PHYSICS 357S - Problem Set #1 - January 2004

Distributed **12th January**. Due to be handed in by **21st January** at class. After this date it should be handed to Jean-Francois. Please be careful handing work in. Try to give it to Jean-Francois personally. Lost work cannot be given credit.

This problem set counts for 10% of the grade. For the numerical values of constants, such as masses, (that I may have forgotten to give you!) , you should use the Appendices at end of the text book starting on page 576 (or just ask me). If you don't understand a question ask me about it. If you think there is a bug (error, typo, etc) in a question..tell me. You might be right!

1) In the alpha decay (α-decay) of a nucleus, one can imagine that α particles form semi-stable subsystems within a nucleus. A nucleus of element having chemical symbol *C*, and with *Z* protons and *A-Z* neutrons is represented by the notation: ${}^{A}_{Z}C$. *Z* is the nuclear charge and *A* is the atomic mass number. An α-particle is a helium nucleus ${}^{4}_{2}He$ and forms a particularly tightly bound system. So sometimes some of the neutrons and protons inside a nucleus come together to form an α particle. This α particle then sees itself sitting inside a potential well formed by the other neutrons and protons. As this is a quantum mechanical system there is a probability for the α particle to tunnel through the potential wall formed by the other protons and neutrons. This is exactly like the problem of tunneling through a potential barrier in simple quantum mechanics;. you must have studied this already. Anyway, you don't need to know any of that to solve the following simple problem.

An α particle has a kinetic energy of 4 MeV.

i) What is its velocity if you assume that this is a non-relativistic system?

ii) How large an error do you make by ignoring relativistic effects?

iii) Why is it OK to treat 4 MeV α particles as non-relativistic. Answer this by comparing the *total energy* and the *kinetic energy.*

(If you did want to understand a bit more about alpha decay, you could look into Nuclear & Particle Physics – W.S.C. Williams (Oxford) Chapter 6.

2) A pion (π) is an example of a type of particle known as a meson. It actually consists of a quark and an anti-quark bound together by the strong, or colour, force. The μ is a lepton, as far as we know it is a point particle with no internal structure; just like a quark, but it does not see the color force, just the electromagnetic and weak forces. The μ can carry either positive or negative electric charge of the same magnitude as the charge carried by the electron (which is also a lepton). The neutrino ν_μ is also a lepton, but it is electrically neutral. Notice the subscript μ . This shows that the neutrino we are talking about is one which is associated with the μ . Each *flavour* of lepton (e , μ , τ) has its own kind of neutrino associated with it. So sometimes a pion can decay into a muon and a neutrino. We write this according to the following symbolic form.

$$\textit{Initial State} \rightarrow \textit{Final state}$$

of in this specific case:

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

Note that electric charge is conserved in this decay process. The superscript on the particle symbol gives the electric charge of the particle.

Assume that a π travelling at a speed v decays this way. If the ν_μ emerges at 90° to the original pion direction, show that the μ comes off at an angle of

$$\tan\vartheta = \left(1 - m_\pi^2 / m_\mu^2\right)$$

This problem is described in the LAB frame. m_π and m_μ are the rest masses of the pion and muon, while the neutrino is massless. *(Neutrinos are mot massless actually. They have tiny masses, as discovered at Sudbury (Ontario) Solar Neutrino Observatory.*

3) In class we discussed how some of the kinetic energy of a *beam* particle colliding with a *target* partcile can be transformed into the masses of new particles in the final state. Assume that a beam particle A of total energy E collides with a target particle B *(remember the LAB is defined as the frame where the target is at rest.)* New particles C_1, C_2 ... are produced in the final state. We write this according to the notation:

$$A + B \rightarrow C_1 + C_2 + \ldots + C_n$$

Show that the minimum energy E for A is

$$E = \frac{M^2 - m_A^2 - m_B^2}{2m_b} c^2 \text{, where } M \equiv m_1 + m_2 + ... + m_n$$

This minimum energy is know as the *Threshold Energy* for producing the final state $C_1 + C_2 + ... + C_n$.

4) We can imagine producing new particles by firing a beam of protons or pions into a target of liquid hydrogen. A liquid hydrogen target is just a target of stationary protons. Many new bound states of the various quarks were discovered in this way. Use the result of question **3)** to determine the minimum momentum for a the beam particles in the following experiments. Note that I miss out the "+" signs between the lists of particles in the initial and final states.

a) $pp \to pp\pi^0$
b) $pp \to pp\pi^+\pi^-$
c) $\pi^- p \to p\bar{p}n$
d) $\pi^- p \to K^0\Sigma^0$
e) $pp \to p\Sigma^+ K^0$

You need to know the masses of the various particles. Look them up in the text book, so that you learn how to use the tables. To help, I also give them here:

p is the symbol for the proton , mass $= 938\, MeV/c^2$.
n is the symbol for the neutron, mass $= 939.6\, MeV/c^2$.
π^0 is the symbol for the neutral pion, mass = $135\, MeV/c^2$
$\pi^\pm$ is the symbol for a charged pion., mass = $140\, MeV/c^2$
K^0 is the symbol for a neutral Kaon, mass $= 498\, MeV/c^2$. The Kaon is a meson, like the pion.
Σ is the sigma particle it is like a heavy proton, the mass is $1189\, MeV/c^2$ if it is charged, and $1193\, MeV/c^2$ if it is neutral.
$\bar{p}$ is the anti-particle of the proton, known as the anti-proton. Anti-particles have the same masses as particles, but opposite electric charges. So the $\bar{p}$ has the same mass as the proton, but has a negative electric charge.

5) The new particles produced in these experiments are often unstable, and rapidly decay. Consider a particle A at rest *(i.e. consider the particle in its rest-frame, or CM {centre-of-mass, or centre-of-momentum} frame)* decaying according to the scheme:

$$A \to BC$$

Show that the energy of B is:

$$E_B = \frac{m_A^2 + m_B^2 - m_C^2}{2m_a} c^2$$

and also show that the outgoing momentum are given by

$$|\vec{p}_b| = |\vec{p}_c| = \frac{\sqrt{\lambda\left(m_A^2, m_B^2, m_C^2\right)}}{2m_A} c$$

where

$$\lambda(x, y, z) \equiv x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$$

6) Use the result of problem **5)** to find the energy in the CM frame of each decay product in the following reactions.

a) $\pi^+ \to \mu^+ \nu_\mu$
b) $\pi^0 \to \gamma\gamma$
c) $K^+ \to \pi^0 \pi^+$
d) $\Lambda \to p\pi^-$
c) $\Omega^- \to \Lambda K^+$

The μ has a mass of $106\, MeV/c^2$
The photon γ is the quantum of light, and is massless.
ν_μ is massless

The Λ particle is in some way like a heavy neutral proton. It only appears with no electric charge, so the superscipt is omitted. The mass is $1116\, MeV/c^2$.

The Λ is distinguished from the proton by carrying one unit of a quantum number called *strangeness*. The Ω^- carries three units of this strangeness quantum number. It has a mass of $1672\, MeV/c^2$.

7) Here are some questions about Feynman diagrams. When you draw them, use the ones I did as examples. Remember to put in little arrows showing the direction of particles and antiparticles, also label each line. Also remember that electric charge is conserved at each vertex of a Feynman diagram. Make sure that you understand what is a virtual particle line, and what is a freely propagating particle line. Virtual lines do not have arrows on them .... why is that?

a) Sketch the lowest order Feynman diagram representing *Delbruck Scattering*, $\gamma\gamma \to \gamma\gamma$, which is the scattering of light by light. Could this process happen in classical electrodynamics (in a vacuum)? Why?

b) Draw the Feynman diagram for pair production,

$$\gamma \to e^+ e^-$$

and for the process of Bremsstrahlung, which is when an electron radiates a photon, in the field of a heavy nucleus.

$$e^+ \to \gamma e^+$$

Why do neither of these processes occur in free space, i.e. why do they only occur in the field of a nucleus? Justify your answer by using the equation

$$E^2 = p^2c^2 + m^2c^4.$$

c) Compton scattering is the scattering of photons off of electrons. Draw all the 4th order diagrams (ones with four vertices) for this process. There are 16 of them! These represent very small corrections to the leading order process. *Remember that all the lines have to be connected; except the ends of the incoming and outgoing particles.*

d) In the notes there is a Feynman diagram for the decay of a muon via the weak interaction. **(i)** Explain why there are two neutrinos involved. **(ii)** Why is it possible for this decay to occur, but not the "decay" $e^+ \to \gamma e^+$ **(iii)** A b-quark can decay to a c-quark via exactly the same kind of diagram. Draw this diagram. The fact that this diagram exists in bature tells you something about the relative mass of the b and c quarks. What is it?