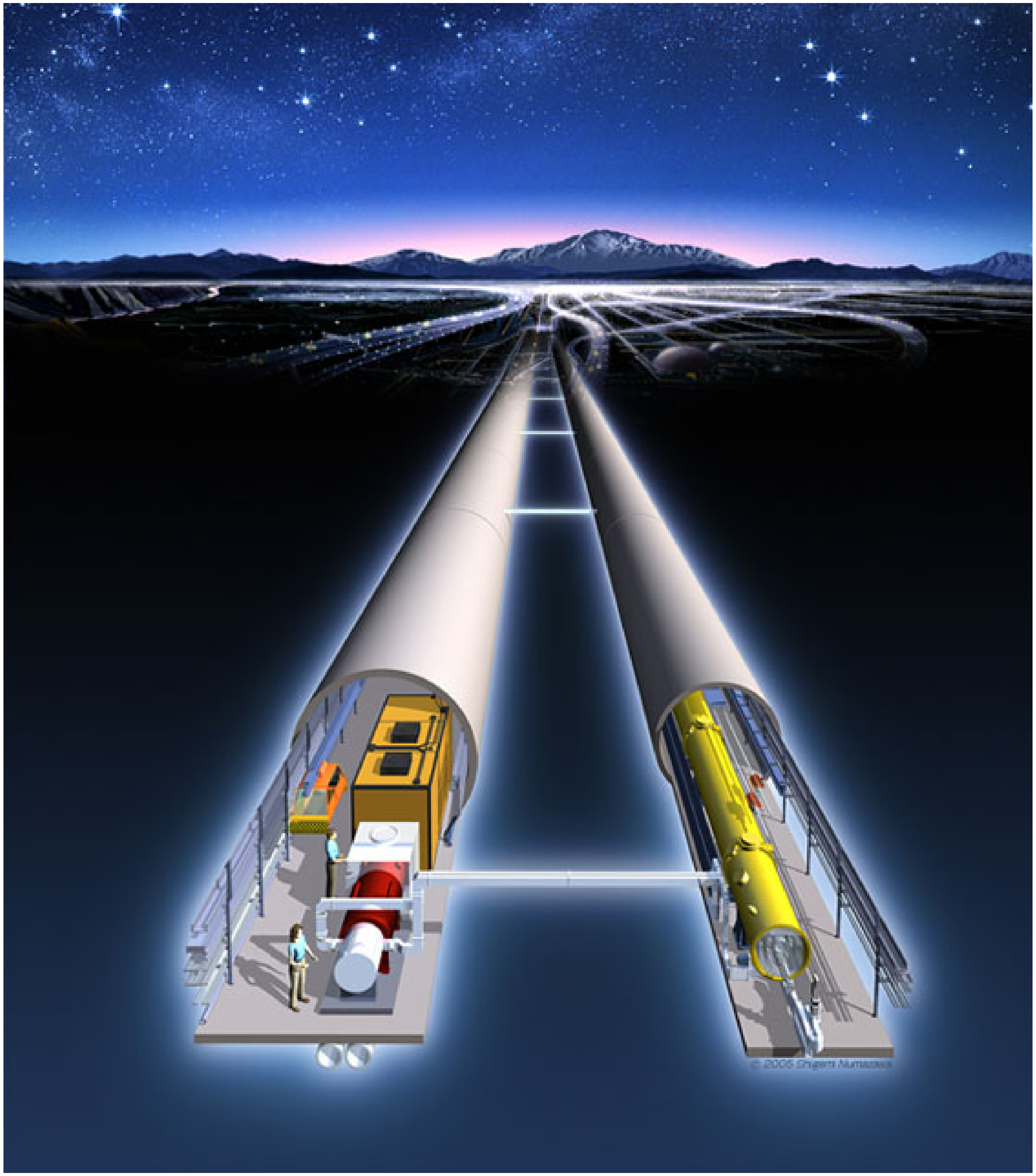
Yale 22-MV tandem.

Daresbury folded tandem
(20 MV in a 230-ft tower).

The ISAC 150-keV/u RFQ linac



500 keV electron LINAC for Cancer Therapy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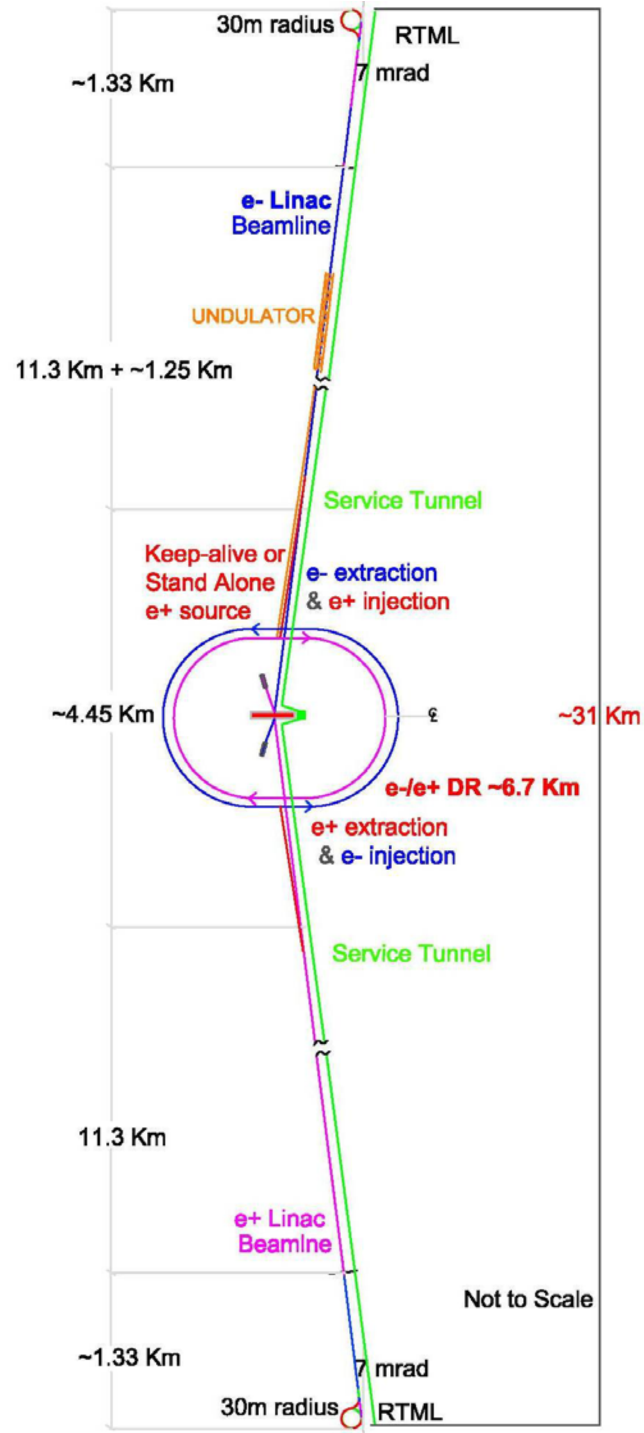
30m radius
RTML
~1.33 Km
7 mrad
e- Linac
Beamline
UNDULATOR
11.3 Km + ~1.25 Km
Service Tunnel
Keep-alive or
Stand Alone
e+ source
e- extraction
& e+ injection
~4.45 Km
~31 Km
e-/e+ DR ~6.7 Km
e+ extraction
& e- injection
Service Tunnel
11.3 Km
e+ Linac
Beamline
Not to Scale
~1.33 Km
7 mrad
30m radius
RTML